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PRACTICAL PHYSICS

E. M. SOMEKH
M.Sc., A.Inst.P.
Head of Physics Department,
The Grammar School, Burton-upon-Trent

WITH A FOREWORD BY
F. C. BROWN
M.A., B.Sc., F.R.I.C.

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LONDON
FOREWORD TO THE SERIES

BY

F. C. BROWN

University of London Institute of Education;
formerly Senior Science Master, The Perse School, Cambridge

Science advances, these days, at a bewildering pace and teachers of science are forced to adopt fresh viewpoints and new techniques. The last few years have witnessed a fresh impetus to the teaching of science. The Association for Science Education has produced highly stimulating proposals for new syllabuses, the Nuffield Foundation is generously sponsoring an exhaustive survey of new teaching material and methods, and many Examination Boards are making considerable revision to their syllabuses in science.

Such re-appraisal is invigorating to all concerned, but the world of textbooks must also take account of such changes. This is a complete new series of texts, in physics and chemistry, which accepts the challenge of this revolution in teaching methods and is designed to meet—both now and for the next few years—the needs of schools and of the student about to proceed to University.

One of my tasks, as Editor of the series, has been to seek out gifted and enthusiastic teachers able to communicate the modern approach, new developments, the basic simplicity and symmetry underlying so much of their subject, and the elegance and vital importance of practical work integrated with theory. Whatever criticisms are made of these books, they will not be condemned as ‘the mixture as before’—each has been written afresh out of the author’s experience.

In each subject a related set of texts is to be published. Each set will comprise:

1. A pair of books designed to cover a complete course comfortably up to the standard of G.C.E. O level. Each book is richly leavened with elegant and practicable experiments for both student and teacher.

2. A theoretical book giving a thoroughly up-to-date treatment of the subject up to G.C.E. A level, or any comparable examination.

3. A theoretical book taking the subject a stage further than the A level book and, therefore, well suited to the requirements of the student who is proceeding to University or contemplating such examinations as the A.R.I.C.

4. A practical book of experiments matched to both the advanced books described above. This will contain both standard experiments and a wide selection of more original problems.

Although the texts are related to each other, each text is complete in itself. We hope that teachers and students, having found satisfaction in one text, will turn confidently to others in the series for an extension of their interests and activities.
MOTOR-GENERATOR SET

*(see Experiment 54)*

This plate shows a simple motor-generator set suitable for schools. Several series of experiments suitable for GCE O level and A level can be made with this apparatus. One such series is given in Experiment 54.

The set is trunnion mounted so that the cage of the motor carrying the field coils is freely pivoted, and the torque exerted when the motor is running can be measured by a spring balance shown at the top left hand.

The motor-generator is mounted on a rigid base four feet high, made of 2½ inch slotted angles, and is therefore easily visible. Perforated hardboard of suitable shape is bolted to the front of the base and carries all the ancillaries.
PREFACE

In this book an attempt has been made to set down an experimental course in physics, based on many years of teaching and examining in the subject. The selection of experiments is based on some of the recommendations contained in the report of the Science Masters' Association (1961) and on the more recent modifications in examination syllabuses.

Every experiment in the book has been tried out in the laboratory and repeated by the students themselves. While commercially obtainable instruments are prescribed wherever desirable, a number of less orthodox devices have been introduced to serve various purposes. Such devices are either readily available or are easily assembled in the laboratory or workshop, and it is believed that a valuable contribution to the training of the scientist arises from the devising of apparatus for specific purposes.

No apology is made for the omission of a lengthy introduction on experimental procedure—a grammar of rules and regulations which many people tend to disregard, forget, or even fail to apply correctly. Instead an introductory practical section is included consisting of 16 experiments based on Ordinary level work. This section serves as a link between Ordinary and Advanced level work and attempts to introduce or revise some fundamental ideas, techniques of measurement and evaluation of results.

Experiments 17–68 form the main section of the book, covering most of the requirements for Advanced level work. It is hoped that the Comments on each experiment will help to explain its significance and will frequently suggest further experiments or measurements. Some students may well be advised to leave part of those Comments alone, together with some of the more difficult experiments in this section.

The last section caters for the Scholarship student or the first year university undergraduate, covering some of the fundamental experiments which were previously thought beyond the scope of the school curriculum. It offers a wide selection for the would-be specialist, fewer details of procedure are included and it is hoped that this section will be read profitably even if some of the experiments cannot be performed.

E. M. S.
ACKNOWLEDGMENTS

I wish to express my thanks for the help I have received from the following manufacturers and organizations: Advance Components Ltd, Mullard Education Service, A.E.I., Griffin & George Ltd, Philip Harris Ltd, Unilab Division of Rainbow Radio (Blackburn), and Panax Equipment.

I also express my thanks to:

Dr Davis of Keele University, Professor Lipson of Manchester College of Science and Technology, Dr Owen of The Cotton, Silk and Man-Made Fibres Research Association, Mr David Brown of Pembroke College, Cambridge, Mr E. O. Bodger of Balliol College, Oxford, to Mr G. Hindle, my laboratory steward, for his patience and excellent workmanship and to Mr Llowarch for the idea for Experiment 74.

I should like to thank the Editor, Mr F. C. Brown, for his encouragement, comments and assistance at all stages. I am also indebted to Mr G. H. Nelson for helping me with the final proof reading.

My final thanks are to my own family whose help and encouragement were indispensable in writing this book.

I apologize for any omissions and will always be grateful for suggestions and criticism.

E. M. S.
INTRODUCTION

It is said that Joule, perhaps one of the greatest experimenters of all time, could send a direct record of his experiments without revision straight to the printers. Very few possess such clarity of mind, but one can acquire the habit of the orderly recording of experimental observations, so that laboratory records can still be intelligible long afterwards.

Some thought should always be given initially to the planning of the experiment, beginning by quickly reading through the whole experiment and then listing all the necessary measurements and tabulations required. Nothing is more irritating than to find, on reviewing some experimental records, that a vital measurement has been overlooked! One cannot always run to the laboratory and hunt in the wastepaper basket for the wire, having somehow forgotten to measure the diameter before discarding it. With electrical experiments it is essential to draw a circuit diagram which should be followed when the circuit is being assembled. To work without a circuit diagram is not only bad experimentation, but can be very costly, for valuable equipment may be damaged as a result.

Lastly, one cannot exaggerate the value of a ‘trial run’ before the start of an actual experiment. Very often certain initial adjustments have to be made which remain unaltered during the experiment and one has to be sure that such adjustments will give the full range of observations. To take a little extra care in the preparation of the experiment can save endless time at a later stage.

Now for some simple aids for drawing good graphs. Necessary tools are a sharpened 2H pencil, transparent ruler, flexicurve and a thick capillary rod or a glass slide for drawing the normals and hence the tangents to curves (see diagram). A slide rule or simple calculator is useful, though it
INTRODUCTION

is hoped that the student is able to evaluate an expression by simple approximations (or contracted multiplication and division) to one significant figure and the appropriate decimal place. Sufficient attention must be given to arithmetical accuracy, for, however impeccable the experimentation, it is no consolation when one discovers that the final result is incorrect due to an arithmetical error.

NOTATION

(1) B.S.I. recommendations for symbols, signs and abbreviations have been followed as closely as possible and the negative index has been used throughout. In distinguishing between kilogram as a unit of mass and the weight of a kilogram as a unit of force, the abbreviations 'kg' and 'kg wt' have been adopted. It is felt that the B.S.I. recommendations for the use of 'kg' to denote kilograms weight and 'kgf' to denote kilograms force have not yet gained general acceptance to justify their use in this book.

(2) Logarithms to base 10 are shown as 'log'; natural logarithms as 'ln'.

(3) Both C.G.S. and M.K.S. systems have been used throughout the book.

(4) The unit of heat is expressed in joules throughout and therefore no determination of the mechanical equivalent of heat has been included.
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Illustration showing a Motor-Generator Set faces page 7.
INTRODUCTORY EXPERIMENTS
EXPERIMENT 1

To determine the refractive index of glass

APPARATUS

A rectangular glass block (approximate dimensions 6·5 cm × 11·5 cm), a reel of fine copper wire (40 S.W.G.), a reel of Sellotape, a plain postcard glued to a square piece of wood so that the card is held in the vertical plane with its long edges horizontal, a pair of dividers, sharp 2H pencil, transparent ruler.

METHOD

Wind the wire tightly round the block so that at least six turns, equally spaced, lie across four faces. Hold these turns in place with strips of Sellotape, making sure that the wires covering the vertical faces are parallel to the vertical edges of those faces. Snip away, with scissors, all spare wire, leaving the parallel wires on face DHGC and one only, XO, on the opposing face AEFB. The final appearance is clearly shown in Fig. 1.

Mark the outline of the rectangular block on a flat sheet of paper. Set the postcard vertical and touching the side EH. Looking into face DHGC of the block, slide the postcard along until the more distant vertical edge of the card Z appears to be in line with the image of O and in turn with each of the wires P₁, P₂, . . . P₆ (Fig. 2). Mark the points Z₁, Z₂, . . . Z₆, and so on, as accurately as possible. (Each observation should be checked and should agree to within 1 mm.)
**Construction**

Remove the glass block after marking carefully points $O, P_1, P_2, \ldots P_6$. Draw $OO_1$ perpendicular to $HG$ and join the points $Z_1P_1, Z_2P_2, \ldots Z_6P_6$, etc., cutting the line $OO_1$ at $I_1, I_2, \ldots I_6$. Measure, using a pair of dividers and the transparent ruler, the distances $OP_1, I_1P_1 : OP_2, I_2P_2$, etc., tabulating your results as follows:

**Specimen results**

<table>
<thead>
<tr>
<th>OP mm</th>
<th>IP mm</th>
<th>OP/IP</th>
</tr>
</thead>
<tbody>
<tr>
<td>125.5</td>
<td>84.1</td>
<td>1.49</td>
</tr>
<tr>
<td>123.5</td>
<td>82.5</td>
<td>1.49 (5)</td>
</tr>
<tr>
<td>120.5</td>
<td>81.5</td>
<td>1.48</td>
</tr>
<tr>
<td>118.2</td>
<td>79.8</td>
<td>1.48</td>
</tr>
<tr>
<td>116.3</td>
<td>78.7</td>
<td>1.48</td>
</tr>
<tr>
<td>116.0</td>
<td>78.0</td>
<td>1.48 (5)</td>
</tr>
<tr>
<td>115.3</td>
<td>77.0</td>
<td>1.50</td>
</tr>
</tbody>
</table>

**Theory**

Consider the ray of light along $OP_6$ (Fig. 3); this is refracted along $P_6Q$ which is the continuation of $Z_6P_6$. The refractive index $\mu = \frac{\sin \phi}{\sin \theta}$. But $\theta = O_1OP_6$ and $\phi = O_1I_6P_6$ (corresponding angles).

The refractive index $\mu = \frac{\sin O_1I_6P_6}{\sin O_1OP_6} = \frac{O_1P_6}{I_6P_6} \div \frac{O_1P_6}{OP_6} = \frac{OP_6}{I_6P_6}$.

**Deduction of $\mu$.** It may appear simple to calculate $\mu$ from the table by working out the ratio $OP/IP$ in each case, as it is done in the third column, but closer examination shows that the ratio is not constant. This is due to accidental errors which are likely to occur at random and are generally due to the observer. The effect of such accidental errors can be reduced by repeating observations. However, some sort of averaging is required in evaluating the most probable value of any physical quantity or algebraic relationship (for example, a physical law which, in this case, is a simple ratio $OP/IP$). Statistical methods of finding averages are accurate but laborious. A graphical method, such as used here, is less accurate though simpler to operate.

It is desirable to plot such a graph during the course of the experiment. This enables one to reject an accidental observation which is obviously a ‘mistake’ (for example, due to writing a figure or reading a scale wrongly) and to check the observation before the apparatus is dismantled.

In this experiment plot $OP$ as ordinate and $IP$ as abscissa; the points should lie on the line $OP = \mu \times IP$, from which the gradient can be deduced.

In choosing units for the axes, you should make full use of the graph paper (though not to the extent of choosing awkward scales such as representing 7 units by 10 squares). Finally, where the relationship is of the form $y = mx$, as in the above example, the origin must be included in the graph even if it means grouping the plotted points close together. The
origin represents the one certain point, which can act as the pivot for the line of best fit (for example, a line drawn evenly between the points). This is best done with the help of a good transparent ruler.

Fig. 4

In Fig. 4, the origin has not been included, the deviations have been unjustifiably exaggerated so that the uncertainty in drawing the line of best fit has been increased; while in Fig. 5 by including the origin the task is made easier. The gradient is found by choosing two points on the graph as far apart as possible. In this case the origin can be one of the points.

Fig. 5
INTRODUCTORY EXPERIMENTS

COMMENTS

Looking at Fig. 3, you will note that the refracted rays, when produced backwards, intersect and envelop a curve, called the caustic curve, whose 'cusp' lies near the point $I_1$, which is the virtual image of the object point O when looked at in the direction O, O. This point is difficult to determine graphically because you require to trace an exceedingly narrow pencil of light from O. However, by using a travelling microscope with a suitable objective, you can determine the 'apparent depth' of the block.

Can you then check the refractive index of the glass?

EXPERIMENT 2

To discover whether (a) enclosed dry air, (b) enclosed ether vapour, obey Boyle's Law at room temperature

APPARATUS

Two sets, as shown in Fig. 1, one containing air, the other ether vapour. The pressure exerted on the enclosed gas can be varied by tightening or unscrewing the G-clamp and the maximum pressure should be about twice the minimum pressure.

METHOD

First read the barometric pressure $\pi$ to the nearest millimetre. Starting with air, at maximum pressure and levels steady, read off the scale the levels of A, B and C (to nearest mm). Reduce the pressure, wait till levels are steady again and repeat the observation. This is repeated as many times as possible until the lowest pressure is reached. Tabulate your results as follows:

ALL MEASUREMENTS IN CM

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<th>3</th>
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<th>7</th>
<th>8</th>
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<tr>
<td>Level A</td>
<td>Level B</td>
<td>Level C</td>
<td>$l = AB$</td>
<td>$h = BC$</td>
<td>$hl$</td>
<td>$p = (\pi + h)$</td>
<td>$pl$</td>
</tr>
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Note that $h$ is negative when level $B$ is higher than $C$, which means that the pressure on the enclosed gas is now less than atmospheric.

Repeat the experiment with ether vapour but choose a place where the temperature, which should be noted, is least likely to change (ideally the ether tube should be water-jacketed). The same tabulation is employed, but omitting column 6. Note the appearance of the mercury meniscus at $B$, when each observation is taken, to see whether a film of liquid ether is visible on it, or not.

**Theoretical and Calculation**

You may assume that the volume of the gas is proportional to $l$, so that to verify Boyle’s Law you have to show that $p$ (equals $\pi + h$) times $l$ is a constant, provided the temperature remains constant. It would be instructive to present the data for air graphically, say by plotting $p$ against $l$. This gives a smooth curve when drawn evenly between the points, in fact a rectangular hyperbola. However, it is not easy to be sure at a glance that it is a rectangular hyperbola, since one smooth curve is so much like another. The physicist often prefers to plot his data so that they can be represented by a straight line.

Now if Boyle’s Law is obeyed $l(\pi + h) = \text{constant} (K)$, whence:

\[
\frac{1}{l} = \frac{1}{K}(\pi + h)
\]

Writing $1/l$ as $x$, $h$ as $y$, one gets $y = \frac{1}{K}(x + \pi)$. This is of the form $y = mx + c$, which is a straight line equation. When $y = 0$, $x = -\pi$, so that one is not only verifying Boyle’s Law by the linearity of the graph, but also checking the accuracy of the observations by deducing the atmospheric pressure.

Intercepts on straight line graphs are unreliable—often more unreliable than the gradient. It may not always be convenient to include the origin, which makes the results even more unsatisfactory. Rewrite the equation as $lh = K - lx$. If you plot $lh$ as $y$, $l$ as $x$, you get a straight line with a negative slope, but now you have to decide how to draw the line of best fit, since there is no origin to use as a pivot.
INTRODUCTORY EXPERIMENTS

It is shown in the Appendix that the most probable line must pass through the point $(\bar{y}, \bar{x})$ where $\bar{y}$ and $\bar{x}$ are the arithmetical means of the ordinates and abscissae respectively

\[
\text{(in this case: } \frac{\text{total of column 6}}{\text{number of observations}}, \frac{\text{total of column 4}}{\text{number of observations}})\text{.}
\]

Plot this point as well as the other observations and use it as a pivot (similar to Experiment 1, where the origin was the pivot) to draw the line of best fit. The gradient is again found by choosing two suitable points, on the line, but far apart. The value of this gradient is $\pi$, the atmospheric pressure.

Conclude your account of the experiment by referring to the table of results for ether. In the last column is $pl$ a constant?

COMMENTS

An ideal gas is the one which obeys Boyle’s Law at all temperatures, but the above experiment is important even if carried out at one temperature only. It can be extended to other gases like carbon dioxide and sulphur dioxide, if they can be trapped in a long capillary tube (internal diameter 2 mm) by a long index of mercury (25 cm long). The pressure is varied by tilting the tube or inverting it so as to get the same range of pressures as in the above experiment.

Ether vapour is not likely to obey Boyle’s Law and by plotting $p$ against $l$ you get what is called an isothermal. From the graph estimate the saturated vapour pressure of ether at the temperature of the experiment and check it against the values given in a reference book. Check, also, from your results, the onset of saturation and compare it with your own observations.

EXPERIMENT 3

To study the factors affecting the sensitivity of a beam balance

APPARATUS

A 50 cm ruler, a stout copper wire pointer 15 cm long pointed at its lower end, the top end flattened out, bent over and Sellotaped to the middle of the ruler (Fig. 1), a brass connecting terminal to slide up and down the pointer, a triangular prism with very fine emery paper Sellotaped over the upper edge, a 9 in stout block of wood or brick, small weights, stopwatch.

METHOD

Set up the apparatus as shown in Fig. 1, the purpose of the emery paper being to provide sufficient friction to prevent the ruler sliding off when tilted.
Without the connecting terminal the ruler is in unstable equilibrium and the slightest force will tip it over to one side. The lowering of the centre of gravity of the ruler is achieved by clamping the brass terminal on to the pointer.

Find the deflection for different (and measured) positions of a small weight $\delta w$, say one gram, placed on the ruler on both sides of the pivot. Tabulate your results as follows:

<table>
<thead>
<tr>
<th>TABLE 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance of weight from pivot $l$ cm</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

Keeping the same distance $l$, find the deflection $x$ for different values of $\delta w$, tabulating your results as in the above table.

Now keep both the distance $l$ and the weight $\delta w$ the same and vary $d$ (Fig. 1), determining, for each position of $d$, both the deflection $x$ and the time for 10 swings of the pointer, tabulating your results as follows:
TABLE 2

<table>
<thead>
<tr>
<th>d cm</th>
<th>Right-hand side ( x_1 ) mm</th>
<th>Left-hand side ( x_2 ) mm</th>
<th>Average ( x )</th>
<th>Time (sec) for 10 swings</th>
<th>( \frac{1}{x} )</th>
</tr>
</thead>
</table>

**Theory and Calculation**

Consider the diagram shown in Fig. 2, representing the forces acting on a beam balance, \( \hat{w} \) being the actual weight of the body plus the weight of the pan, \( w \) being an extra weight on the other pan causing the beam to tilt by an angle \( \theta \). For equilibrium

\[
(W + w) \cos \theta = W_0 h \sin \theta + \hat{W} l \cos \theta
\]

where \( W_0 \) is the weight of the beam and \( h \) is the distance of its centre of gravity from the point of suspension, so that

\[
w l \cos \theta = W_0 h \sin \theta
\]

and

\[
w = \frac{W_0 h \tan \theta}{l}
\]

Fig. 2

\[
\Rightarrow \quad \frac{W_0 h \theta}{l} \approx \frac{W_0 h x}{la} \quad \ldots \quad \ldots \quad (1)
\]

The sensitivity of the balance is expressed in milligram/division or

\[
\frac{w}{x} \approx \frac{W_0 h}{la} \quad \ldots \quad \ldots \quad (2)
\]

so that the smaller \( w/x \), the more accurate will the weighing be.

The effect, on the sensitivity, of \( l \) is shown in Table 1 and, as one would expect, the longer the arm of the balance the greater is the sensitivity. From your second set of results you should learn whether our approximations \( \tan \theta \approx \theta \approx x/a \) are justified. In the third set of results shown in Table 2, two main points stand out: firstly, by increasing \( d \) you are lowering the centre of gravity or increasing \( h \); secondly, by decreasing \( d \) you are increasing the period of swing—is this a good thing?

If \( M \) is the mass of the rule and pointer, \( m \) the weight of the connector, and \( h_0 \) the very small distance of the centre of gravity of the rule from the pivot (equals less than half the thickness of the rule), then

\[
W_0 = M + m, \quad w = \delta w
\]

and we have \((M + m)h = md - Mh_0 \) or \( h = \frac{md - Mh_0}{M + m} \).
Equation (1) becomes \[ \delta w = \frac{(md - Mh_f)x}{la} \]

or \[ \frac{1}{x} = \frac{1}{\delta w la}(md - Mh_f) \]

Plot \(1/x\) against \(d\) and you should get a straight line, establishing the relationship between the sensitivity and \(h\).

**Comments**

Examining the expression for the sensitivity, equation (2), one notes that the beam of a balance must be light (small \(M + m\)), long, yet rigid. Table 2 shows that, as \(h\) decreases, the time of swing increases, making the weighing operation slow and tedious. Yet \(h\) should be small. In recent designs the motion of the balance is damped without affecting the sensitivity.

With an ordinary balance, there is no certainty that arm lengths \(l\) are equal, so that double-weighings are recommended with the weights and the object to be weighed interchanged to give two values, \(w_1\) and \(w_2\). The true value \(w = \sqrt{w_1w_2}\) (called the geometric mean of \(w_1\) and \(w_2\)). Discuss the reason for this method of averaging the two results.

---

**EXPERIMENT 4**

**To calibrate a simple accelerometer**

**Apparatus**

A vertical, trapezium-shaped board with 12 inclined numbered shelves, covered in front by a thin sheet of cellulose acetate. The inclination of each shelf increases in steps of 1° so that the bottom shelf is inclined by 13° to the horizontal. Each shelf holds a small ball-bearing which can roll up the incline when the apparatus is accelerated to the left (see Fig. 1), dropping into a well when it reaches the top. The length of each shelf can be adjusted by means of a small stop. The balls are 'reset', by means of a small magnet, before each operation.

The horizontal base of the accelerometer, which carries a small spirit level, is hinged to another smooth horizontal board, the latter being fixed by two screws to the top of a trolley. The base of the accelerometer can be tilted by means of another screw, thus extending its range. Slotted weights up to 750 g and a holder, pulley and cotton thread, ruler.

**Introduction**

As the apparatus is accelerated to the left, the ball-bearing starts to roll up the incline when the component of the acceleration up the inclined plane exceeds the downward component of the acceleration due to gravity.
by a small acceleration $f_0$, so that the distance $s$ it rolls up depends on the
time $t$ the acceleration lasts:

$$s = \frac{1}{2}f_0t^2$$ \hspace{1cm} (1)

It is convenient to keep the distance $d$ moved by the accelerometer the
same, which means that the trolley is always started from the same point,
and if a chair or stool is placed beneath the accelerating slotted weights,
the acceleration stops when the weight reaches the chair.

If $f$ is the actual acceleration of the trolley, then the accelerating time $t$
is related to $f$ and $d$ as follows:

$$d = \frac{1}{2}ft^2$$ \hspace{1cm} (2)

Dividing equation (1) by (2) we get $f_0/f = s/d$ or $s = f_0d/f$ so that the
lengths of the inclined shelves are not equal but get shorter as the accelera-
tion $f$ increases.

**Method**

Set the base of the accelerometer horizontal by adjusting the screw and
make sure that the top of the bench is level. Adjust the length of the string
so that the accelerator 'run' is a convenient distance (remember you have
to stop the trolley by hand when the string goes slack but a rubber stopper
should be placed in front of the pulley just in case).

Now add weights to the holder gradually until the first ball goes over,
repeat, as a check, and record your observations as shown in table overleaf.
This is continued until the 12th ball is over.

Now tilt the base of the accelerometer by 2° (this requires a little trigo-
nometry and the use of a ruler) and repeat the experiment. If our reasoning
is correct the ball number 3 will require an accelerating force to go over
corresponding to that of number 5 in the above table. Investigate further
by increasing the angle more. Another way of checking on the behaviour

<table>
<thead>
<tr>
<th>Shelf no.</th>
<th>Accelerating force g wt</th>
<th>Check</th>
<th>Average</th>
<th>Acceleration $f$ cm sec$^{-2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

of the accelerometer is by removing some weights from the trolley, record-
ing the mass of the trolley and the accelerometer each time in addition to
the table shown above.

**THEORY AND CALCULATION**

Ignoring friction, if $mg$ dynes is the accelerating force and if $Mg$ is
the mass of the trolley with the accelerometer, then the acceleration
$f = mg/M$; complete the above table.

The effect of tilting the base of the accelerometer should be to displace
approximately all readings by one place for each degree increase in tilt.

**COMMENTS**

It is interesting to investigate the relation between acceleration $f$ and
the length of the shelf, since the cellulose acetate front can easily be
removed and the stop for each shelf moved, enabling the length to be
varied.

It is possible to adapt the above accelerometer to measure the efficiency
of car brakes by the deceleration they produce. From your investigation
above you can extend the range of the accelerometer for the larger
accelerating distances which would be required in the case of a car. Under
ideal conditions a car with good brakes should decelerate by as much as
900 cm sec$^{-2}$ (or 0.9 g).

**EXPERIMENT 5**

**To determine the acceleration of a body falling freely under the
action of gravity**

**APPARATUS**

An A.C. timer with paper tape, 8 V A.C. supply, rheostat, tapping key,
200 g weight, clamp and stand, G-clamp and Sellotape.
INTRODUCTORY EXPERIMENTS

Method

 Clamp the timer vertically and connect it up to the low tension A.C. supply as shown in the diagram. Adjust the rheostat to give a suitable amplitude of vibration. Cut a length of paper tape of about 80 cm, loop one end of the tape round the weight and Sellotape the loop thus formed. Thread the tape through the two wire guides and pull through about 50 cm of the tape, holding it against the upper edge of the base of the timer as shown in the diagram. Start the vibrator by pressing the key and at the same time let the weight fall freely by lifting off your finger. A series of dots should be traced on the tape as shown—otherwise either the amplitude of vibration is too small or a hole has been made in the carbon paper, both of which can easily be put right. The experiment should be repeated after turning over the tape.

Theory and Calculation

You have learnt from your elementary work that the distance $s$ a body travels from rest when accelerated uniformly in a straight line is $\frac{1}{2}at^2$,
where $a$ is the acceleration and $t$ is the time in seconds. It is easy to apply this formula to the trace on the tape, as the time between two consecutive dots is equal to the period of vibration of the vibrator and that, if adjusted correctly, is equal to $1/50$th of a second (as the alternating supply is 50 cycles per second).

---

**Fig. 3**

TRACE ON TAPE

Number each dot and measure the distance $s$ between the first dot O and each of the succeeding 17 dots—18 altogether. It is possible to miss a dot, especially where they are close together at the beginning of the trace, but it is easy to check from your results whether you have done so or not. Tabulate your results as follows:

<table>
<thead>
<tr>
<th>Number of vibrations $n$</th>
<th>Total distance $s$ cm</th>
<th>$\sqrt{s}$ cm$^{1/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

To represent these results by a linear graph we cannot plot the distance $s$ against $t^2$ as we are uncertain, by a fraction of a vibration, of the actual starting time, and also with $t^2$ we are likely to get a gentle curve. On the other hand, taking the square roots of both sides, thus $\sqrt{s} = \sqrt{\frac{1}{2}} a \cdot t$ and plotting $\sqrt{s}$ against $t$ we get a straight line which may not pass through the origin and the gradient of which is $\sqrt{\frac{1}{2}} a$; from this the acceleration can be calculated.

Plot $\sqrt{s}$ as ordinate and the number of vibrations $n$ as abscissa and you should get a good straight line whose equation is $\sqrt{s} = \sqrt{\frac{1}{2}} a \cdot (n/50) + \text{constant}$, and the gradient of which is $\sqrt{\frac{1}{2}} a/50$, from which $a$ can be deduced.

**Comments**

Although the facilities of the school laboratory are inadequate for an accurate determination of $g$ by the free fall method, in actual fact it is this method which is capable of the highest degree of accuracy. An accurate value of $g$ is necessary for the absolute determination of pressure, forces, etc.

The above method is not seriously affected by friction between the vibrator and the paper tape provided the weight used is not too small. The accuracy of the method depends on how close the frequency of the
supply is to 50, as it is known to vary, especially at peak hours. An accuracy of 2% has been found feasible.

A more accurate method is to use a higher frequency, say 1000 cycles per second, and to use a high speed counter, for example a dekatron counter, for timing the fall (see Experiment 99).

**EXPERIMENT 6**

To investigate by simple experiments the basic principles involved in the determination of \( g \) by the simple pendulum

**Introductory experiments**

**Apparatus**

Two parallel half-inch round steel rods 6 ft long, separated from each other by a \( \frac{1}{4} \) in gap and fastened to a long hardwood beam. A similar pair of parallel rods, about 3 ft long, curved, with a radius of curvature of about 20 ft. Two 1 in steel ball-bearings. Clinometer (if available), stopwatch, metre rules.

**Method**

Tilt the straight pair of parallel rods so that they make an angle of about 1° with the horizontal. This can be measured with a clinometer or calculated from the values of \( h \) and \( s \) (see Fig. 1). Find the time taken by

![Fig. 1](image)

one of the ball-bearings to roll down this 'track' for a distance of at least 5 ft. (The path of the centre of the ball-bearing is shown diagrammatically in Fig. 1.) As the time it takes is less than 10 seconds several checks should be carried out and the average taken. Increase the tilt by about 1° at a time, and repeat the experiment five times to give six observations. Record the results as shown in table overleaf. This experiment was first carried out by Galileo—without a stopwatch!

You have learnt from your elementary work that an object rolling down an inclined plane accelerates uniformly so that if \( s \) is the distance moved, \( t \) the time taken and \( a \) the acceleration, \( s \) equals \( \frac{1}{2}at^2 \). Thus, to find what
happens if the inclination is varied, plot a graph with the acceleration, \(2s/t^2\), as ordinate and the angle of inclination to the horizontal, \(\theta\) 

<table>
<thead>
<tr>
<th>Distance rolled (s) cm</th>
<th>(h) cm</th>
<th>(h/s = \sin \theta)</th>
<th>Time (t_1) sec</th>
<th>Time (t_2) sec</th>
<th>Time (t_3) sec</th>
<th>Average (t) sec</th>
<th>(t^2) sec(^2)</th>
</tr>
</thead>
</table>

\((\sin \theta \approx \theta \text{ when } \theta \text{ is small})\), as abscissa. You should get a straight line passing through the origin proving, at least for small angles, that the acceleration is \(k\theta\), where \(\theta\) is in radian measure and \(k\) is a constant related to \(g\).

![Diagram of Centre of Curvature and Equilibrium Position](image)

Now consider the curved pair of rails, Fig. 2. The inclination varies from zero (horizontal) at the mid-point to a maximum at the ends of the rails; furthermore the angle of inclination varies directly with the distance along the curve \(s\) from the mid-point. Therefore as \(\theta\) is small the acceleration experienced by a rolling sphere at every point is directly proportional
to its distance, $s$, from the mid-point. (The path of the centre of the sphere is shown diagrammatically in Fig. 2.) We want to reason out what happens to a sphere, when rolled from a point, distance $s_0$ from the centre. The force of gravity down the curve causes it to accelerate and thereby gain kinetic energy. Now consider Fig. 3. The abscissa is the displacement, $s$. The ordinate is the force experienced by the rolling sphere at any given point which is equal to mass $\times$ acceleration = mass $\times$ $k \theta$, but it is clear from Fig. 2 that $\theta = s/l$ where $l$ is the radius of curvature of the curved rails. Thus the force at any given point, distance $s$ from the centre, is $mks/l$. Hence a straight line graph is obtained.

![Diagram](image)

**Fig. 3**

The area of the triangle OAB is the work done by gravity on the sphere as it passes from a point $s_0$ away to the centre (work done = average force $\times$ distance) and is equal, by the law of conservation of energy, to the maximum kinetic energy gained by the sphere ($\frac{1}{2}mv_{\text{max}}^2$). Therefore

$$\frac{1}{2} \times mks_{\text{max}} \times s_0 = \frac{1}{2}mv_{\text{max}}^2$$

therefore

$$ks_0^2/l = v_{\text{max}}^2$$

therefore $v_{\text{max}} \propto s_0$.

If you double the distance $s_0$ (provided that $\theta$ is still kept small) then the velocity of the rolling sphere when it reaches the centre is doubled. If two spheres are released simultaneously from two points distance $s_0$ and $2s_0$ respectively from the centre then it can be shown (see ‘comments’ later) that the speeds of the two spheres at any subsequent time are always in the ratio of $1:2$. It follows therefore that the two spheres should reach their maximum velocities at the same time (i.e. as they pass the mid-point). Thus unlike the one on the straight rails, a sphere on the curved rails takes the same time to roll to its equilibrium position irrespective of its starting point, provided $\theta$ is small.

Confirm this by experiment; you will note that after the sphere reaches its equilibrium position it continues to roll until it uses up all its kinetic energy and then reverses at a point roughly as far from the centre as its
starting point, provided no energy is lost. This to and fro motion is an approximate example of simple harmonic motion (see Experiment 19).

If a heavy bob is hung on a long string suspended from the ceiling and made to swing, its subsequent motion is very similar to that of the rolling sphere. Here again, the time it takes to do a complete oscillation is independent of the initial displacement, provided the angle is small. Such motion is termed isochronous and was discovered by Galileo at 21 years of age. The motion of the pendulum, like the rolling sphere, is the result of the action of the force of gravity and, because it is isochronous, it is used as a timing device in clocks.

**Pendulum experiment**

**Apparatus**

A bob, a length of cotton thread, a retort stand, clamp and boss, two pennies, G-clamp, stopwatch.

**Method**

Clamp the base of the stand to a rigid bench and grip the cotton thread between two pennies in the clamp, as shown in Fig. 4. Tie the bob on to the end of the thread so that the length $l$ hanging from the clamp is about 180 cm. This length can best be measured using two metre rules, from the point of suspension A to a suitable point on the bob B (not necessarily the centre of gravity), provided B is the same point each time.
INTRODUCTORY EXPERIMENTS

Now set the pendulum in motion, keeping the angular amplitude less than 5°. Time a suitable number of oscillations, the timing being started when the bob is moving at its fastest (i.e. when it passes its equilibrium position). It is convenient to use the vertical stand as a reference line, as shown in Fig. 4. One complete oscillation about the equilibrium position is shown in the inset in Fig. 4. When timing a number of oscillations, the total time should never be less than 10 seconds as the error in starting and stopping the watch, together with the observer’s personal error, would seriously affect the estimation of the period. It is also always necessary to repeat a timing observation, not only as a check against miscounting oscillations, but also to reduce the random effects due to draughts, etc. If the two timings differ by more than 0.2 second, a third timing should be done, unless the timings are inordinately large, and the average of the three should be taken.

Now reduce the length of the pendulum $l$ and repeat the experiment. Continue until six sets of observations are obtained. Tabulate the results as follows:

<table>
<thead>
<tr>
<th>$l$ cm</th>
<th>$t_1$ sec for $n$ oscillations</th>
<th>$t_2$ sec for $n$ oscillations</th>
<th>$t_3$ sec for $n$ oscillations (if necessary)</th>
<th>average $t$ sec</th>
<th>average period $T = t/n$ sec</th>
<th>$T^2$ sec$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Theory and Calculation**

It is shown in your theory books that the period of oscillation $T$ of a simple pendulum is related to the length of the pendulum $l$ by the formula $T = 2\pi\sqrt{\frac{l}{g}}$. To verify this formula experimentally it is best to plot a linear graph. Therefore plotting $T$ against $l$ will not help, but on squaring both sides and plotting $T^2$ as ordinate and $l$ as abscissa, as $T^2 = (\frac{4\pi^2}{g})l$, we get a straight line, which only passes through the origin if $l$ is measured from the centre of gravity of the bob (assuming that the diameter of the bob is small compared to $l$). The gradient of the graph is $4\pi^2/g$, from which $g$ can be deduced (units cm sec$^{-2}$).

**Comments**

This method of determining $g$ is more accurate than other methods in elementary physics such as the ‘free fall’ method, but here again the accuracy is limited by that of the stopwatch where an error of $\frac{3}{4}$% would lead to an error of 1% in $g$ ($g = \frac{4\pi^2l}{T^2}$).

Referring to Fig. 3, the area ACDB is equal to the gain in kinetic energy of the rolling sphere. The area of ACDB is the difference between
the areas of the triangles OAB and OCD, therefore
\[\frac{1}{2}mv^2 = \frac{1}{2}mk\dot{s}^2/l = \frac{1}{2}ms^2/l\]

hence \(v^2 = (k/l)(s_0^2 - s^2)\). This is an expression for the velocity at any subsequent moment after release of the ball and it represents the equation

of a circle radius \(s_0\) (see Fig. 5). The subsequent motion of the ball-bearing can be represented as the projection of a particle along the \(s\) axis, the particle moving at a constant speed of \(v_{\text{max}} = \sqrt{(k/l)s_0}\). A complete oscillation is represented by a complete revolution of the particle along the imaginary circle. An increased amplitude does not affect the period as it only means a larger circle and hence a faster speed, as the two are related by the formula already shown. The period of the oscillation is

\[\frac{\text{circumference}}{\text{speed}} = 2\pi s_0/v_{\text{max}} = 2\pi s_0/\sqrt{(k/l)s_0} = 2\pi \sqrt{l/k}.
\]

For the simple pendulum, \(k = g\).

Calculate the period you would expect from a pendulum with length equal to the radius of the curved rails. How does this agree with the period of the rolling sphere? How do you explain the difference? Try the experiment with rolling spheres of different radii; find their periods; how do you explain the results?
EXPERIMENT 7

To verify the relation between object and image distances for a converging lens and to deduce the focal length by the method of no-parallax

APPARATUS

Transparent ruler, two pins and holders, converging lens and holder, enlarging lens suitably mounted (focal length 10 cm), large pin and stand, plane mirror, viewing mirror, 'distance piece', rulers, two spring clips, rubber band, a piece of plasticine.

INTRODUCTION

Mount the transparent ruler on the bench, using the plasticine as shown in Fig. 1, and put one of the pins at O about 10 cm from the ruler. Mark an arrow on the ruler at about the 15 cm mark, as shown, so that

![Diagram]

Fig. 1

on looking in direction AO, the arrow and the pin appear in line. As you move your head to the left, say in direction BO, the far-away pin appears to be displaced relative to the arrow and appears to lead in the direction of the motion of the head. Confirm this by moving your head to the right in direction CO. This apparent displacement is called parallax and appears to be bigger the greater the distance between the two objects viewed and the bigger the angular displacement of the observer. We shall
make use of the parallax to locate a real image formed by a lens and to determine the precise position of the image when the parallax between the locating pin and the image pin is absent.

**Method**

Find roughly the focal length of the lens by using it to obtain the image of a distant object (say a window) on a piece of paper and measuring the distance between the paper and the lens. This can be checked more accurately as follows. Place the lens on the plane mirror and, using the large pin, which can slide up and down a standard retort stand above it as shown in Fig. 2, adjust the pin height to be roughly equal to the focal length of the lens. Look down at the pin and its reflection from a height of about 30 cm and move your head to the side as you did in the introductory experiment. If there is any parallax, decide whether the pin or its reflection is the one nearer to you. Should the pin appear to be nearer, lower it slightly and check for parallax again until finally it disappears. (In practice, because of what is termed spherical aberration, all pencils of light from the object are not refracted to the same point, so that one cannot hope for a complete absence of parallax. If the diameter of the lens is small compared with its focal length, say 1 to 5, much less spherical aberration occurs.) Measure the distance \( a \) between the principal focus, \( F \), and the top of the lens and the distance \( b \) between \( F \) and the plane mirror. The average of these distances is the focal length. Check by turning over the lens and repeating. An agreement of between 1 and 2 mm should be possible.

For the main experiment use the arrangement shown in Fig. 3, bedding the lens in its holder with a little plasticine. Adjust the height of the 'distance piece' to coincide with the centre of the lens, then similarly adjust the heights of the object and locating pins. Place the object pin so that its distance from the lens is greater than \( f \) and look down at the viewing mirror for the image of the locating pin and that of the object pin, tilting
the lens slightly about either axis if necessary until the two images appear to touch, tip to tip. Do not disturb the lens once adjusted. Now move the locating pin (and the viewing mirror) until there is no parallax between it and the image of the object pin. Measure the object and image distances.

**Fig. 3**

**Fig. 4**
as shown in Fig. 4. If \( 2a \) = thickness of lens, \( d \) = length of ‘distance piece’, then \( x_1 - x_2 = 2a + d \) and hence \( a = (x_1 - x_2 - d)/2 \).

Therefore

\[
\begin{align*}
\text{OL (object distance } u \text{)} &= x_4 - x_1 + a \\
\text{OI (image distance } v \text{)} &= x_2 - x_3 + a.
\end{align*}
\]

Repeat five times, varying the values of \( u \) from \( 2f \) to \( 4f/3 \), and tabulate as follows:

<table>
<thead>
<tr>
<th>All in cm</th>
<th>All in cm⁻¹</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ( x_1 )</td>
<td>2 ( x_2 )</td>
</tr>
</tbody>
</table>

**Theory and Calculation**

Plot \( 1/u \) against \( 1/v \) (remember that \( u \) and \( v \) are interchangeable and called ‘conjugate’, so that you should have altogether 12 points on your graph). You should get a straight line with a negative slope with equal intercepts on the axes, equal to \( 1/f \). Check also that, in column 9, \( (1/u) + (1/v) \) is constant and equal to \( 1/f \). Deduce an average value for \( f \) (correct to the nearest mm).

It frequently happens that you need to find the focal length of a converging lens system consisting of several lenses, for example an enlarging lens. The above method cannot be used because you do not know from where to measure the object or image distance.

A method we owe to Newton is to measure the distance of the object and image from the corresponding principal focus (see Fig. 5). One can

![Fig. 5](image-url)
easily show that \( x \cdot y = f^2 \). Hence \( f \) can be found. To find the focal length of the enlarging lens use the same method as in Fig. 3, except that you measure \( O \) and \( I \) from the principal foci which are determined by the plane mirror method (the principle of which was shown in Fig. 2). This time, however, the mirror is held vertically as shown in Fig. 5. Use the same notation as shown in Fig. 3. Then \( x = x_4 - X_1, y = X_2 - x_3 \). Tabulate your results as follows:

\[
\begin{align*}
X_1 &= \\
X_2 &= \\
\end{align*}
\]

<table>
<thead>
<tr>
<th>( x_3 )</th>
<th>( x_4 )</th>
<th>( x = x_4 - X_1 )</th>
<th>( y = X_2 - x_3 )</th>
<th>( 1/y \text{ cm}^{-1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Plot \( x \) against \( 1/y \). You should get a straight line, passing through the origin, the gradient of which is equal to \( f^2 \). Deduce \( f \).

**Comments**

Parallax is frequently met with in practical physics in such observations as pointer readings, displacements, etc., and can introduce serious in-

![Diagram of light from fixed star to Earth](image)

\( \Theta_2 - \Theta_1 = 1.5' \)

\( \frac{1.5}{3600 \times 57} \) radians

**Fig. 6**

accuracies. In the field of astronomy parallax plays a very important part in the measurements of astronomical distances. One of the nearest stars
to our solar system is ‘Alpha Centauri’ which is 4.3 light years away (one light year is equal to $9.5 \times 10^{12}$ km). This enormous distance can be estimated by measuring the angle this star subtends with any of the ‘fixed’ stars at the earth and repeating it, say six months later, when the earth has moved in its orbit round the sun through a known distance. The two angles differ by $1.5''$ of an arc (see Fig. 6) from which the distance $D$ of Alpha Centauri can be estimated. For

$$
1.5'' = \frac{1.5^\circ}{3600} = \frac{1.5}{3600 \times 57} \text{ radian} = \frac{\text{diameter of the earth’s orbit}}{D}.
$$

This parallax, though extremely small, constitutes a real proof of the motion of the earth round the sun and if only the instruments of Galileo’s time could have measured it much unpleasantness would have been avoided!

You can estimate distance by measuring angular parallax, using the telescope of a simple spectrometer; choose two suitable points, say 15 ft apart, and measure the angle subtended by a distant chimney or flagpole at each of them, say 200 ft away, and a very distant object on the skyline, say a pylon. Deduce the difference in angles and estimate the distance of the near object to the nearest foot. Check if you can.

**EXPERIMENT 8**

**To find the specific heat of steel**

**Apparatus**

One kilogram of steel ball-bearings about 4 mm diameter, a stout cardboard tube about 80 cm long and 6 cm diameter, each end enclosed by a stout cork. One cork has a hole drilled through it to take a wooden plug (a short length of pencil will do). A second similar plug has a hole down its centre and through this passes a copper-eureka junction, soldered to a small copper disc which ‘caps’ the inner end of the plug. The arrangement is shown in Fig. 2. Microammetter, switch, 50°C thermometer (not total immersion) to read to 0.1 deg C, short copper cylinder with a hole drilled into it to take the thermometer. Vacuum flask, clock, 200 ohm resistance box, and two felt jackets to cover both ends of the tube.

**Method**

The principle of the method should be clear from Fig. 1. Grip the ends of the cardboard tube, which are insulated by felt jackets, with two hands; and invert it rapidly 100 times. The kinetic energy of the fallen balls is converted into heat, and as the rise of the temperature is rather small (about 1 deg C) great care must be taken to measure the temperature rise as accurately as possible.
Measure the temperature by means of a copper-eureka thermocouple, with the hot junction soldered to a small copper disc and the cold junction kept in water at room temperature in a corked up vacuum flask (see Fig. 2). The thermocouple circuit is connected to a microammeter set at its highest sensitivity, a 200 ohm resistance box and a switch.

First, to calibrate the thermocouple, rest a copper cylinder on top of the small disc, with a spot of glycerine between them. Moisten a 50°C thermometer with glycerine and insert it in the cylinder. Adjust the resistance in the circuit so that the microammeter gives a convenient deflection and record the ammeter readings against the corresponding temperature as the cylinder cools. Tabulate your results in the usual manner.

Now insert the temperature-sensitive plug into the cork and take the
initial ammeter reading of the temperature of the ball-bearings. Replace the dummy plug. Take a note of the time on the clock and start inverting the tube, say 100 times, noting the time again when you finish. Replace the sensitive plug and wait until the equilibrium temperature is reached, noting the ammeter reading and the time. Allow the tube to cool for \( \frac{1}{2} \) (time taken for 100 inversions + time to reach equilibrium temperature). Note the change in the ammeter reading. Measure by means of a metre rule the height \( h \) through which the ball-bearings fall each time the tube is inverted.

Calibrate the thermocouple again as a check.

**Theory and Calculation**

Plot a graph of ammeter reading against temperature and draw the line of best fit. Use the graph to find the temperature rise of the ball-bearings, also the temperature drop during cooling of the tube, the sum being the corrected temperature rise \( \theta \) deg C.

The potential energy lost by the falling ball-bearings is \( Mgh \times 100 \) joules where \( M \) is the mass of the ball-bearing in kg, \( h \) is the height in metres and \( g \), the acceleration produced by gravity, is 9.81 m sec\(^{-2}\).

If \( s \) is the specific heat of steel in J g\(^{-1}\) deg C\(^{-1}\) then \( 1000 \times Ms \times \theta = Mgh \times 100 \) and \( s = gh/10 \). Repeat the experiment at least once and average the results.

**Comments**

The result obtained from the above experiment can be checked as follows:

Heat some of the ball-bearings inside a steam jacket and find their temperature by means of a thermometer inserted among them. Quickly transfer them to a calorimeter containing some cold water, a disconnected electric immersion heater and a 50°C thermometer, the temperature of which had been recorded previously. Stir the mixture using the immersion heater as a stirrer, and record the highest steady temperature.

Allow the calorimeter with its contents (without the ball-bearings) to cool to the initial temperature and switch on the electric heater for a measured time to give the same final temperature as before. In practice one cannot reproduce exactly the temperature rise, but one can deduce by proportion the time \( t \) sec needed to give the same temperature rise.

The power of the immersion heater \( P \) is found using a voltmeter and ammeter or, if it is a mains-operated heater, using a domestic electric meter and timing the rotating disc.

If \( M \) g is the mass of the ball-bearings and their temperature drop \( T \) deg C, then the specific heat is \( Pt/MT \) J g\(^{-1}\) deg C\(^{-1}\).
EXPERIMENT 9

To measure the room temperature, using a constant-volume air thermometer

APPARATUS

Constant-volume air thermometer with special three-way tap as shown, special no-parallax mirror, 50°C thermometer reading to 0.2 deg C, large copper calorimeter, crushed ice, Bunsen burner.

It is most important that the air contained in the thermometer is dry and also that the mercury is clean and free from moisture, otherwise many of the precautions that are recommended for this experiment will be useless.

METHOD

First, take the barometric reading \( \pi \) (to the nearest tenth of a millimetre) and immerse the bulb of the thermometer for some time in the calorimeter, which is filled with water at room temperature. When the immersed bulb has reached the temperature of the water, turn the tap so that the bulb is open to the atmosphere and the levels of the mercury in the manometer are the same. Adjust the mercury levels by altering the position of the reservoir tube on the right so that the mercury level just touches the glass index. Slide down the no-parallax mirror so that the cursor line just touches the top of the curved meniscus and read off the level, estimated to a fraction of a millimetre. As the air is at atmospheric pressure and the diameters of the manometer tubes are equal, then this reading is the same as that of the mercury column in contact with the glass. Insert the 50°C thermometer vertically into the calorimeter, and when conditions are steady, take the temperature, using a magnifying glass with which you should be able to estimate it to within 0.1 deg C. Remove the thermometer. Now close the clip on the drying tube and turn the tap through half a revolution so that the enclosed air is isolated from the outside.

Lower the reservoir tube and replace the water in the calorimeter by crushed ice which should completely cover and surround the bulb. Adjust the reservoir tube until the mercury in the left limb of the manometer touches the glass index again, thus ensuring that the volume is kept constant. Wait for a full five minutes to check whether the levels of the mercury remain unchanged and the conditions are steady. Re-adjust if necessary. Read the level of the mercury on the right-hand side, again using the
no-parallax mirror as explained before. Replace the ice with water at room temperature and record both the temperature and the new level of mercury. Now remove the thermometer and heat the water until it is boiling gently. Raise the reservoir tube until the mercury touches the glass index and wait until the conditions become steady as previously explained. Adjust if necessary and re-read the level of the mercury with the aid of the no-parallax mirror. When the experiment is finished and the bulb begins to cool, lower the reservoir tube right down to prevent 'suck-back' of the mercury. Record your observations as follows:

<table>
<thead>
<tr>
<th>Temperature °C</th>
<th>Level of mercury in the reservoir tube</th>
<th>Difference between level of mercury in reservoir tube and tube on left h cm</th>
<th>*Pressure of gas (π + h) cm mercury</th>
</tr>
</thead>
<tbody>
<tr>
<td>Room temperature (1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Room temperature (2)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0°C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Boiling point</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Remember that h is negative when the level in the reservoir tube is lower than that on the left.

**Theory and Calculation**

Temperature is defined by the gas thermometer using the relation \( \frac{t}{100} = \frac{p - p_0}{p_{100} - p_0} \) where \( t \) is the temperature, say that of the room, \( p_0 \) the pressure at the ice-point, \( p_{100} \) that at the steam-point (i.e. where steam is in equilibrium with pure water boiling under a pressure of 760 mm Hg), and \( p \) the pressure at temperature \( t \).

In your experiment, the atmospheric pressure is not necessarily 76·0 cm and so the boiling point \( x \) is not 100°C, but it is easy to estimate it by a simple formula, namely \( x = 100 + \frac{\pi - 76·0}{2·7} \). Calculate the room temperature, using the value of the boiling point that you have found in the relation \( \frac{t}{x} = \frac{p - p_0}{p_x - p_0} \). Compare your answer with the thermometer reading. Also calculate the pressure coefficient \( \beta = \frac{p_{b.p.} - p_0}{(x) \times p_0} \) which, for an ideal gas, should be \( 2\frac{1}{3} \) deg. C\(^{-1}\).
The above experiment is of fundamental importance as all thermometers are calibrated against the gas thermometer, but it does suffer from two failings. First, not all the contained gas is heated to the same temperature (only that contained in the bulb), the effect of this being that the recorded pressure at the steam-point is slightly less. We will now attempt to estimate this small amount (a few millimetres). Using the displacement method, an estimate is made of the volume of the gas which is neither heated nor cooled and is contained partly in the manometer and in the tube leading to the bulb (the so-called dead space). This is usually about 2 ml and represents more than 1% of the total volume of the gas. A short glass tube with an internal volume equal to the dead space is inserted between the tap outlet and the drying column as shown in the figure. No correction is needed at room temperature, as all the gas is approximately at the same temperature. At the ice-point, after the reading has been taken, turn the tap through half a revolution so that the air in the bulb is connected to the enclosed air in the side tube. You will note that the pressure will show a small rise which should be measured accurately on readjusting the mercury level. This difference, which is probably in the order of 7 mm, is now subtracted from the original $p_0$. Adjust the height of the reservoir tube until the mercury levels are equal, i.e. at atmospheric pressure, and turn the tap back through half a revolution so that the gas is disconnected from the side tube.

Repeat this procedure at the boiling point, but this time you will notice that on turning the tap and connecting the unheated air in the side tube, the level will register a fall and the difference when found should be added to the $p_0$ to give the corrected pressure.

The second failing in this experiment is due to the fact that the container expands, and the volume of the gas is not kept exactly constant. The correction here is a small one and can only be justified if the levels are read accurately (to 0.1 mm). The actual definition of the gas scale is given by the formula

$$\frac{t}{100} = \frac{(p\nu)_t - (p\nu)_0}{(p\nu)_{100} - (p\nu)_0}$$

where $p$ is the pressure and $\nu$ the volume of the gas at the stated temperatures. If $\lambda$ is the linear coefficient of expansion of the container, then the volume at any temperature $t$ is $\nu_0(1 + 3\lambda t)$. Therefore the above relation becomes

$$\frac{t}{100} = \frac{p_t(1 + 3\lambda t) - p_0}{p_{100}(1 + 300\lambda) - p_0}$$

$p_0$ and $p_{100}$ here are the corrected pressures at $0^\circ$ and $100^\circ$ (or, as in the above case, the latter is at the boiling point).
EXPERIMENT 10

To determine: (a) the cubical coefficient of expansion of liquids, and (b) the linear coefficient of expansion of a hollow metal rod

INTRODUCTION

Thermal expansion is the result of an increase in the amplitude of vibration or the motion of molecules due to increased thermal energy. As we have seen in Experiment 9 the increase in pressure at constant volume of a gas due to molecules gaining thermal energy is used to define temperature; most gases in fact behave very nearly in the same way, but this is not the case with liquids or solids. With liquids the only thermal expansion it is possible to measure is that of volume, with solids the most convenient thermal expansion to measure is that of length. Both expansions are small and pose interesting problems in their measurement.

(a) The cubical coefficient of expansion of liquids

APPARATUS

U-tube apparatus shown in Fig. 1, various liquids (for example, aniline and mercury).

METHOD

Do this experiment close to a tap with running water, which should at first be made to flow through both jacket-tubes. Check the thermometers, note any difference, and then connect a long, vertical glass tube to the T-piece joining the two arms of the U-tube. By means of levelling screws make sure that the centimetre lines are horizontal. Fill the U-tube with the liquid to be tested through the long vertical glass tube, at first only to a height of roughly 2 cm from the bottom of each arm. Adjust both movable Terry clips (see Fig. 1, one attached to a long pointer sliding on the outer tube on the right, the other carrying a small millimetre scale on the left) so that the top of each Terry clip is just level with that of the liquid. Read the position of the pointer and record it.

Fill up the tubes with more liquid until its level is roughly 37 cm from the bottom, re-adjust the Terry clips and again record the pointer position. Empty one of the jacketing tubes of water and pass steam through instead until conditions are steady; adjust the Terry clips, read the position of the pointer and deduce the expansion $\epsilon_1$ to 0.1 mm. Use the screw clips attached to the T-piece to empty some of the liquid until the level in the cold tube is again roughly 2 cm from the bottom, wait until conditions are steady, adjust the Terry clips, take the pointer readings and deduce the expansion $\epsilon_2$ of the short column liquid. Record both temperatures. Clean the inside of the U-tube thoroughly and repeat the experiment with another liquid.

THEORY AND CALCULATION

The cold vertical column of the liquid, 35 cm high, is balanced by a slightly higher column of heated liquid ($35 + \epsilon_1 - \epsilon_2$) cm high because
of the reduced density of the heated liquid, due to expansion. If $a$ is the cubical coefficient of expansion of the liquid and $\rho_0$ is the density at $0^\circ$C, the density at $t^\circ$C is $\frac{\rho_0}{1 + at}$. As the two columns of liquid are balanced, then

$$\frac{35\rho_0}{1 + at} = \frac{(35 + e_1 + e_2)\rho_0}{1 + at_2}$$

$t_1$ and $t_2$ being the temperatures of the cold and hot columns respectively. Hence $a$ (given to two significant figures and as a ratio, per deg C).

(b) The linear coefficient of expansion of a hollow metal rod

**Apparatus**

Hollow rod (brass or copper) mounted with an optical lever as shown in Fig. 2, lamp and scale, steam generator, 100°C thermometer, travelling microscope.

**Method**

Set up the apparatus as shown in Fig. 2, adjust the horizontal tube so that the terminal knife-edge just touches the fine horizontal groove on the optical lever. Read both the temperature and the position of the light spot on the scale, also measure the distance $D$ between the scale and the mirror on the optical lever. Pass steam through the tube and when conditions are steady re-read the temperature and the deflected position of the light spot. Remove the optical lever and find $d$ (see Fig. 2) as accurately as you think justified, depending upon how well you succeed in getting the knife-edge to touch the horizontal groove.
The theory and calculation

If $e$ is the expansion of the rod, then the angle tilted by the optical lever is $e/d$ radians and the angle turned by the reflected light is $2e/d$ radians. If $x$ is the displacement of the spot then $x/D = 2e/d$. Hence calculate $e$. The linear coefficient of expansion $\lambda$ is $e/l$ where $l$ is the length of the rod and $t$ the temperature rise (express $\lambda$ in the same way as $a$, above). Deduce $\gamma$, the cubical coefficient of expansion ($\gamma$ is approximately equal to $3\lambda$).

Comments

You already know that, from the definition of temperature, the cubical coefficient of expansion of a perfect gas is $\frac{4}{3} deg C^{-1}$. How do the expansions of liquid and solid compare? Also note the considerable differences between the expansions of liquids (say mercury and aniline) and of solids (invar and aluminium).

It was stated in the introduction that the thermal expansion is dependent on the thermal energy given to the molecules (i.e. the specific heat, $s \times$ the temperature rise, $t$) and one should expect for any given substance that the ratio of the two would be a constant. This was found to be nearly the case for solids where $\lambda/s = constant$ (Grüneissen's Law) is obeyed.

In the second part record the position of the spot and the temperature as the rod cools. Should the expansion be strictly proportional to the temperature rise? Should the expansion be strictly proportional to the length of the rod?

In the first part the expansion is not deduced by measuring the height of each of the two columns and finding the difference. Instead, the difference is measured directly. Why? This is called a 'differential method' and it is used whenever the measured quantity is a small difference between two comparatively large quantities. (Can you think of other examples in physics where this method is desirable?)

The optical lever in the second experiment is a device for magnifying a small displacement (which can be a rotation as in the case of a suspended coil galvanometer) and magnifies the displacement by the ratio $2D/d$. The accuracy is limited usually by $d$ as it is a small quantity, so that a small error due, say, to the finite thickness of the knife-edge can produce a proportionately large error in measuring $d$.

Experiment 11

To find the velocity of sound in air and also to find the frequency of a tuning fork

Apparatus

Set of six tuning forks, five of known frequencies and one unknown, rubber striker, ruler, long glass tube of internal diameter 5 or 6 cm
clamped vertically and supplied with water (Fig. 1), short glass tube of smaller diameter to fit loosely into the top of the wide tube (Fig. 2), electric motor with revolution counter and variable speed (up to 1800 rev/min), slotted circular disc (24 slots), small lamp, suitable electrical supply for motor and lamp, stopwatch.

**Fig. 1**

**Fig. 2**

**Introduction**

If a vibrating tuning fork is held over the top of a long vertical tube, sound waves are sent down the tube and are reflected back like echoes. If the length of the tube is adjusted so that these echoes are being constantly reinforced by the vibrating fork, the air in the tube will be set into energetic vibration and the tube will resonate loudly.

**Method**

Fill the wide tube slowly, blow across its open end. The pitch of the note emitted will rise as the tube fills up. Adjust the length of the air column approximately so that its pitch is equal to that of one of the tuning forks, then hold the vibrating fork above it and determine accurately the resonating length of air column by very slowly emptying and filling the tube with water. Repeat with the other forks, completing the following table:
Find in the same way the mean length of the resonant air column for the fork with the unknown frequency. Record the temperature of the air inside the tube. Place the short and narrow tube in position—do you find this affects the resonant length of the air column? Check with each tuning fork. Do you find it harder or easier to locate resonance with the narrower tube?

To find the frequency of the unknown tuning fork, use the simple stroboscope consisting of a rotating disc in front of the lamp; the rotating disc interrupts the light 24 times each revolution, and if the speed of the motor is adjusted so that the tuning fork, illuminated by this flashing light, appears stationary, then the rate of flashing is equal to the frequency of the tuning fork or a multiple of it. (If the speed of the motor changes, the limbs of the fork appear to snake forward or backward. Have you an explanation for this?)

Using the stopwatch, find the average number of revolutions per second. Check at least once.

Investigate fully the effect on the frequency of the fork when it is struck hard.

**Theory and Calculation**

The shortest length of the air column which would resonate with the tuning fork is one quarter of a wavelength ($\lambda$), but your experience with
the narrow tube should have convinced you that this is only approximately true as the diameter of the tube does affect \( l \) slightly; in fact \( \frac{\lambda}{4} = l + a \) where \( a \) is an 'end' effect depending on the diameter of the tube.

The velocity of sound \( v = f \lambda = 4f(l + a) \) and to plot a straight line we write \( \frac{1}{f} = (\frac{4}{v})(l + a) \).

Plot \( 1/f \) against \( l \), find the gradient and deduce the velocity of sound in the tube at room temperature.

From your graph deduce the frequency of the fork of unknown frequency as a check on the stroboscope method.

**Comments**

What should the second resonant length of the air column be in terms of \( \lambda \)? Verify your statement, if possible. Do you find locating the second resonance more difficult? Why?

Can you give other examples of resonance?

Sound waves are transmitted in air by the molecules and under normal conditions one cannot expect sound in a gas to travel faster than its molecules, so that the velocity of sound gives a lower limit to the speed of the molecules.

As the kinetic energy of the molecules is proportional to the absolute temperature, how does the speed of the molecules vary with absolute temperature? Hence deduce the velocity of sound at 0°C.

---

**EXPERIMENT 12**

**To verify Ohm's Law**

**Apparatus**

20 Copper-eureka thermocouples in series (practical details, see Appendix), Pye Scalamp galvanometer, (0–200 ohm) variable resistance box of the plug type, switch, large vacuum flask filled with crushed ice, large beaker, tripod, Bunsen burner.

**Method**

Connect circuit (as shown in Fig. 1, where only 10 thermocouples are indicated, though it is preferable to have 20), place all the 20 unmarked junctions in the beaker which is half filled with water, and the remaining marked 20 junctions in the vacuum flask. Check, by choosing suitable sensitivity of the Scalamp, that thermoelectric current is flowing the right way in the galvanometer.

Heat the water until it is boiling gently; adjust both the sensitivity of the galvanometer and the resistance \( R \) to about 50 ohms so as to give almost a
full-scale deflection of the galvanometer. Enter the galvanometer reading in the following table:

<table>
<thead>
<tr>
<th>Number of hot junctions</th>
<th>Galvanometer deflection</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Without altering resistance $R$ transfer 3 unmarked junctions from the boiling water to the vacuum flask, wait till conditions become steady and record the galvanometer readings as before. This is repeated till all but 2 of the unmarked junctions are transferred into the flask.

Transfer back all the unmarked junctions to the boiling water, and check that, when conditions are steady, the galvanometer reading is the same as before. Inconsistent results are invariably due to immersing too much of the thermo-junctions into the water, for you must not allow the
cotton-covered insulation to get wet. (Ideally a waterproof insulation should be used.)

In the second part of the experiment you should vary the resistance $R$ in series with the thermocouples, keeping the e.m.f. constant. Record your observations in the table at foot of previous page.

**Theory and Calculation**

In the first part of the experiment the e.m.f. in the circuit is altered by varying the number of hot junctions while the circuit remains unchanged. If you plot the number of hot junctions as the abscissa and the corresponding current as the ordinate, you should get a straight line passing through the origin. What does this prove?

For the results of the second part of the experiment we need to extend Ohm's Law to a complete circuit. If $I$ is the current, $r$ the resistance of the thermocouples, $R$ the series resistance, then $I = \frac{E}{R + r}$ or $\frac{1}{I} = \frac{1}{E} (R + r)$.

Plot $1/I$ as the ordinate and $R$ as the abscissa and you should get a straight line of which the gradient is $1/E$ and the intercept on the $R$ axis is $-r$.

**Comments**

In the last part, an intercept had to be deduced from a graph. It frequently happens that in deducing the intercept, the graph may have to be produced backwards to meet the appropriate axis at a point off the graph paper. It is comparatively easy to deduce the intercept without either extending the graph or even doing a tedious calculation. It is only necessary to include the zero on the axis where the intercept is to be found. For example, in the above case mark the resistance axis, 0, 1, 2, ... , etc., to...
give a graph as in Fig. 2. Read off point P, double it and read off the graph the corresponding value of R. This is the numerical value of the intercept. It sometimes happens that you cannot read double the value of P from the graph, as this is off the graph. The easiest course is then to use similar triangles. In Fig. 3, \[
\frac{\text{intercept}}{25.1} = \frac{9.0}{6.9}.
\]

The above experiment is similar to the one which Ohm published in 1826 and from which Ohm's Law is deduced. A null point method using a centre zero galvanometer (4–0–4 V and 2–0–2 mA ranges), a 0–9 V grid bias battery, in steps of 1.5 V, a 10 ohm resistor, rheostat (0–1 A), ammeter, plug switch, two-way switch and 12 V supply can be employed in an alternative method using the circuit shown in Fig. 4. Use only one cell (1.5 V) for the grid bias battery, and use the centre zero galvanometer as a voltmeter. Adjust the rheostat to give zero deflection (this can be
confirmed by opening the switch K connecting the voltmeter into the circuit. Now turn over the two-way switch so as to use the galvanometer as a milliammeter and adjust the rheostat more accurately to give a zero current reading. Note the value of the current in the ammeter. This is repeated using two cells, three cells, and so on. Plot the number of cells against the current reading and you should get a straight line passing through the origin, verifying Ohm's Law for metallic conductors.

EXPERIMENT 13

To measure an unknown resistance $X$ using a metre (or half-metre) bridge

Apparatus

2 ohm resistor, unknown resistor $X$, centre zero milliammeter and voltmeter, metre or half-metre bridge, accumulator, switch, rheostat, tapper and length of bare eureka wire (28 S.W.G.).

\[
\begin{align*}
\frac{X}{2} &= \frac{V_{AB}}{V_{BC}} & \text{or} & & X &= 2 \cdot \frac{V_{AB}}{V_{BC}} \\
\end{align*}
\] (1)

Calculate $X$. The accuracy of $X$ as calculated in this way is not high. The voltmeter, unless it has a very high resistance, is likely to draw some current from the circuit and the p.d. measured is thus affected. Further, unless the scale of the voltmeter is large, with a no-parallax mirror, the needle very fine and the scale linear, one cannot hope for a high degree of accuracy. Now consider the circuit in Fig. 2, $R_1$ being a fixed resistor and $R_2$ a variable resistor. If we adjust the value of $R_2$ until the p.d. across $R_1$
is the same as the p.d. across $X$, as checked with the voltmeter, then it
would follow that the p.d. across $R_2$ would be the same as the p.d. across
the 2 ohm resistor. Now if we were to connect the voltmeter between B
and D, it would read zero; but as the voltmeter is only a galvanometer
with a high resistance in series and since the p.d. between B and D is zero

![Fig. 2]

(or nearly so) we can now connect the milliammeter terminals direct to
B and D and get a finer adjustment of $R_2$ to give zero milliammeter reading.

Now, $V_{AB} = V_{AD}$, $V_{BC} = V_{DC}$ or $\frac{V_{AB}}{V_{BC}} = \frac{V_{AD}}{V_{DC}}$. Since the same current
flows in $X$ and the 2 ohm resistor and in $R_1$ and $R_2$ therefore
$\frac{V_{AB}}{V_{BC}} = \frac{X}{2}$
(as before), $\frac{V_{AD}}{V_{DC}} = \frac{R_1}{R_2}$, therefore

$$\frac{X}{2} = \frac{R_1}{R_2} \quad \text{or} \quad X = 2\frac{R_1}{R_2} \quad \cdots \quad \cdots \quad (2)$$

The accuracy of the value of $X$ calculated by this method is certainly
higher, as resistors can be made to a high degree of accuracy and the
method does not depend on the doubtful accuracy of an instrument read-
ing. It is a great improvement on the measurement of resistance (it is
called a null-point method) as the galvanometer is used to detect the
current but not to measure it. This circuit was first used by Wheatstone
and is called after him.

Now consider equation (2) again: $\frac{X}{2} = \frac{R_1}{R_2}$. For this the values of $R$
and $R_2$ are not required but only their ratio. If $R_1$ and $R_2$ are to be made
of one uniform resistance wire, then $\frac{R_1}{R_2}$ would be equal to the ratio of their
lengths, say $\frac{l_1}{l_2}$, and the circuit would be modified as in Fig. 3. The balance
point is found by moving a contact D (a tapper) along the wire until the
galvanometer reads zero. Thus we need only one known resistor to enable
us to find $X$.

**Method**

Connect up the circuit as shown in Fig. 3 (the rheostat in series with the
battery is to reduce the current flowing through the metre wire; it is not
necessary if the wire has a sufficiently high resistance). The resistors $X$ and 2 ohm should be connected by short leads to the bridge and all terminals should be firmly screwed down. First use the galvanometer as a voltmeter (for safety's sake) and tap either end of the metre wire, which should give opposite deflections on the voltmeter. Find also the approximate balance-point, when the voltmeter gives zero deflection. Next connect the galvanometer as a milliammeter and determine more accurately the balance point, moving the tapper 1 mm on either side of the balance point which should thus just give opposite deflections on the milliammeter (this is called 'bracketing' the balance point). Record $l_1$, $l_2$ is $100 - l_1$ (or $50 - l_1$). Now interchange resistors and repeat, remembering that $l_2$ is now on the right-hand side of the balance point. Record $l_1$ and $l_2$ again. Calculate $X$ from each set of observations and average.

**Comments**

The principle of the Wheatstone bridge is extensively used in electrical measurements and it is not confined to the measurements of resistance. Now you can use the circuit in Fig. 3 to measure the resistivity of the eureka wire provided, by putting it in the place of $X$. Vary the lengths of wire connected across the terminals, starting with the full length and reducing it by stages until you have made six observations altogether. Keep one end of the wire permanently fixed to one of the terminals and mark with ink the point on the wire where it joins the other terminal. Unscrew the terminal and measure the length of wire $l$ between the terminals (i.e. the distance between the ink mark and the fixed terminal) using a ruler. This does not necessarily measure the actual length of eureka wire used, but provided the same procedure is carried out each time, it will not affect the final result, as we will see later. Tabulate your results and in your table calculate your resistance $R$ for each length of wire $l$.

Now plot a graph with $R$ as ordinate and $l$ as abscissa and you should get a straight line not necessarily passing through the origin as the length $l$ may differ from the true length by small constant length each time. The gradient of the line is the resistance per cm of the wire. Now use the micrometer screw gauge, after checking and recording the zero error (if any), to
measure the diameter of the wire at two separate points at least, mutually at right angles, along the wire. Work out the average diameter and calculate the area of cross-section of the wire. The resistivity $\rho = Ra/l$; hence $\rho = \text{area of cross-section} \times \text{gradient of graph}$. The unit of resistivity is ohm-cm (for M.K.S. $l$ is in metres and $\rho$, ohm-metre), the answer being given to two significant figures only.

**EXPERIMENT 14**

*To verify Faraday's Law of electromagnetic induction and to apply it in order to estimate the angle of dip*

**APPARATUS**

General purpose motor (6 or 12 V) with revolution-counter, bicycle dynamo mounted on the same board as the motor and connected by flexible drive to the motor, 0–10 V A.C. voltmeter, stopwatch, 0–1 A A.C. ammeter, two rheostats, an earth inductor with commutator segments and brushes, Pye Scalamp.

**Verification of Faraday's Law of electromagnetic induction**

**METHOD**

Connect up the circuit shown in Fig. 1, adjusting the rheostat so that the motor rotates slowly. Count the number of revolutions shown in 30 seconds, recording the mean voltmeter reading, and check at least once, adjusting the rheostat to keep the output voltage constant. Repeat five times, increasing the speed each time to reach finally the maximum speed possible. Tabulate your results as shown in table at top of following page.

**THEORY AND CALCULATION**

The cycle dynamo consists of a permanent magnet rotating between fixed coils so that the faster the magnet rotates, the bigger is the induced e.m.f. recorded by the A.C. voltmeter. As the field of the magnet is fixed,
then the rate of cutting of magnetic flux by the coils is proportional to the number of revolutions in a fixed time, i.e. \( V \propto f \). Verify this relationship by plotting \( V \) against \( f \). You should get a straight line passing through the origin, thus establishing the truth of Faraday's law.

**Estimation of the angle of dip**

**Method**

This is not an easy experiment and you must have a partner to help you with it. Connect the brushes of the earth inductor to a Pye Scalamp galvanometer using long leads so that there is a large distance between the two (Fig. 2). Mark the direction of the magnetic meridian on the bench and place the earth's inductor so that the plane of the coil is vertical and at right angles to the meridian (do not move the base of the inductor once it has been set).

Keeping the base fixed, rotate the plane of the coil into a horizontal position, so that the coil itself can be rotated about a horizontal axis, parallel with the magnetic meridian. One of you now rotates the coil at a steady rate (a speed of 2 rev/sec is convenient), checked by the other using a stopwatch (a metronome, if available, is useful to help maintain a steady rate of turning). Set the galvanometer at a suitable sensitivity so that a large deflection is produced, the mean, \( x_1 \), being recorded. Now turn the plane of the coil back to its original position, and keeping to the same rate of turning, checked by a stopwatch, record the new mean reading \( x_2 \) on the galvanometer. These observations are checked several times until consistent results are obtained. Average all the values of \( x_1 \) and \( x_2 \).

**Theory and Calculation**

The earth inductor is used as a D.C. dynamo and the output in volts is proportional to the rate of cutting of magnetic flux, which in turn depends on...
on the magnetic flux density $\times$ rate of turning of the coil. As the latter is kept constant, then $x$, the galvanometer deflection, is proportional to the flux density which in turn is proportional to the field strength. When the plane of the coil is horizontal, the rotating coil will be cutting the magnetic flux in a vertical direction, thus $x_1$, is proportional to the vertical component of the earth’s magnetic field $V$. Similarly $x_2$ is proportional to the horizontal component $H$. The resultant of the earth’s magnetic field makes an angle $\theta$ with the horizontal, given by the relation $\tan \theta = V/H = x_1/x_2$, hence $\theta$ which is the angle of dip. This method can be used to estimate the angle of dip to within a degree or two.

**Comments**

The obvious application of electromagnetic induction, of which Faraday was aware, is the generation of electric power through the use of the dynamo. A simple study of the cycle dynamo provided can be made, using the circuit shown in Fig. 3, where the output of the dynamo is connected to an external circuit so that the current taken can be varied. Keep the speed of the motor constant, so that the voltage recorded when the key is open is just over 6 V (check on the speed regularly with a stopwatch during the experiment). Record in a tabulated form the voltage and the corresponding current for six observations, varying the current by means of a rheostat, over a wide range. Insert, in an additional column in your table, the electric power $(V \times I)$ and plot it in a graph against the current $I$. Also plot $V$ against $I$, and comment on both graphs.

**Experiment 15**

To study the conduction of electricity by electrolytes

**Apparatus**

Water voltameter filled with acidulated water, copper voltameter, 0–15 mA milliammeter with 0.15 A shunt, switch, rheostat, potentiometer and tapper, simple conductivity cell with copper electrodes (see Fig. 2), distilled water, copper sulphate crystals, stopwatch, three accumulators, thermometer, wooden stand and clamp.

**Method**

This circuit is connected up as shown in Fig. 1; adjust the rheostat so that the current flowing is about 0.15 A. Fill the collecting tubes over the
platinum electrodes with the electrolyte by opening the taps and raising
the reservoir tube on the left until both tubes are completely immersed
in electrolyte (see Fig. 1). Close the taps and then lower the left-hand tube.
Remove the cathode of the copper voltameter, dry it, clean it with sand-
paper and weigh it as accurately as possible, then replace it.

Fig. 1

Fig. 2

Start the experiment proper by switching on the current and starting
the stopwatch, keeping the current steady by adjusting the rheostat. At
10 minute intervals, switch off the current and lower the reservoir tube
until the level of the liquid is the same as in the tube collecting the gas over
the cathode. The volume of gas evolved is read from the graduation on the
tube. Resume the experiment for a total of 50 minutes. Record your results
as follows:
At the end of the experiment remove the cathode of the copper voltameter, wash gently with water, rinse with methylated spirit and then dry carefully in the hot air rising from a Bunsen flame. Weigh it and deduce the mass of copper deposited on it.

Now connect up the circuit shown in Fig. 2, discarding the shunt of the ammeter. Dissolve a measured quantity of copper sulphate (about 2 g) in 50 ml of pure water to make up the electrolyte. Pour this into the conductivity cell. Keep the distance $l$ between the electrodes constant, vary the applied potential between the two electrodes by tapping different points on the potentiometer wire, and record both the length $d$ of wire tapped and the current in mA. Tabulate your results as follows:

Repeat the above experiment after halving $l$.

**Theory and Calculation**

The results of the first half of the experiment should convince you that the carriers of electricity in an electrolyte are the charged atoms or groups of atoms. (Your first set of results should make it clear that the volume of hydrogen—check that it is hydrogen—evolved at the cathode is proportional to the charge passed, i.e. to the time during which a constant current is passed.)

Secondly, the same total charge liberates different masses of copper and hydrogen. The mass of hydrogen evolved can be calculated from the total
volume \( V \), the temperature \( t \), the pressure of the saturated water vapour \( h_1 \) mm of mercury, the barometric pressure \( h \) mm of mercury and the density of hydrogen (2 g for every 22.4 l. at S.T.P.). The volume \( V \) of hydrogen at pressure \( h - h_1 \) at \((273 + t)\)° absolute will occupy

\[
V_0 = \frac{V(h - h_1) \times 273}{760 \times (273 + t)} \text{ ml}
\]

at S.T.P. and the mass of hydrogen evolved is therefore

\[
V_0 \times \frac{2}{22.4 \times 1000} \text{ g}.
\]

The ratio of the masses of copper and hydrogen liberated should be the same as the ratio of their chemical equivalents, if the atoms are in fact the carriers of electricity—do you find that this is so?

The second part of the experiment is intended to show you the similarity between the conduction of electricity in solids and liquids. The applied potential to the cell is the difference of the potential between the ends of the tapped wire \( d \) and the potential between the milliammeter leads, which is not negligible. If the total length of the potentiometer wire is 100 cm and the e.m.f. of the accumulator is 2.0 V (or 2000 mV) then 1 cm corresponds to 20 mV. If the resistance of the ammeter is \( r \) ohm (usually 5.0 ohm) and the current is \( I \) mA, then the total p.d. is \( (20d - rI) \) mV. Now complete the second table, commenting on the last column; is Ohm’s Law obeyed? How does the resistance of the electrolyte vary with \( l \), the distance between the two electrodes?

**Comments**

When the separation between the two electrodes in the second part of the experiment is kept the same, and the potential between them is varied, the current is found to be proportional to the potential difference (Ohm’s Law).

There are two possible explanations for this, either the number of charged atoms (ions) present varies directly with the potential difference, or the speed of the ions increases with increasing potential giving a larger current. It is possible to measure directly the speed of ions (see Experiment 98), and it is found that the second explanation is the correct one.

When the distance between the electrodes is varied, keeping the potential difference the same, the current is found to vary inversely with \( l \). This suggests that the speed of the ions is proportional to potential difference divided by \( l \) or to the potential gradient.
EXPERIMENT 16

To study the variations or deviations of actual observations, to deduce a measure of their probability and to make use of it in the inference of the most probable value of an observation

INTRODUCTION

In the limited time available for practical work in schools, there is little need to apply much of this, but the knowledge gained is useful in the study of measurements of important physical quantities and it may be useful to the specialist of the future.

We have learnt that the measurement of many physical quantities is subject to random errors by the observer, inaccurate measuring instruments and variable conditions. These cannot be completely controlled. This makes it necessary to repeat experimental observations, to record them and to infer from them the most probable value of the physical quantity measured.

It is not always necessary to repeat an observation, for example measuring the length of a thin rod or weighing a ball-bearing, unless it is to check for a possible mistake and there is nothing gained by recording such repeated observations. On the other hand measuring and recording the diameter of a cylindrical rod in mutually perpendicular directions is not wasted repetition but is a check against the non-circular cross-section of the rod.

Repeating an observation is not only a way of obtaining greater accuracy by averaging, but it also gives further information about the reliability of such an observation; this is the reason why you should always record all observations and not just the average. When a number of such observations shows large variation, there is a higher degree of uncertainty in the average than that of an equal number of closely grouped observations.

APPARATUS

Curved channel (see Fig. 1 and Experiment 17), 20 \( \frac{3}{8} \)-in ball-bearing, G-clamp, large drawing board, large strip of graph paper, Sellotape, carbon paper, rulers and plumb-line.

METHOD

Use the G-clamp to fix the curved channel firmly to one corner of the bench, with the horizontal portion sticking over the edge of the bench and the long side AB parallel to the other edge of the bench, as shown in Fig. 1. Use the plumb-line to locate the point vertically below point B on the floor. Draw a line on the floor, parallel to the edge of the bench where the edge of the channel overhangs, and through point B. Then draw another line parallel to the first line and at a distance, away from the channel, equal to the radius of the ball. Place the drawing board with a long edge just touching the second parallel line. Allow a ball-bearing to roll
down from the highest point C and locate the point of impact on the board. Sellotape a long strip of graph paper (this could be made by joining accurately several 6-in sheets, checking with a ruler that the graph lines are straight) along the length of the board, parallel to the long edge, so that the located point of impact lies about halfway across the width of the graph paper. Put the board back into position. Cover the target area with carbon paper, carbon side down, and roll from C each of the 20 ball-bearings (it is quicker if you have a partner for this experiment, one of you

rolling the balls and the other retrieving them). Now move the board sideways about 3 cm at a time keeping the long edge parallel to the marked line and each time roll down the 20 balls. Do this about 30 times.

**Theory and Calculation**

Divide the target area into 12 equally spaced strips, using the graph lines to help you (see Fig. 2). Count the number of impacts in each strip, and where the impressions lie on a dividing line, add ½ to each of the two adjoining strips. Measure the distance of the mid-point of each strip from the reference line (horizontal range X cm), tabulating your results as in table overleaf.

Find the arithmetical mean $\bar{X}$ by dividing the total of column 3 by the total number of impacts $N \left( \bar{X} = \frac{\Sigma nX}{N} \right)$. Now calculate the deviation from the arithmetical mean for each row (column 4) which may be negative or positive, square the deviation (column 5), multiply the square of the deviation by the corresponding frequency (column 6). The square root of the (total of column 6 divided by $N$) is called the standard deviation $\sigma$ $\left( \sigma = \sqrt{\frac{\Sigma n(X - \bar{X})^2}{N}} \right)$ which is a measure of the scatter of the observations from the mean and is expressed in the same units as $X$. 
<table>
<thead>
<tr>
<th>1</th>
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<tr>
<td>Range $X$ cm</td>
<td>Frequency $n$</td>
<td>$nX$</td>
<td>$X - \bar{X}$</td>
<td>$(X - \bar{X})^2$</td>
<td>$n(X - \bar{X})^2$</td>
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<tr>
<td>Totals</td>
<td>$N$</td>
<td>$\Sigma nX$</td>
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<td>$\Sigma n(X - \bar{X})^2$</td>
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Plot $n$ as ordinate, $X$ as abscissa and draw a smooth curve between the points. This curve is called the Normal Distribution Curve or the Gaussian Law of Error, which can only be drawn with accuracy if the number of observations is very large. (It is not certain that all types of random experimental errors follow this law, but sampling errors tend to do so. See Experiment 100.)

Now, the standard deviation is a measure of the uncertainty of $\bar{X}$; the smaller $\sigma$, the more accurate is $\bar{X}$; also the larger the number of observations $N$, the smaller is the uncertainty of $\bar{X}$, as we are using a large sample. The yardstick by which we can denote our uncertainty is the standard error $\sigma/\sqrt{N}$, which gets smaller when $\sigma$ decreases or $N$ increases. It is usual to state that the true horizontal range of the ball-bearing is likely to lie between $\bar{X} + (\sigma/\sqrt{N})$ and $\bar{X} - (\sigma/\sqrt{N})$ or simply the horizontal range is $\bar{X} \pm (\sigma/\sqrt{N})$. Calculate the limits from your results. Draw on your target sheet a line at a distance $\bar{X}$ from the reference line and two other lines parallel to it on either side of it, distance $2\sigma$ away. Count the number of hits outside the area included by the two outer lines. What percentage is this of the total?

What is the probability of a ball hitting the target sheet with the range greater than $\bar{X} + 2\sigma$?

**Comments**

You will note that the Gaussian (or normal) curve of error is symmetrical about the mean (i.e. + or − errors are equally likely to occur) and that small errors are more frequent than large ones. Note also that the two points on the curve where the slope is steepest are at a distance $\pm \sigma$ from the mean.

Many statistical measurements on large but homogeneous populations seem to show normal distribution, such as of height, collar size, etc. (the standardization of intelligence tests is based on the hypothesis that degrees of intelligence are normally distributed).

Fig. 3 shows a simple apparatus for studying the distribution of the diameters of lead shot. It consists of two thin brass rods (or hacksaw blades
titled to give a cross-section thus \( \setminus \uparrow \) inclined to the horizontal and with the gap between the rods (or blades) widening gradually towards the bottom end (the gap increases by about 1 mm). Lead shots are rolled down between the rods (or blades) and drop through the gap at various points, being collected by paper-partitioned slots (about 12 in number) as shown in the diagram. The gap between the rods (or blades) is previously adjusted so that the maximum frequency of fall-through occurs halfway down the rods.

Plot a frequency distribution curve and comment on it.
ADVANCED LEVEL EXPERIMENTS
EXPERIMENT 17
To verify the law of conservation of linear momentum

APPARATUS

Small electromagnet supported by two long inclined copper wires (30 S.W.G.) attached to the ceiling as shown in Fig. 1, two ball-bearings (about 150 g and 100 g respectively), half-metre and metre rules, number of large drawing boards, large sheet of carbon paper made up by joining several carbon papers together, plumb-line, 12 ohm rheostat, switch and 12 V supply. Curved channel as illustrated in Fig. 2 (see also Experiment 16), G-clamp.
Method

Find and record the mass of the two ball-bearings (to the nearest gram). Adjust the height of the electromagnet so that when supporting the larger of the two balls (ball 1, mass M g), the ball should just clear the top of the table and should just touch the other ball-bearing (ball 2, mass m g) which should be placed balanced over the edge of the table (a tiny pellet of plasticine helps). Chalk on the floor a line perpendicular to the edge of the table at the point where ball 2 is balanced, and in the vertical plane of oscillation of the electromagnet. Spread out the drawing boards at the foot of the table, cover them up with drawing paper fixed with Sellotape, and use your plumb-line to locate the foot O of the perpendicular to the edge of the table where ball 2 is placed. Draw a line on the board parallel to the edge of the table and passing through O. Now cover up the papered boards with the carbon paper, carbon side down.

Pull aside the energized electromagnet carrying ball 1, along the direction of the chalked line, and measure the height of the centre of gravity of the ball above the bench using the clamped vertical half-metre rule, shown in Fig. 1. Release the electromagnet and ball 1 will hit ball 2 and both will drop on to the floor (if ball 1 is not released, you have to reduce the current in the electromagnet so that the ball is just supported in the displaced position of the electromagnet). Remove the carbon paper and number the points of impact P₁ and Q₁ of the two balls 1 and 2 respectively (using two balls of different masses helps to distinguish between the two impact marks, but a visual check is always helpful when in doubt).

Measure the height of the centre of gravity of ball 1 in its equilibrium position when supported by the electromagnet, and deduce the vertical drop h of ball 1 before striking ball 2. Measure also the distances x₁y₁ and X₁Y₁ and the vertical drop H (all shown in Fig. 1), tabulating your results as follows:

<table>
<thead>
<tr>
<th>H</th>
<th>h</th>
<th>x</th>
<th>y</th>
<th>X</th>
<th>Y</th>
<th>MX</th>
<th>mx</th>
<th>MY + my</th>
<th>2M√hH</th>
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</table>

Repeat your observations using different values of h and different angles of contact.

Theory and Calculation

The velocity V of ball 1 on impact can be calculated from the law of
conservation of energy, thus
\[ \frac{1}{2} M V^2 = M g h \quad \text{and} \quad V = \sqrt{2 g h} . \]
Let the velocities after impact of the two balls be \( v \) and \( u \) respectively; as air resistance is small these velocities remain constant in flight. The time taken for a freely falling body from rest through a vertical height \( H \) is \( \sqrt{2 H/g} \) sec, so that the ranges of the two balls are \( R = v \sqrt{2 H/g} \) and \( r = u \sqrt{2 H/g} \).

Therefore \[ v = R \sqrt{\frac{g}{2H}} \quad \text{and} \quad u = r \sqrt{\frac{g}{2H}} . \]

The law of conservation of momentum states that
\[ M V = M v \cos \phi + m u \sin \theta , \]
or
\[ M \sqrt{2 g h} = M R \cos \phi \sqrt{\frac{g}{2H}} + m r \cos \theta \sqrt{\frac{g}{2H}} , \]
\[ 2 M \sqrt{h H} = M Y + m y \quad \ldots \ldots \ldots \ldots \quad (1) \]
Also as the transverse momentum before impact is zero then the total transverse momentum after impact is also zero. Therefore
\[ M R \sin \phi \sqrt{\frac{g}{2H}} = m r \sin \theta \sqrt{\frac{g}{2H}} . \]
or
\[ M X = m x \quad \ldots \ldots \ldots \ldots \quad (2) \]
Verify both relations in the above table. You should get a fairly good agreement (within 5%) between columns 9 and 10 and columns 7 and 8.

Comments

The law of conservation of momentum follows directly from Newton's third law of motion and is applicable where impulsive forces are concerned, such as in collisions between nuclei. Though you can verify in this experiment that energy is not conserved, in nuclear collisions both the laws of conservation of mass-energy and momentum are assumed.

A simpler version of the above experiment is the curved channel shown in Fig. 2; ball 2 rests on a special screw, adjustable both in its height and its position at the end of the horizontal 'shoot'. The initial momentum of ball 1 is found by letting it roll down the curve and fall on to the floor without any other impact, as its horizontal range is a measure of its velocity on leaving the 'shoot'. You can show by this that its range is proportional to the square root of its vertical drop as it rolls down the curve. The experiment is then carried out as discussed above, but the agreement is perhaps not as good.
EXPERIMENT 18

To show that the acceleration of a body moving in a circle radius $R$, with a constant angular velocity $\omega$ radians sec$^{-1}$, is directed towards the centre and equal to $\omega^2R$

APPARATUS

Stopwatch, half-metre ruler, lightweight pan of known mass, a box of weights. A spring-wound gramophone with variable speed. The brass cylinder M (Fig. 1) is about 90 g and is soldered to curtain rollers which glide on a horizontal curtain rail. The rollers should be well oiled. M is attached by a short spiral spring to a vertical metal peg which coincides with the axis of rotation of the turntable. When the turntable revolves, M moves outwards extending the spring and pulling out with it a cotton thread. The thread passes through a hole in a vertical strip of stiff paper glued to the turntable close to the peg.

Fig. 1

METHOD

Pull out the thread as far as it will go, set the turntable at its slowest speed, wind up the gramophone and start the table turning. Allow the table to reach a constant speed and time twenty revolutions with the help of a chalk mark, tapping the side of the turntable, since the roller tends to stick and to move outwards in jerks. Stop the turntable gently. Carefully press the thread with your finger against the peg, while pulling out the cylinder M until the thread is taut. Measure with the help of a companion the distance $R$ (to the nearest mm) of the centre of the roller from the peg. Repeat the whole operation. The time of twenty revolutions should be within 0·2 sec of the previous reading, the distance within 1 mm. Repeat once more, if necessary.
Now increase the speed of the motor and carry out the experiment (including the checks) again. Proceed, with increasing motor-speeds, until six sets of observations are obtained.

Tabulate your results as follows:

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius $R$ cm</td>
<td>Time for 20 rotations $T$ sec</td>
<td>Acceleration towards centre $(\frac{40\pi^2}{T}) R$ cm sec$^{-2}$</td>
<td>$Wg/M$</td>
</tr>
<tr>
<td>$R$</td>
<td>$R$</td>
<td>$R^*$</td>
<td>Average $R$</td>
</tr>
</tbody>
</table>

* If necessary

Column 3 gives the inward radial acceleration of the mass $M$ in its uniform circular motion. This radial acceleration is produced by the tension in the spiral spring, which we have to find.

On the gramophone box is a small fixed pulley (not shown in Fig. 1) at the same level as the second hook on cylinder M. Align the curtain rail with this pulley and tie to the second hook a cotton thread which passes over the pulley and supports the lightweight pan. Add various weights to the pan, tapping gently the side of the gramophone box each time, and measure the corresponding distance between the centre of the roller and the peg. Check each observation, tabulating as in Table 2 overleaf. Now an estimate has to be made of the small force $a$ g wt required to overcome the friction between the rollers and the rail and that of the small pulley. This is done by unhooking the spring, removing the pan and attaching instead a small paper tray. As $a$ is only in the order of a few grams (2 or 3), it can be quickly estimated by adding small weights, tapping gently, as before, the gramophone box until the cylinder just begins to move. Finally weigh the mass $M$ to the nearest gram.

**Calculation**

Plot the corrected force $W$ (g wt) against the extended distance $X$. Except for the smallest value of $W_1$, the points should lie on a straight line. Use the graph to read off the accelerating force (in g wt) corresponding to each distance $R$ in Table 1. This should be converted into dynes and the acceleration it produces on mass $M$ calculated in cm sec$^{-2}$. Insert this value in column 4 of Table 1. Do the entries in columns 3 and 4 agree?


TABLE 2
Weight of pan = \( w \) g

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight added ( W_1 ) g</td>
<td>Distance ( X ) cm</td>
<td>Total weight ( W_1 + w ) g</td>
<td>Force (corrected for friction) ( W = W_1 + w - a ) g wt</td>
</tr>
<tr>
<td></td>
<td>( X_1 )</td>
<td>( X_2 )</td>
<td>Average</td>
</tr>
</tbody>
</table>

Comments

The above experiment is a simple and direct example of uniform circular motion. The topic of the experiment is an important one because circular motion finds many useful applications in the modern world. For example, ultra-centrifuges, man-carrying centrifuges, satellites, etc. Other experiments can be done, although they involve further calculations. In one such experiment a U-tube is made with a long horizontal portion. The tube is fixed on the turntable so that one vertical arm is at the axis and the other at the rim, with the horizontal portion lying along a radius. Liquid is poured into the tube. When the turntable is rotating rapidly the levels of the liquid are unequal. Devise a method of measuring the difference in levels when the table is rotating. Using the usual notation, the difference in levels is \( \omega^2 R^2 / 2g \); try to prove and check this relationship experimentally.

EXPERIMENT 19

The study of simple harmonic motion

Apparatus

Fletcher's trolley with glass plate, A.C. timer and tape (see Experiment 5), spiral spring (this can be bought from a sports shop as part of a chest expander and extends by about 14 cm per kg wt), long flexible chain (such as is used to hang lamp bowls), stout fishing line, clamps and stand, reel of Sellotape, G-clamps.
**Method**

Screw a steel plate, bent to an angle of 45°, to the underside of the trolley-bed and clamp the plate to the bench support so that the trolley-bed is firmly held in the position shown in the diagram. Clip a smooth glass plate on to the upper face of the trolley-bed, rest the trolley on the glass and firmly attach to it the long spiral spring. Fix the upper end of this spring to a stand firmly clamped to the bench. Line up the spring so that it is parallel to the trolley-bed (a boss on the clamp will keep the spring in position). Arrange that the vibrator brush rests over the mid-point of the trolley and that it has a period of vibration of 1/15th sec or less (this can be checked by a stroboscope or by the A.C. timer). If the wheels are well oiled, the trolley should make, after being displaced, at least twenty oscillations with a period of about 1.5 sec.

Fix a long strip of white paper to the upper face of the trolley with Sellotape. Ink the vibrator brush. Draw the trolley down and release it, so that the brush draws a straight centre line on the paper. Now draw the trolley down again, set the vibrator going and then release the trolley. Pull the vibrator aside when the trolley is just short of completing a full oscillation. You may find it necessary to make several attempts before you are successful, and it is best to work with a partner.
Theory and Calculation

The traces made on the paper look puzzling at first sight (Fig. 2) as they are made up of almost two sets of vibrations, but the start is unmistakable, and you should be able to identify the vibrations made in the first swing of the vibrating trolley because of the big amplitude of the vibration and its thick ink. Measure the distance $s$ from the starting point to the end of each vibration (which should be numbered), tabulating your results as follows:

<table>
<thead>
<tr>
<th>$s$ cm</th>
<th>Number of vibrations $n$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Now identify the vibrations made in the second swing, completing the numbering of the vibrations as before and completing the above table. Plot $s$ as ordinate and $n$ as abscissa and you will get a smooth curve (Fig. 3)

which should be drawn carefully with a flexicurve. The maximum value of $s$ represents twice the amplitude of oscillation ($= 2s_0$). Find the gradient at every point (this is proportional to the velocity at the point) and complete the following table:
Plot again \((ds/dn)\) against \(n\) and you should get another smooth curve (a sine curve, as shown in Fig. 4). Again find the gradient at every point \(\frac{d^2s}{dn^2}\) which is proportional to the acceleration. Complete the following table, where \(s_0\) is the displacement from the equilibrium position (Fig. 3) and \(s\) is found from the first table:

<table>
<thead>
<tr>
<th>(\frac{d^2s}{dn^2})</th>
<th>(n)</th>
<th>(s - s_0)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Plot \(\frac{d^2s}{dn^2}\) against the displacement \((s - s_0)\) and you should get a straight line passing through the origin, proving that the acceleration is proportional to the displacement, but as the line has a negative slope the acceleration is directed towards the origin (i.e. the equilibrium position). This is an example of simple harmonic motion, and if the gradient \(k\) is deduced from your graph you should verify that the period \(T = 2\pi/\sqrt{k}\). The period can be checked from the number of vibrations in one complete oscillation of the trolley.

**Comments**

There are many examples of simple harmonic motion, where the restoring force is proportional to the displacement from an equilibrium position. Fig. 5 gives another example where a hanging chain replaces the spring in producing the restoring force. The bed of the trolley is tilted so that a suitable part of the chain balances the component of the weight of the trolley down the inclined plane. The trolley again has to be at rest with its mid-point below the brush (by adjusting the length of the thread). The period this time is long, and only one swing is necessary and this is best done by pulling down the trolley till the brush is above the upper end,
and releasing both the vibrating brush and the trolley. Stop the trolley when it reaches its highest point and carry out the same graphing and calculations. The effect of the frictional force (assumed to be constant) is greater in this experiment while the restoring force per unit displacement is so much smaller (hence the longer period). Investigate the effect of friction on the period and the simple harmonic motion.

**EXPERIMENT 20**

**To verify Hooke’s Law for a spiral spring and to study the motion of various masses attached to a vibrating spring**

**Apparatus**

Spiral spring (similar to the one used in Experiment 19, extending by about 14 cm per kg wt), large scale pan supported by three short lengths of fishing line (not nylon) capable of supporting a 12 lb weight, a long length of similar line carrying a perspex cylindrical lens with cursor line as shown in Fig. 1, a half-metre rule, a stout hook, fixed to rigid beam or support, about six 200 g weights, stand and clamp, stopwatch.

**Method**

Attach the scale pan to the lower end of the spring by a length of fishing line, so that the distance from scale pan to lower end of spring is about 70 cm. Attach one end of the spiral spring to the stout hook. Support the half-metre rule vertically, so that the curved part of the lens is touching the scale. The lens should be about halfway between scale pan and spring.
Read off the position of the cursor line on the vertical scale, then add a 200 g weight to the pan and read the new position of the cursor line, completing the table as shown below (Table 1). Take four more observations with increasing loads on the scale pan. Check the observations again with decreasing loads. Very close agreement should be obtained.

Add 200 g wt to the empty scale pan, pull down the pan by a distance not exceeding the extension for 200 g (see Table 1) and release. Time 20 vibrations of the oscillating pan, using one of the graduations of the ruler as a reference line. Repeat the operation until times agreeing to 0·1 sec are obtained. Carry out the experiment five times more with increasing masses, as before, completing the table as shown overleaf (Table 2). Weigh the scale pan and the lens. Also weigh the spiral spring and record both observations.

**Theory and Calculation**

It is clear from Table 1 that within the experimental error, the extension is proportional to the load, provided the elastic limit of the spring is not exceeded; this is the reason why one checks the extensions with the decreasing loads.

**Table 1**

<table>
<thead>
<tr>
<th>Increasing load</th>
<th>Decreasing load</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total weight added</td>
<td>Cursor reading cm</td>
</tr>
<tr>
<td>Mg wt</td>
<td></td>
</tr>
</tbody>
</table>

to find whether permanent extension in the spring is produced. This precaution is always taken in experiments concerning elastic properties.

Plot the extension $E$ as ordinate and the load $M$ g wt as abscissa, and draw the line of best fit. It should pass through the origin. Deduce $\lambda$, the
TABLE 2

<table>
<thead>
<tr>
<th>Mass in pan g</th>
<th>Time for 20 oscillations ( t_1 ) sec</th>
<th>Check time ( t_2 ) sec</th>
<th>Recheck time if necessary ( t_3 ) sec</th>
<th>Average time ( t ) sec</th>
<th>Period ( T = t/20 ) sec</th>
<th>( T^2 ) sec²</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

reciprocal of the gradient, which is the force in gram weight per unit extension.

When the mass \( M \) attached to the spring is set into vertical oscillation then, at any point, if the extension of the spring is \( x \) cm from the position of equilibrium of the mass \( M \), there will be a restoring force of \( \lambda x \) g weight or \( \lambda x \) dynes. Then the restoring acceleration will be \( \lambda g/M \) cm sec\(^{-2}\). We have learnt from Experiment 19 that such motion is simple harmonic, and that the period \( T \) is \( 2\pi/\sqrt{\lambda g/M} \) or \( 2\pi \sqrt{M/\lambda g} \). Therefore \( T^2 = 4\pi^2(M/\lambda g) \). Plot \( T^2 \) against \( M \). You should get a straight line, the gradient of which \( m \) is \( 4\pi^2/\lambda g \). Hence \( g = 4\pi^2/\lambda m \). Deduce a value for \( g \) (which should be given correct to two significant figures).

Comments

Robert Hooke, who discovered the law of behaviour of springs, was as a young man employed by Robert Boyle, the discoverer of the behaviour of ‘the spring of the air’ (Boyle’s Law).

The importance of the spring is that it provides us with a definite force when stretched by a definite distance, so that equal forces can be applied, in turn, to different masses. The acceleration of each object will then be inversely proportional to its mass, and thus we have a means of measuring masses in terms of a standard mass. The last experiment in fact could be used to compare masses in this way, though the above experiment could not be used in a satellite where all objects are weightless. (We are rather spoilt by having a gravitational field on the earth’s surface, for it makes the comparison of masses easier, we simply have to compare ‘weights’, as on a beam balance, the weight being proportional to the mass with equal gravitational field strength.) In the absence of a gravitational field two fully-stretched springs are required, pulling in opposite directions on a ring to which the masses to be compared could be fastened in turn.

Supposing two springs similar to the one you used in the above experiment were involved, calculate the period of oscillation of 1 kg mass attached inside a satellite in the manner just described.
EXPERIMENT 21

The measurement of frictional forces opposing motion

To measure the frictional forces opposing motion when a solid slides over a solid

APPARATUS

The apparatus is as shown in Fig. 1, with at least two sets of 'shoes' (shown in inset) to give three types of sliding surface (the block can slide either on a wooden surface or a glass plate top). Sets of weights up to 4 kg, some polystyrene beads.

Fig. 1

METHOD

Using the spring balance find the weight of the slide, letting it hang freely from the fishing-line. Make a mark at the side of the long board, which should be level, so that the slide can be pulled forward from the same point each time. Wind on the handle and record the average force indicated by the balance as the slide is pulled along at uniform velocity. Repeat the operation a number of times, varying the number of weights carried on the slide and tabulating your results as shown in table overleaf. Vary the conditions by altering the 'shoes' or the surface on which the slide is pulled and repeat the above experiment. Finally let the slide be pulled resting on some polystyrene beads lying on top of the board.
Weight of slide and added weight \( W \text{ g wt} \) & Frictional force \( F \text{ g wt} \) \\

|     |     |     |

Theory and Calculation

You will have noted that the initial frictional force just before the slide is set into motion is higher than when the slide is in motion. In this experiment however we are only concerned with sliding friction, and you may have already noted that the frictional force is proportional to the total force pressing the surfaces together (normal reaction). Plot \( F \) against \( W \) (include the origin) and draw a line of best fit passing through the origin, deducing the gradient \( \mu_s \) (a pure number usually less than one) which is the coefficient of sliding friction for the two surfaces under test. You will note the pronounced drop in \( \mu_s \) when polystyrene beads are used. Does \( F \) in this case vary proportionately with \( W \) over the whole range of \( W = 0 \) to \( W = 4 \) kg wt?

To measure the frictional forces opposing motion when a solid moves in a fluid

Apparatus

The apparatus is as shown in Fig. 2, with a set of ball-bearings of equal diameter, stopwatch, powerful magnet to retrieve the ball-bearings, short length of glass tube, hydrometer, thermometer, micrometer screw gauge.

Method

Weigh a few ball-bearings, record their average diameter \( d \) cm and mass \( m \) g. Find also the specific gravity of the castor oil (using the hydrometer provided) and its temperature.

Holding the short glass tube vertically above the surface of the liquid, drop a ball-bearing through it so that it falls centrally into the liquid. Find the time \( t \) it takes to fall through the distance \( h \) marked by two black threads tied to the tube on the outside. Repeat this at least twice and take the mean of \( t \). Measure the distance \( h \), and take the temperature of the oil again.

Theory and Calculation

Motion of a solid in a fluid is usually complicated, but in the case of a sphere the frictional force \( F \) was shown by Stokes to be \( F = 6\pi a \eta v \) where \( a \) cm is the radius of the sphere, \( \eta \) is the coefficient of viscosity in poises and \( v \) cm sec\(^{-1} \) is the velocity. When the sphere reaches a constant velocity then \( F \) balances the weight of the sphere less the upthrust of the liquid on it.
i.e. \[ mg \left(1 - \frac{a}{\rho}\right) = 6\pi a \eta v \]

where \( \rho \) is the density of steel (which can easily be ascertained) and \( \sigma \) is the density of castor oil.

Hence, calculate the viscosity, \( \eta \), of the castor oil at the mean observed temperature (\( \eta \) is expressed in poises and calculated, here, to two significant figures).

**Comments**

In the first experiment does the area of contact affect \( \mu_a \)? How would you investigate this? Can you explain your results?

Stokes' Law was used by Millikan in determining the radius of small oil drops (see Experiment 62) falling through air and is only applicable when the motion is streamlined. Also note that \( \eta \) is very sensitive to temperature changes (see Experiment 74), and this is one of the major sources of error in this experiment.
EXPERIMENT 22

To study the efficiency of storing energy in a coiled spring and a rotating fly-wheel

APPARATUS

Gramophone motor mounted as shown in Fig. 1, holder and weights up to 2 kg, strong fishing line, rulers, wall-mounted heavy fly-wheel, pair of vernier callipers and a stopwatch. The apparatus is used as shown in Fig. 1(a). The vertical disc which replaces the winding handle has a grooved edge with a thread wound on it supporting the holder and weight.

Energy in a coiled spring

Start with the spring unwound and find the minimum weight \((W \text{ g})\) needed to make the vertical disc turn.

Now wind on the disc through a known number of complete revolutions and find \(W\) again; as you would expect \(W\) increases with the tension on the spring. Tabulate your results in the Table 1(a) as shown on following page. When the spring is almost fully wound up, turn over the box so that the spindle is horizontal and projecting over the table. Tie through the hole drilled into the spindle a length of fishing line and start the spindle rotating, lifting a known weight from the floor through a measured height (the weight must not be small or else it gains considerable kinetic energy which is not being measured). This is repeated with reduced weight if necessary until the spring is fully unwound, recording the weight \(w\ \text{g},\) and the height \(h\ \text{cm} \) lifted each time as shown (in the Table 1(b)). Measure the circumference \(c\ \text{cm} \) of the winding disc.
TABLE 1(a)  

<table>
<thead>
<tr>
<th>No. of turns</th>
<th>$W$ g wt</th>
<th>Work done in winding ergs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>$n_1 \left(\frac{0 + w_1}{2}\right)gc$</td>
</tr>
<tr>
<td>$n_1$</td>
<td>$w_1$</td>
<td>$n_2 \left(\frac{w_1 + w_2}{2}\right)gc$</td>
</tr>
<tr>
<td>$n_2$</td>
<td>$w_2$</td>
<td>$n_3 \left(\frac{w_2 + w_3}{2}\right)gc$</td>
</tr>
<tr>
<td>$n_3$</td>
<td>$w_3$</td>
<td>etc.</td>
</tr>
</tbody>
</table>

TABLE 1(b)  

<table>
<thead>
<tr>
<th>Height lifted $h$ cm</th>
<th>Weight lifted $w$ g</th>
<th>Ergs $kw$</th>
</tr>
</thead>
</table>

THEORY AND CALCULATION

Plot $W$ against the total number of turns and comment on your graph. Calculate the total work done in winding up the spring (the input) by adding the products of the number of turns, the circumference of the disc and the mean force in dynes (totalling column 3 in Table 1(a)). Find also the total output work from column 3 (Table 1(b)); and calculate the efficiency $\left(\frac{\text{output work}}{\text{input work}}\right)$.

Rotating flywheel

Tie to the axle of the wheel a length of fishing line attached to a 500 g weight $W$ so that the weight just touches a wooden board placed on the floor vertically below (Fig. 2). Wind the fishing line on to the axle evenly and note the highest position of the weight $h_1$ on a vertical ruler. Allow the weight to fall and find the time it takes to reach the board on the floor and note the highest position $h_2$ it ascends to as the fishing line gets wound on again. Repeat at least twice, and repeat with increasing weight, tabulating your results as in Table 2 overleaf.

Fig. 2

THEORY AND CALCULATION

The energy lost from friction at the axle and from stopping the weight $W$ is $Wg(h_1 - h_2)$, so that the rotational kinetic energy of the flywheel
when the weight descends is

\[ Wgh_1 - \frac{Wg(h_1 - h_2)}{2} = Wg\left(\frac{h_1 + h_2}{2}\right). \]

Complete column 6 (Table 2) and estimate the efficiency \(\frac{h_1 + h_2}{2h_1}\). Does it vary with \(W\)?

**TABLE 2**

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5 (v = \frac{2h_1}{t}) cm sec(^{-1})</th>
<th>6 (Wg\left(\frac{h_1 + h_2}{2}\right)) ergs</th>
<th>7 (\frac{1}{2}\omega^2 = \frac{1}{2}\left(\frac{4h_1}{td}\right)^2)</th>
<th>8 (I = \frac{\text{column 6}}{\text{column 7}}) g cm(^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(W) g</td>
<td>(h_1) cm</td>
<td>(h_2) cm</td>
<td>(t) sec</td>
<td>(v = \frac{2h_1}{t}) cm sec(^{-1})</td>
<td>(Wg\left(\frac{h_1 + h_2}{2}\right)) ergs</td>
<td>(\frac{1}{2}\omega^2 = \frac{1}{2}\left(\frac{4h_1}{td}\right)^2)</td>
<td>(I = \frac{\text{column 6}}{\text{column 7}}) g cm(^2)</td>
</tr>
</tbody>
</table>

**Comments**

Energy is often stored in systems, whether mechanical or electrical (for short or long periods); among the former are clock springs and flywheels and the latter, capacitors (for example, in electronic flash) and accumulators (where efficiency as high as 80% is claimed).

The rotational kinetic energy of the flywheel, analogous to the kinetic energy of translation \(\frac{1}{2}mv^2\), must depend on \(\frac{1}{2}\omega^2\) where \(\omega\) is the angular velocity (radian sec\(^{-1}\)). This can be estimated from the above experiment because if \(v\) is the velocity of the weight when reaching the floor, then \(\omega = 2v/d\) where \(d\) is the diameter of the axle, assuming no slipping takes place. If the weight accelerates uniformly as it descends then \(\frac{h_1}{t} = \frac{0 + v}{2}\) or \(v = \frac{2h_1}{t}\) and \(\frac{1}{2}\omega^2 = \frac{1}{2}\left(\frac{4h_1}{td}\right)^2\). Complete column 7 (Table 2) and you will note that the kinetic energy of rotation divided by \(\frac{1}{2}\omega^2\) is a constant (column 8). This constant plays the same part in rotational energy as inertia in translational energy, its dimension is g cm\(^2\) and it is called moment of inertia \((I)\). By finding the mass \(M\) of the flywheel show that \(I = \frac{1}{2}MR^2\).

If you time the ascent \((h_2\) cm\) of the weight, do you see any relation between the times of descent and ascent and the respective distances moved?
EXPERIMENT 23

The study of the use of the venturi meter and the verification of Bernoulli’s theorem

APPARATUS

Glass filter pump, simple mercury manometer (Fig. 1), small Woulfe’s bottle, glass T-piece, screw clip, short glass capillary tube, simple Rotameter, stopwatch, small venturi (for details see Fig. 2), Edwards’ RBF1 vacuum pump, water manometer reading up to 30 cm water pressure.

The use of the venturi meter

![Diagram of venturi meter]

Fig. 1

METHOD

Connect up the apparatus as shown in Fig. 1 and turn on the tap gently so that the mercury in the manometer is only about 1 cm above the level in the dish. Measure \( h \). Time the flow of water for 10 seconds, collecting the water in the beaker provided and measuring the volume of water using the measuring cylinder. Measure \( h \) and time the flow of water again for 10 seconds as a check. Record your results in the table overleaf. Increase the rate of flow and wait till conditions are steady (note the change in the appearance of the water jet and its sound as more air is evacuated from the vessel. When conditions are steady, not only is the level of the mercury in the manometer steady, but the water jet is clear and quiescent). Repeat the observations as carried out before, and continue until a maximum value, for \( h \), of about 40 cm is reached.
The Woulfe's bottle acts as a stabilizer and it isolates the manometer in case of a 'suck-back' due to sudden fall in the pressure. This fluctuation in pressure could be serious and further re-checks may be necessary.

**Theory and Calculation**

The filter pump sucks air out because of the reduction in pressure when the water flowing from the tap is forced into the narrow tube (visible through the glass), and is thus accelerated to a high velocity. Bernoulli's theorem states that in a streamlined non-viscous flow the total of the potential, kinetic and pressure energies of a fluid is a constant. If you ignore the changes in potential energy then \( \frac{1}{2}v^2 + (p/\rho) \) is a constant \( (K) \). If you plot \( v^2 \) against \( p \) you should get a straight line, but as the cross-sectional area of the throat of the narrow tube does not change during the experiment, then \( v^2 \propto Q^2 \), where \( Q \) is the volume of water flowing per second. Complete the table given above and plot \( Q^2 \) against \( h \) and you should get a straight line, proving that the higher the kinetic energy of 1 g of water flowing, the lower is the pressure.

**Verification of Bernoulli’s theorem**

**Method**

This is similar to the previous method except that air replaces water and also an attempt is made to verify the numerical values of Bernoulli’s theorem.

<table>
<thead>
<tr>
<th>Level of water on right</th>
<th>Level on left</th>
<th>Difference in levels</th>
<th>Rotameter reading</th>
<th>Volume of air ( Q ) litres per min</th>
<th>( Q^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_1 ) cm</td>
<td>( h_2 ) cm</td>
<td>( h ) cm</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The apparatus used is connected up as shown in Fig. 2. Suck air through the apparatus and measure its rate of flow (in litres per minute) with the rotameter, the pressure at the throat of the venturi being measured by a water manometer. Both the rotameter-float level and the manometer level fluctuate slightly, but mean values can be found and entered in the table at foot of the previous page. Alter the rate of flow by gently screwing down the clip and repeat as many times as possible. Measure carefully the diameter \( d \) cm of the throat of the venturi using a travelling microscope.

**Theory and Calculation**

The same Bernoulli theorem is used here again, \( \frac{1}{2}v^2 + (p/\rho) = K \) (constant) where \( p \) is the pressure of the air at the throat, \( \rho \) the density of air at the temperature of the experiment and \( v \) cm sec\(^{-1} \) the velocity of the air. Now from the graph provided with the rotameter read off the corresponding rate of flow of air \( Q \) in litres per minute for every float reading, and complete the table shown above. The area of the throat of the venturi is \( \pi d^2/4 \) and the velocity of the air
\[ v = \frac{1000Q \times 4}{60 \times \pi d^2} = \frac{200Q}{3\pi d^2} \text{ cm sec}^{-1}, \quad \rho = hg \text{ where } h \text{ is the manometer height.} \]

Therefore
\[ \frac{1}{2} \frac{(200)^2Q^2}{(3\pi d^2)^2} + \frac{hg}{\rho} = K \]

Plot \( Q^2 \) against \( h \) (remembering that \( h \) is negative) and you should get a straight line the gradient of which is \( \frac{9g\pi^2d^4}{2 \times 10^4\rho} \). Verify.

**Comments**

We have seen that the venturi meter, based on Bernoulli’s theorem, is a form of flow-meter; in fact the venturi which you have just calibrated in the previous method can be used in further experiments as a flow-meter. Calculate the velocity of air at the throat of the venturi using the relation \( v = \frac{200Q}{3\pi d^2} \) and also at the outlet of the pump where the diameter \( D \) is slightly larger, and should of course be measured. This then replaces \( d \) in the above relation.

Place a table tennis ball at the outlet of the pump and note its behaviour. Alter the speed of the air at the outlet (by screwing down the clip) and note any changes in the height of the ball. Make suitable assumptions about such behaviour and verify approximately your theory using the mass of the ball, its diameter, and any other data you may have acquired in the above experiment.

**EXPERIMENT 24**

**To study the behaviour of various wires or filaments when stretched**

**Apparatus**

Searle’s extensiometer (Fig. 1) with two steel wires (26 S.W.G.), at least two metres long, slotted weights up to 4 kg, rulers, micrometer screw gauge, two metres of bare copper wire (30 S.W.G.), long thin strip of balsa wood, pin, clamps, two metres of nylon fishing line (monofilament), microscope, stopwatch.

**Method**

Examine the wires for any visible kinks after hooking up suitable weights on to each hook. (The hook at the bottom of the experimental wire carries the weight-holder.) Adjust the screws until platform \( P \) is level. Ascertain the screw pitch (usually 0.5 mm) and note the number of divisions marked on the drum (if 50, then one division turned by the drum would represent
a vertical movement of \( \frac{1}{160} \) mm). Record the readings from the short vertical scale and the drum itself. Add 500 g to the stalk, re-adjust the screw and note the new reading; recording your observations as follows:

<table>
<thead>
<tr>
<th>Load kg wt</th>
<th>Increasing load readings</th>
<th>Decreasing load readings</th>
<th>Extension</th>
<th>Average extension</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Repeat until you have added all the weights provided, then decrease the load by 500 g each time, recording the readings as before. If, with zero load, readings do not agree closely, then the wire may have been stretched beyond its elastic limit, though it is more likely that the wire was not kink-free or the wire may have slipped from its chuck. In the event of such a disagreement the observations should be rejected and the experiment repeated. Measure the length \( l \) of the wire and its mean diameter (measured at right angles at different places).

Attach one end of the copper wire to an iron girder or any other firm support by winding several turns of the copper wire, and clamping it between two pieces of wood so as not to damage the wire. Tie the other end to the stalk and mark its position by means of the long narrow piece of balsa wood tapered to a point at one end and the other just touching the bottom of the stalk (Fig. 2). The position of the pointed end of the balsa lever is read from a vertically clamped metre rule. Now add 500 g and read the new position of the pointer. Remove the weight and check on the reading of the pointer.

Repeat the experiment several times, and find the mean displacement of the pointer. Find the length \( l \) of the wire and its mean diameter as before. Measure the arms \( a \) and \( b \) of the lever.

Now remove the lever and read the position of the bottom of the holder (alone) directly on the vertical rule now moved close to it. Add the 500 g and read the new position. Now increase the load carefully by steps of a 100 g each time, recording your observations as before. You will note after a time that \( (a) \) the extension of the wire does not come about immediately,
as it appeared to do with the steel wire, but tends to 'creep' slowly, (b) the
extension has suddenly become much larger with the same increasing load,
and, when all the load is removed, the wire does not regain its former
length. Continue the experiment until the wire breaks. Examine the broken ends
under the microscope, making sketches and recording any peculiarities you ob-
serve.

Replace the copper wire by the nylon monofilament, noting the position of the
stalk as before on a vertical rule placed close to the stalk. Add 500 g, timing the
position of the stalk every 15 seconds at first and then every minute. You will note
that the 'creep' is even more pronounced, and that when the load is removed, the
fibre does not regain its former length for possibly several hours. Now, starting with
the stalk alone, obtain a set of load/extension values, adding a 100 g weight at intervals of 2 minutes. Measure
the length of the filament and its diameter as before.

**THEORY AND CALCULATION**

Plot load (kg wt) against extension for all the three materials used. Deduce the gradients \( m \) (in the case of nylon, find the initial gradient) and from these calculate Young's modulus \( Y \) for each material.

\[
Y = \frac{\text{load (dynes)}}{\frac{\text{area of cross-section}}{\text{extension}}} = \frac{\text{load}}{\text{extension}} \times \frac{l}{\text{area}}.
\]

The gradient \( m \) is in kg wt per unit extension; converting this to dynes per unit extension and substituting in the above equation we get

\[
Y = m \times 1000 \times g \times \frac{l}{\text{area}} \text{ dyne cm}^{-2}.
\]

Give each answer correct to 2 significant figures. Deduce the breaking stress of copper (dynes cm\(^{-2}\)).

**COMMENTS**

How do the various elasticities compare? Have you noticed the 'plastic'
flow in the wire when it seems to slip, when loaded, past the yield point?
How would you explain this, particularly in the light of your observations
on the broken ends? How would the shape of the load/extension graph
differ from a stress/extension graph?

How does the load/extension diagram for nylon compare with that for steel?
EXPERIMENT 25

To find the surface tension of various liquids by (a) an absolute method, (b) a comparison method

Introduction

Few of the numerous methods used in measuring surface tension are absolute (give a value of the surface tension directly from the observations without reference to another liquid or specialized tables). The capillary rise method, where the liquid wets the tube, is an example of an absolute method, though for still higher accuracy the use of tables would be necessary. For the comparison method the 'drop' weight method is selected because (a) the principle is used in other experiments in the book, and (b) it is quick and simple to perform.

Apparatus

A suitable thin-walled glass tube from which a capillary can be drawn, a travelling microscope with six-inch objective, pin and holder, pipette, spray bulb, mercury, small beaker (lipless), watchglass, fish-tail Bunsen burner. Stainless steel hypodermic needle (no. 14 is suitable, the end filed off flat and smooth), cleaning fluids, small funnel and connecting alkathene tube, wetting agent (Johnson's 326).

Method

Heat the middle of the glass tube using the fish-tail Bunsen burner and when the glass is soft and red hot pull it out quickly. When cooled, break off a length of about 9 in from the middle portion, tie it to a clean glass rod at two places (to keep it straight) and dip it into the water-filled lipless beaker as shown in Fig. 1. Wet the inside of the capillary by lowering it well down into the beaker and then raising it; adjust the pin so that its tip is level with the top of the beaker and add, using the pipette, more water
until the tip of the pin just touches the surface, which should rise a little above the top of the beaker (owing to surface tension). Find the capillary rise $h$ (measured from bottom of the meniscus) using the travelling microscope with the six-inch objective (the field of view is sufficiently wide to include both the pin and the capillary). Raise or lower the capillary tube so that the meniscus occupies different parts of the capillary and repeat as many times as possible.

Dry the capillary by washing out with methylated spirits and blowing air through it, and then with the help of the spray bulb draw a small index of mercury into the length of the capillary which was previously occupied by the liquid. Measure the length of the index and the mass $M$ of the mercury using a watchglass.

Clean the hypodermic needle, attach it to the stem of the funnel and mount it over a dry, weighed beaker as shown in Fig. 2. Let the needle pass through a hole in a stiff paper lid, to prevent evaporation. Put water in the funnel and allow it to drip slowly from the lower end of the needle into the beaker. Collect 100 drops and re-weigh the beaker. Repeat the operation as a check.

Make up solutions by mixing known weights of the wetting agent and of water to give about 100 g of mixture. Use a little of each solution to rinse out the funnel and the hypodermic needle before repeating the above ‘drop weight’ experiment with that solution. Use solutions of increasing strength until the last set of observations is made with pure wetting agent. Tabulate your results as follows:

<table>
<thead>
<tr>
<th>Weight of wetting agent in 100 g mixture with water</th>
<th>Weight of 100 drops of mixture</th>
<th>Check</th>
<th>Average weight of a single drop</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Finally find the ‘drop weight’ for the pure wetting agent.

**Theory and Calculation**

The surface tension $S$ can be calculated from the capillary rise $h$ and radius $R$, $S = \frac{R \rho g}{2} \left( h + \frac{R}{3} \right)$ dynes cm$^{-1}$ ($\rho$ being the density of the liquid).

Now $R$ is found from the mass of the index and its length $l$ cm. $M = 13.6 \pi R^2 l$ where the density of mercury is 13.6 g/ml.

Give the answer correct to two significant figures.

For the ‘drop weight’ method, if $w_1$ is the ‘drop weight’ when pure water was used and $w_2$ the ‘drop weight’ when the wetting agent was
added then \( \frac{S_1}{S_2} = \frac{w_1}{w_2} \) where \( S_1 \) and \( S_2 \) are the surface tensions respectively.

Plot a graph connecting the weight of wetting agent added and the resulting surface tension and comment on your results.

**Comments**

Surface tension is a striking example of molecular attraction (calculate the force required to separate two sheets of glass held together by a film of water only 0.001 mm thick). Besides the inter-molecular attraction in the liquid (called cohesion) there are the attractions between the molecules of the liquid and (a) the molecules of the air, (b) the molecules of the glass (called adhesion). The relative strengths of these three types of forces decide what is called the angle of contact; for water this is virtually zero (i.e. it wets the glass), but for turpentine it is not. Use the apparatus shown in Fig. 3 to find the angle of contact between turpentine (not substitute) and glass. Fill the glass beaker to the brim, vary the tilt of the glass slide, so that on viewing the level of the liquid with the travelling microscope it is parallel to the horizontal crosswire right up to the glass slide as shown in Fig. 3. Use any two points on the side of the slide as marks A and B and find the horizontal and vertical displacements \( x \) and \( y \) respectively between them, the angle of contact \( \theta = \tan^{-1} \left( \frac{y}{x} \right) \) to the nearest degree.

Use the capillary rise method to find the surface tension of turpentine,

\[ S = \frac{Rg\rho h}{2 \cos \theta} \]

Variation of surface tension with temperature can be investigated using Jaeger’s method. The apparatus for this is shown in Fig. 4. The clean piece
of capillary tube must be carefully dipped into the liquid in the beaker and care taken to maintain its lower end at the same distance from the surface —even when that surface rises through expansion of the liquid on heating. Allow water to drop slowly from the tap funnel D. This will cause the formation of an air-bubble on the lower end of C. Allow about two minutes for bubble formation, reading the highest difference in levels observed on the manometer. This difference $h$ is a measure of the surface tension of the liquid in the beaker. Raise the temperature of the liquid and find the corresponding values of $h$. Comment on your results in the light of the kinetic theory of matter.

The effect of the wetting agent, like that of a detergent, is to lower the surface tension and thereby alter also the angle of contact. Explain how it is used in photography in rapid drying of films without leaving drying marks, and how a detergent alters the surface tension of water so that fatty globules of dirt roll off the fibres of a fabric and so clean it.

**EXPERIMENT 26**

**To find the refractive index by the critical angle method**

**Apparatus**

Semi-circular glass block, a quadrant of which is painted black as shown in Fig. 1, ground-glass sheet with black line drawn across it, transparent ruler, clamp and stand, hollow glass cube, air-cell, elbow telescope, 'light pin' (straight filament in tubular glass envelope, 6–12 V), collimating lens, filters, various liquids.
**Method**

First, clean the flat surface of the block which will rest on the sheet when the block is in position, as in Fig. 1. Place the block on the sheet so that the marked line coincides with the boundary of the painted quadrant, as in Fig. 1. On looking in the direction of the arrow, through the curved surface of the block, you will see in the bottom part of the field of view the internally reflected image of the edge of the blackened part of the block. The boundary appears slightly coloured and, if looked at from a sufficiently distant point, sharp. Clamp the transparent ruler vertically about 15 cm away from the block, and estimate to the nearest millimetre the position on the ruler which marks the point where the dark boundary exactly coincides with the line on the ground-glass sheet. Repeat twice using the ruler at different distances from the block, tabulating your results as follows:

<table>
<thead>
<tr>
<th>Horizontal distance $x$ cm</th>
<th>Vertical distance $y$ cm</th>
<th>$\tan \theta = \frac{x}{y}$</th>
<th>$\theta$</th>
<th>$\csc \theta = \mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Smear a drop of water on to the face of the block in contact with the plate and repeat the experiment, noting that the value of $y$ is consistently lower than before, and tabulating your results in the same table. Repeat with another liquid, say paraffin.

**Theory and Calculation**

When the boundary appears to coincide with the mark (which is clearly set at the centre of the semi-circular glass block), then OP (Fig. 1) represents the path of a critically refracted ray of light after falling at grazing incidence on the flat diameter of the semi-circle. Thus the angle POR is the critical angle $\theta$, which is connected with the refractive index by the relation $\mu = \frac{1}{\sin \theta} = \csc \theta$. Now complete your tabulation and verify that the values of $\csc \theta$ agree within experimental error. Average in each case.

When a liquid is used, $\csc \theta$ is the refractive index between glass and liquid, i.e. $\mu_0/\mu_L$, therefore $\mu_L = \mu_0 \sin \theta$, where $\theta$ is the critical angle POR in each case. Give the answer for each $\mu$ correct to two decimal places.

**Comments**

The above method is only suitable when the refractive index of the liquid under test is less than that of glass. To measure the critical angle from
liquid to air, you need to form an air-film within the liquid which clearly, owing to the properties of liquids, can only be done by inserting into the liquid two parallel sheets of glass separated by a thin film of air, and sealed tightly. Fortunately—as can be shown—the presence of the glass does not affect the critical angle and you can use the arrangement shown in Fig. 2.

![Fig. 2](image)

The lens is arranged to produce a parallel beam of light and the elbow telescope focused to give a sharp image of the filament. A suitable liquid of high dispersive power can be used, making sure that it does not attack the cement of the glass cube or the sealing of the air-cell.

Rotate the plane of the air-cell and determine accurately (to a tenth of a degree) the two positions at which the image of the filament is just extinguished. Note the change in colour of the image due to the difference in refractive index for different colours and use red and blue filters to enable you to determine the critical angle for each of the two bands of colour. In each case, measure the angle $2\theta$ as shown, and deduce the angle $\theta$, hence calculating the refractive index. The refractive index for yellow light can be taken to be the mean of the two refractive indices, i.e. $\mu_{\text{yellow}} = \frac{\mu_{\text{red}} + \mu_{\text{blue}}}{2}$. Now deduce the dispersive power of the liquid equals $\frac{\mu_{\text{blue}} - \mu_{\text{red}}}{\mu_{\text{yellow}} - 1}$.

The difficulty with this method lies in the measurement of the angle, and the spectrometer can be adapted, using the collimator and the telescope in place of the lamp and elbow telescope. The air-cell is held centrally above the turntable and rotates with it. The hollow glass block is supported a few millimetres above the turntable, so that it does not turn with it.
EXPERIMENT 27

The determination of the focal length of a converging lens by the magnification method and an elementary study of the effect of a stop on the lens

APPARATUS

Converging lens (about 10 cm focal length, diameter 5 cm), lens holder, two drawing board clips, two metre rules, reflecting mirror, plastic translucent focusing screen ruled with millimetre lines, illuminated object with provision for either fine cross-wire or triangular gauze, magnifying glass, circular cardboard with circles of different radii cut out, their centres lying on the circumference of a circle concentric with the card itself (suitable diameters of holes are 5 cm, 4 cm, 3 cm, 2 cm, 1 cm and 0.5 cm). The card is supported and rotates on a pin, which is clamped so that the centre of each of the circular holes coincides with the centre of the lens (see Fig. 2).

METHOD

Determine the approximate focal length of the converging lens by focusing a distant object (say a window) on the screen. Clamp one of the metre rules to the bench, using the drawing board clip provided. Use the illuminated crosswire as an object. Bring the mounted lens close to it and centre it. Place the screen at a distance from the object, roughly four times the approximate focal length you have found. Adjust the position of the lens to give a sharply focused image on the screen. There are always two positions of the lens which will give a sharp image for any one object-image distance (greater than 4f), one image thus obtained being magnified, the other diminished.

Measure the diameter of each image vertically and horizontally, also the distance between the screen and the lens holder (it is not necessary to measure the actual image distance, as long as measurements are taken from the same points on the lens and the screen each time). Measure the diameter of the object in the two corresponding directions a and b, and tabulate your results as follows:

<table>
<thead>
<tr>
<th>Distance between lens holder and screen Y cm</th>
<th>Vertical diameter of image (a_1) cm</th>
<th>Horizontal diameter of image (b_1) cm</th>
<th>Vertical magnification of image (m_1 = a_1/a)</th>
<th>Horizontal magnification of image (m_2 = b_1/b)</th>
<th>Mean magnification (m = \frac{(m_1 + m_2)}{2})</th>
</tr>
</thead>
</table>
Repeat five times, increasing the object-image distance to $6f$, so that you have altogether twelve observations.

**Theory and Calculation**

The magnification formula is $1 + m = v/f$ where $m$ is the linear magnification $\frac{\text{image height}}{\text{object height}}$, $v$ is the image distance and $f$ the focal length of the lens. If $m$ is plotted as an ordinate and $v$ as abscissa, then one gets a straight line, the gradient of which is $1/f$. In the above experiment the distance between the screen and the lens holder differs from $v$ by a fixed quantity $a$, so that the effect of plotting $m$ against $Y (= v + a)$ is merely to displace the straight line parallel to itself, without altering the slope. Plot all the twelve points, draw the line of best fit and deduce $f$, giving it to three significant figures.

**Comments**

The advantage of the above method is that neither object nor image distance need be known, so that the focal length of a compound lens system can be determined by this method (see also Experiment 7). You will note from your table above, that for any object-image distance the magnifications produced by the two settings of the lens, $m$ and $M$ ($M > 1$, $m < 1$), are such that $m.M = 1$. This can easily be shown, as object and image are conjugate points, and the object and image distances are interchangeable. If $a_1$ and $a_2$ are the heights of the images produced by the two settings of the lens, then $m.M = (a_1/a_1). (a_2/a_2) = 1$ and therefore $a = \sqrt{a_1a_2}$.

This gives an elegant method for determining the size of an inaccessible object, say a filament or a virtual image. Use the lens provided to find the length of the straight filament of a 6 V clear bulb, using this as an object and the ruled transparent screen.

You may have noticed some of the defects of the image cast by the lens. One of the reasons is the fact that rays falling at the periphery of the lens do not focus at the same point as those falling on the centre of the lens (see Experiment 7). This is especially the case with the lens provided in the experiment, where the diameter or aperture of the lens is almost half the focal length, and therefore the angle of incidence (or the angle of refraction) is comparatively large. One of the solutions to the problem is to reduce the aperture of the lens by using a stop (in fact the judicious use of a stop in a strategic position in an optical instrument can improve its performance considerably). To study the consequences of the use of a stop use the arrangement shown in Fig. 2.

Use the two rulers provided as guides for the light-box (with triangular gauze as object) which should be placed about 1 metre from the lens. The reflecting mirror is used to cast an image on the screen, the latter being clamped horizontally and viewed with a magnifying glass. It is best to carry out this experiment with a partner and in a darkened room. Starting with the widest aperture, move the lens to focus the image sharply on the screen. When this has been done, neither lens nor screen should be moved. Place the cardboard diaphragm close to but not touching the lens. Move the light-box forward gradually and then backward, and you will note that the
sharpness of the image is not seriously affected over a certain range of distance, which should be recorded and tabulated. Turn the cardboard disc to bring the next smaller hole into line, and you should note immediately the improvement in the quality of the image produced. Find the new range of positions of the illuminated object giving a reasonably sharp image on the screen. This you will find distinctly greater than with the larger hole. Repeat with the other four smaller apertures. The effect of reducing the aperture is to increase what is termed the depth of focus, which enables a camera to take a tolerably sharp picture of object at different distances from the lens, though at the expense of the reduction of the brightness (which you should have observed). With the smallest aperture you have used, it is possible to focus clearly the images of objects at distances from a few feet to infinity, without altering the distance between lens and film. This is the principle of box cameras and fixed focus ciné-cameras.

In photography the diameter of the aperture (the adjustable iris) is usually expressed as a fraction of the focal length, say f/11, so that different lenses with the same f number will give equal brightnesses on the film, and for each aperture the depth of focus is usually indicated. For lenses of short focal length (5 cm or less) the f number of the smallest stop is never greater than f/16, suggesting that there is a limit to the smallness of the size of the diaphragm, consistent with sharp focus of the image—why?

**EXPERIMENT 28**

**To determine the focal length of a diverging lens and to study some of its uses**

**APPARATUS**

Diverging lens (focal length about 10 cm), two converging lenses (focal lengths 20 cm and 50 cm), three lens holders, two metre rules, illuminated
object (a light box with triangular object and cross-wire), screen, knitting needle, small transparent screen (ruled in millimetres) and holder, plasticine.

**Method**

Put the diverging lens and the more converging of the other two lenses together and hold them above some writing; if the image seen through them is diminished, then the combined focal length of the two is negative and they are diverging. Had the combined lens system been converging, then the self-conjugate method discussed in Experiment 7 could have been used to find the combined focal length $F$ and also the focal length of the converging lens $f_1$, and hence $f_2$ the focal length of the diverging lens (since $1/F = 1/f_1 + 1/f_2$).

The method discussed here is equally suitable whether $f_1$ is greater or less than $f_2$. The converging lens, bedded in plasticine and centred in its holder, is first used to form a real but diminished image of the illuminated object. Read off the position of the screen O on the scale. Insert the diverging lens, accurately mounted, between the converging lens and the screen, a few centimetres from the screen. The image on the screen becomes blurred, and the screen must then be moved farther away to find the new position of the image (see Fig. 1a). Now record the new position of the image I, and the position of the diverging lens M, though the latter does not necessarily correspond to the optical centre of the lens; in order to find the correct position of M place the screen on either side of the lens (see Fig. 1b) so that the knitting needle touches each surface of the lens in turn and the screen. The corrected position of M is halfway between the two positions of the screen, and this is compared with the actual reading made at the base of the holder, and a suitable correction $\alpha$ (usually small and possibly positive or negative) is recorded. The observations are entered in the table on facing page. Repeat the observation five times, increasing the value of $v$ each time by moving the diverging lens closer to the converging lens.

**Theory and Calculation**

The converging lens provides a virtual object for the diverging lens so that the final image formed by the latter is real. Thus, in the table below, $u$ is negative while $v$ is positive, and if you plot $1/v$ against $1/u$ you should
get a straight line making negative intercepts on the two axes, each equal to $1/f$. Deduce $f$ (you can check this by combining the diverging lens with a more powerful converging lens, as discussed at the beginning of the method).

**Comments**

The diverging lens is often used to balance out the defects of a converging lens, as in achromats (or colour-corrected lenses) in telescopes, cameras, microscopes, etc.

To illustrate some of its further uses, use the converging lens with the longer focal length (lens 1) to form a real image of a distant object (say a window) using the transparent focusing screen provided. Measure the height of the image and its distance from the lens, which is approximately equal to the focal length $f$. Now replace this lens with the other converging lens (lens 2) and focus again. Note the smaller size of the image—which you should record—as well as the approximate focal length $f$. If these two lenses were actual camera lenses, it is clear that the use of the one with the longer focal length gives a larger image on the film, so that greater detail can be obtained from the negative, but the disadvantage is having to mount and carry a lens such a long distance from the film. Now return to lens 2, place it at a distance roughly three-quarters of $f$ from the screen and then insert the diverging lens (Fig. 2), moving it along to form a real

![Fig. 2](image_url)
ray diagram of Fig. 2 shows this diagrammatically). This is the principle of the telephoto lens, useful in capturing greater detail of distant objects, where a 'standard' lens would be less successful.

Referring back to Fig. 2, as lens 2 is brought nearer to L, O moves farther away, and when the virtual object distance for the diverging lens L equals its own focal length the rays emerge parallel (Fig. 3). Verify this. It is best to use lens 1 instead of lens 2 as the former gives you a higher magnification. Place your eye close to the diverging lens, as the field of view is very small (in fact the exit pupil is virtual, see Experiment 30); you should see an erect image. This is the principle of the opera glass, where compactness, cheapness, adequate magnification (and narrow field of view) could be reconciled. It is believed to have been first made by the Dutch (the 'Dutch spyglass') but Galileo improved on it, much to the advancement of observational astronomy. It is now called after him.

EXPERIMENT 29
The determination of the focal length of concave and convex mirrors and the study of some of their applications

Apparatus
Laminated semi-silvered concave mirror (focal length about 20 cm), $2\frac{1}{2}$ in square glass slide set at 45° on a slotted square piece of wood as shown in Fig. 1, white screen, knitting needle, two metre rules, light box with the illuminated object in the form of a triangle and a cross wire, mirror holder, magnifying glass (focal length 2.5 cm) mounted in a similar holder. Convex mirror (focal length 10 cm) with only the centre silvered (in a circle 1 cm diameter), the rest clear glass, a concave mirror (focal length 20 cm) with a central circular patch (1 cm diameter) unsilvered, plasticine.

The concave mirror

Method
Find the approximate focal length of the semi-silvered concave mirror by focusing a distant object (say a window) on the screen. Next arrange a simple optical bench as shown in Fig. 1, the glass slide reflecting the light from the illuminated object on to the mirror. With the help of some plasticine centre the mirror accurately in its holder and stick the holder firmly on to the ruler.

The position of the pole of the mirror on the metre scale can be found with the help of the knitting needle. Place the screen in front of and behind
the mirror with the knitting needle just touching each of the two surfaces of the mirror and the screen (Fig. 2). Read off both positions of the screen and find their mean. This should be the position of the pole of the mirror M on the scale of the ruler. To find the position of the object in relation to the ruler, move the glass reflector and the light box (placed at right angles to the ruler as shown in Fig. 1) till a reflected image is formed on the illuminated object, showing that the object and image coincide and that the distance of the object from the mirror is equal to the radius of curvature \((2f)\). Place the screen on the ruler so that a sharp image is formed on it also and read off its position, which should coincide with the scale reading at the far side of the square base of the glass reflector (a small correction may be necessary). It is important that once this is done the distance between the light box and its reflector should not be changed.

Now move the object and its reflector a little nearer and find the position
of the image formed on the screen (which should be moved farther away). Read off the correct position of the object O and the position of the screen I and complete the following table:

<table>
<thead>
<tr>
<th>Position of M</th>
<th>Corrected position of object O</th>
<th>Position of image I</th>
<th>Object distance MO u cm</th>
<th>Image distance IO v cm</th>
<th>$1/u$ cm$^{-1}$</th>
<th>$1/v$ cm$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Remains unchanged during experiment</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Repeat four times, reducing $u$ each time.

Now move the object so that it is inside the focus ($u < f$), and the image is now erect, magnified and virtual. You can locate the image by placing the screen (illuminated if necessary) behind the mirror, as in Fig. 1, and looking in the mirror. You will see not only the erect image but the illuminated screen behind the semi-silvered mirror. Hook a fine vertical wire over the screen and use it to determine by the method of no-parallax the position of the virtual image. This should be repeated with a different value of $u$ at least once, entering the observation as before in the above table, except that $v$ here is negative.

**Theory and Calculation**

Plot $1/u$ against $1/v$. You should get a straight line, the intercepts of which on the axes should each be equal to $1/f$. Deduce $f$.

Now place the glass sheet reflector with one of the sides of its square base coinciding with the principal focus of the mirror so that a real image of a distant object (say a window) is formed on a plane coinciding with one of the sides of the square base (see Fig. 3). Place the magnifying glass so as to focus on that image. You now have a model of the reflecting telescope, first designed by Newton and called after him.

**The convex mirror**

**Method**

Place the concave mirror with the unsilvered patch approximately halfway along the two metre rules, and place the illuminated object (for this experiment the object should be very well illuminated and the experiment if possible carried out in a dark room) facing the mirror, at the end of the ruler (Fig. 4).
Stand the screen on its side so that only half of the mirror is illuminated, and move it along until you get a sharp image of the object. Centre the mirror so that the image lies on the axis before taking a measurement. Read off the position of the screen on the scale. Repeat this measurement for a check, as it is used in all subsequent observations.

Keeping the positions of the concave mirror and the illuminated object fixed, place the convex mirror in its holder a few centimetres nearer to the mirror than the image. Remove the screen and place it behind the concave mirror, quite close to it. Adjust the position of the convex mirror to give a sharp inverted image on the screen. Read off the position of the screen, the image distance \( I \), and the position of the convex mirror. To get the corrected position of the convex mirror (as all distances are measured from the back of it), place the screen between the illuminated object and the back of the mirror, so that the knitting needle touches the back of the mirror and the screen. Read off the position of the screen and add to it the length of the needle and this gives you the corrected position of the convex mirror \( M \). In order to avoid repeating this every time, deduce the necessary correction (i.e. the difference between the corrected observation and the reading of the mirror holder on the scale, which could be negative or positive). The tabulation for this experiment is the same as before, except that the correction must be made to \( M \) each time. Repeat the observation five times, each time moving the screen farther away from the mirror, increasing the image distance \( Mf \) as well as the magnification. Focus by moving the convex mirror carefully, as these movements are only in the order of a few millimetres each time. Another difficulty is that the magnified image becomes less bright and this is when the brightness of the illumination of the object becomes most important.

**Theory and Calculation**

In this experiment the object for the convex mirror is virtual and therefore the object distance \( u \) is negative. Plot \( 1/u \) against \( 1/p \) as before and you should get a straight line, cutting both axes to give negative intercepts, each equal to \( 1/f \). Deduce \( f \). Check this value by placing the convex mirror so that light from the illuminated object is reflected off the unsilvered back of the mirror, and image and object coincide. As the back of the mirror acts as a concave mirror, the distance between the pole of the mirror and the object then being the radius of curvature, or twice the focal length \( (R = 2f) \). Hence \( f \) may be found.
Now move the mirror back to the position it occupied for the very first observation. If the concave mirror is now turned toward a distant object (say a window) a real image is formed almost at the pole of the concave mirror. Place the magnifying glass to view this image and you have another type of reflecting telescope, the Cassegrain telescope (Fig. 5), which has the advantage of compactness (telescopes of this type with high magnifications weigh only a few pounds), and is also useful for spectroscopic and photographic work. Finally, the imperfections in the two mirrors used tend to cancel each other out.

Fig. 5

Comments

The models of the telescopes described above suffer from spherical aberration, as not all the rays from a parallel pencil of light falling on the mirror (objective) come to a sharp point focus. In practice parabolic mirrors replace the spherical mirrors and hyperbolic mirrors replace the convex mirrors. These telescopes have a great advantage over other types because of their freedom from colour distortion (chromatic aberration), the laws of reflection being the same for all colours. The objective mirrors can be supported at the back as well as at the rim. All large diameter telescopes are of this type.

EXPERIMENT 30

The study of the simple astronomical telescope and the microscope, and the determination of their magnifying powers

Apparatus

Three converging lenses of 50 cm, 5 cm and 2.5 cm focal lengths, fixed with Terry clips to small wooden blocks, small focusing screen ruled in millimetres, small white millimetre scale, small opaque strip with circular hole (exit pupil), small split-field periscope made up of a small mirror inclined at 45° and another similar parallel mirror, half of which only is silvered (Fig. 1). Each of the above is equipped with a small Terry clip which can be clipped firmly on to a stout 70 cm rod which in turn can be clamped horizontally in a standard boss and stand. Mains lamp (pearl) and holder, two half-metre rules, wooden stand.

The astronomical telescope

Method

Arrange the experimental telescope as shown in Fig. 1 with the objective A (the lens with the longest focal length) clipped on right at the end of the
rod and place the eye-lens B at a distance greater than the focal length of the objective away. Focus on a distant object (say the wall of a building or ruled equidistant parallel lines on a board) by moving the eye-lens until the image appears sharply defined. Without disturbing the lenses illuminate the objective with the pearl lamp and locate the image of the objective, by moving the focusing screen which also clips on to the rod close to the eye-lens, but on the opposite side to the objective. Read off the diameter of the image of the objective from the millimetre scale on the screen.

![Fig. 1](image)

Now replace the focusing screen by the perforated metal plate so that the hole coincides with the previous position of the exit pupil (i.e. the image of the objective). Place the small split-field periscope between the exit pupil and the eye-lens. Still without disturbing the lenses, turn the telescope back to its former position and view the distant object through the exit pupil. You will see the magnified image side by side with the reflected image of the object, and you should be able to count the number of lines (or courses of bricks) on the object corresponding to one magnified line (or brick width) on the image. Record this observation, the magnifying power of the telescope in normal adjustment. Measure the distance between the objective and the eye-lens. Now remove the lenses and find their focal lengths by the plane mirror method described in Experiment 7. Measure the diameter of the objective.

**Theory and Calculation**

The magnifying power of the telescope, focused on a distant object so that the final image appears to be at infinity (i.e. in normal adjustment), is \( F/f \) where \( F \) is the focal length of the objective and \( f \) the focal length of the eye-lens. Check that this is the case. Also check that the distance between the lenses in normal adjustment is the sum of their focal lengths. The magnifying power can also be accurately calculated as the ratio of the
diameter of the objective to that of the exit pupil. This relation can be verified theoretically from the ray diagram (Fig. 2).

The exit pupil rqp is clearly the image of PQR formed by the eye-lens, as two rays from each of the points P, Q and R intersect after refraction at p, q and r respectively. It is also clear that rr’p’p is a rectangle so that r’q’p’ = rqp.

As the triangles Or’p’ and OPR are similar, therefore

\[ \frac{PR}{R’P’} = \frac{F}{f} = \frac{PR}{rp} \]

Verify from your results that this is so.

**The microscope**

**Method**

Place the screen at a distance of about 25 cm from one end of the rod (Fig. 3) and place close to it the small objective (the lens with the smallest focal length). Adjust the distance between the lens and the screen, so that on looking through the lens a large magnified image of the screen is observed. Locate by the method of no-parallax (see Experiment 7) the position of this image, using one of the 50 cm rulers clamped vertically to a stand and moved along beside the rod. The setting should be such that the image distance is approximately four or five times the object distance. Measure accurately the distances between the lens and the screen and the lens and the ruler. Record your observations. Without disturbing anything, clip the eye-lens (the same one you used with the telescope) on the opposite side of the vertical ruler and move the lens along the rod until a clear image of the ruler is obtained, as well as a clear image of the screen. The ruler can now be removed after recording the distance between it and the eye-lens. Again without disturbing either of the two lenses or the object screen, illuminate the objective and find the exit pupil as before. The object screen will not hinder you, as you can swivel it, without altering its position, through 90°. Place the perforated screen at the exit pupil and
swivel the object screen back to its former position. Place the split-field periscope between the exit pupil and the eye-lens as before. Put the vertical 50 cm ruler close to the rod once more so that it is visible in one half of the periscope's field of view. You should now see both images side by side and you can move the ruler until there is no parallax. Read off and record the magnifying power as before.
Remove the objective and exit pupil (i.e. the perforated plate) and place the object screen at the position previously occupied by its image (i.e. at the first position of the vertical ruler, the distance of which from the lens you have recorded). Looking through the periscope (Fig. 4) now move the ruler until no parallax is again obtained, the ruler in this position being about 25 cm from the eye. Read off the magnifying power of the eye-lens and record it.

**Theory and Calculation**

The overall magnifying power of the microscope is the magnification of the objective $m$ multiplied by the magnifying power of the eye-lens $\mu$. You have already recorded the distances of the object screen and the first image from the objective lens, so that the magnification of the objective is the ratio of these two distances, i.e. $\frac{\text{image distance}}{\text{object distance}}$. The magnifying power of the eye-lens has been found directly in the last part of the experiment, and you have also recorded the overall magnifying power of the microscope. Verify that the overall magnifying power is $m \times \mu$.

**Comments**

The magnifying power of the telescope when used to focus on an object say only 3 yards away can be found using the same method, and can be compared with that calculated from the knowledge of the focal lengths and the distance between the object and the objective. Note that as the distance between the lenses increases, so does the magnifying power.

The exit pupil, as Fig. 2 shows, is the place where the emergent rays are crowded together and is therefore a suitable position at which to place the eye, as it will give a maximum field of view. (Bespectacled persons get less of a 'look-in'!) The field of view could be further increased by placing a fat converging lens (5 cm focal length, 5 cm diameter) between the objective and the eye-piece at the position of the image of the object produced by the objective. This is one of the reasons why in practice the eye-lens is replaced by two or more lenses to form a compound eye-piece.

In the case of the practical microscope, the magnifying power can be increased by pulling out the draw-tube, thus increasing the magnification produced by the objective, though in practice this is done at the expense of the quality of the image. To determine the magnifying power of an actual microscope, the magnification of the objective is very nearly $v/f_o$ as $v$ is usually several times the object distance and therefore $u$ is very nearly equal to $f_o$. The magnifying power of the eye-piece is usually marked on it, say $\times 10$. 
EXPERIMENT 31

To set up and use a spectrometer (a) to examine various spectra, (b) to investigate the relationship between the refractive index of glass and the wavelength \( \lambda \) of light in air

APPARATUS

Spectrometer and 60° prism, sodium lamp, mercury vapour lamp, incandescent light bulb and supply, various Wratten filters. Various salts, platinum wire and holder, hydrochloric acid, Bunsen burner.

METHOD

No details will be given at this stage for levelling the turntable or aligning the axes of the instrument. Get to know the clamping screws and the fine adjustment for the turntable and the telescope, then focus the eye-piece on the cross wire. Focus the telescope at a distant object, then align the collimator and the telescope, illuminating the slit, which should be fairly wide, with sodium light. Focus the collimator. The spectrometer is now set for use.

Place the prism with its refracting edge facing the collimator and without using the telescope look for the reflected images of the slit at both the faces of the prism AB and AC, or else rotate the turntable slowly until they are visible. Clamp the turntable and with the telescope look for these images, which should appear vertical and bisected by the horizontal cross wire in your field of view if the spectrometer has been properly adjusted. Narrow the slit, clamp the telescope and use the fine adjustment to locate the position of each reflected image. Record your readings, and deduce the difference.

Unclamp the turntable, rotating it so that the light from the collimator falls obliquely on one of the faces of the prism, and locate, first without and then with the telescope, the refracted image. Now rotate the turntable slowly in such a direction that the refracted image appears to move closer to the axis of the collimator (i.e. decreasing the deviation), and follow it up with the telescope until the image appears to turn back. Clamp the turntable and use the fine adjustment screw to hold the image at the point when it begins to turn back (i.e. minimum deviation). Clamp the telescope and locate as before and record the position of the image. Remove the prism without disturbing the turntable, align the telescope with the collimator and record the position of the telescope. From the two readings deduce the angle of deviation \( \Delta \). Repeat on the other side of the collimator and average \( \overline{\Delta} \).

Set the prism again at the point of minimum deviation and without
disturbing the telescope rotate the turntable by 5° exactly. You will note that the angle of deviation has increased, and as the image is no longer on the cross wire, record this increase. Rotate the turntable back through 5° and rotate the telescope back to its former position. Repeat this observation, this time rotating the turntable through 5° in the opposite direction, also through 2° in both directions, recording the increases in deviation in each case and tabulating your results as follows (let \( i \) be the angle of incidence of the light when the prism is set for minimum deviation):

<table>
<thead>
<tr>
<th>Angle of incidence</th>
<th>( i + 5 )</th>
<th>( i + 2 )</th>
<th>( i - 5 )</th>
<th>( i - 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Increase in deviation ( \delta D )</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Examine with the spectrometer the spectra of the mercury discharge lamp, of various salts when heated in the Bunsen flame (dipping the platinum wire each time in hydrochloric acid beforehand), and of the incandescent lamp with and without the various filters between lamp and slit (do not let the filters become heated by the lamp). Give a descriptive account of your observations.

**Theory and Calculation**

The angle of the prism \( A \) is half the angle between the two directions of the reflected light off faces AB and AC. The refractive index of the prism for sodium light \( \mu_D = \frac{\sin (A + D)/2}{\sin A/2} \). Deduce \( \mu_D \) to 3 decimal places.

For minimum deviation the angle of refraction, from symmetry, is \( A/2 \), hence \( i = \mu_D \sin A/2 \). Deduce \( i \), complete the above table and plot the angle of deviation against the angle of incidence. Does the curve look symmetrical?

**Comments**

The curve you have just plotted gives a clue as to why the prism is set for minimum deviation, because a small change in the angle of incidence makes no difference to the angle of deviation. The slit itself has a finite width, so that pencils of light from different points across the slit fall with slightly different angles of incidence on the prism and the image appears at its narrowest when the prism is set for minimum deviation (cf. the image when the angle of deviation is large). It is also desirable to let light of different wavelengths suffer the same deviation and this is very nearly the case if the prism is set for minimum deviation for an intermediate wavelength.

Find the angle of deviation for each of the seven prominent lines of the mercury vapour lamp (Fig. 2). Estimate roughly the refractive index for the line in the extreme violet, 4047 Å, using \( \mu = \frac{\sin (A + D)/2}{\sin A/2} \), and this will give you some idea of the
range of $\mu$. If $\mu_D$ is, say, 1.61, calculate the angle of deviation after two refractions, say for $\mu = 1.60, 1.61, 1.62, 1.63$ and 1.64, as $i$ and $A$ are the same for all of them. Plot $\mu$ against deviation and use the values of the deviation you have already observed to deduce from the graph the corresponding values of the refractive indices.

The fact that, in general, $\mu$ decreases with increasing $\lambda$ means that in a dispersive medium the speed of light, which is always less than in vacuo, is not a constant but depends on the frequency of the light, the higher the frequency the lower is the speed (there is an exception, called anomalous dispersion, an example of resonance: see Experiment 79).

Plot $\mu$ against $1/\lambda^2$ and you should get a straight line. This is one of many empirical formulae which enable one to deduce $\lambda$ when a prism spectrometer is used for the study of spectra.

Look through the spectrometer at light reflected from clouds; comment on the type of spectrum you see and explain.

EXPERIMENT 32

To determine the specific heats of solids and liquids

Apparatus

Vacuum flask, special brass electrical calorimeter (for details see Appendix) with heating coil (50 cm of 26 S.W.G. nickel-chrome), ammeter (0–1 A), voltmeter (0–12 V), rheostat, two switches, Scalamp microammeter, 10 ohm resistor, ice in a beaker or vacuum flask (referred to, below, as 'flask'), 50°C thermometer, low tension supply, stopwatch. Heating coil (100 cm of 26 S.W.G. nickel-chrome), soldered to two stout copper leads through a cork which fits the flask provided (Fig. 2). The cork also carries a small stirrer operated by a 2 V toy motor and is bored for a thermometer. Motor lubricating oil or paraffin (kerosene).

Specific heat of a solid

Method

It is first necessary to calibrate the thermocouple used in the electrical calorimeter. Join the thermocouple to a Scalamp set for a high sensitivity in series with a switch and 10 ohm resistor. Keep one junction of the thermocouple permanently in the beaker or flask of ice. Immerse the brass calorimeter in water, the temperature of which is varied. Plot a calibration curve connecting temperature and Scalamp deflection. Take care to ensure that temperatures are steady before both thermometer and Scalamp readings are recorded. Dry the calorimeter carefully and place it in the apparatus as shown in Fig. 1, resting on a cork at the bottom of the flask.

Connect the calorimeter to the circuit shown and adjust the current to
about 1 A. The p.d. then should be about 3 V. Switch off, and when the temperature is again steady, record the Scalamp reading. Then switch on the supply, starting the stopwatch at the same time, and allow the current to flow for a full five minutes. Switch off, but wait a full minute and record the maximum deflection of the Scalamp. Allow the calorimeter to cool for a further three minutes and take the reading again. Remove the calorimeter and immerse it in cold water, finding the temperature, when steady, as a check on the calibration curve. Then dry the calorimeter, transfer it back to the flask and repeat the experiment with the current reduced to 0.75 A.

**Theory and Calculation**

If $V$ and $I$ are the p.d. in volts and the current in amps, then $VI$ is the number of joules supplied to the calorimeter as heat every second, and the total calories supplied is $VI \times 300$. Using your calibration curve, deduce the initial, maximum and final temperatures: $\theta_1$, $\theta_2$, and $\theta_3$ respectively. The mass of the calorimeter $M$ should have been recorded before it was assembled, and should be known to within 0.1 g. As a first approxi-
mation, the temperature drop $\theta_s - \theta_s$ is the cooling correction which should be added to the maximum temperature $\theta_s$, to allow for the heat lost to the outside. This, though small in itself, leads to a significantly lower temperature, because of the small thermal capacity. The final corrected temperature is $2\theta_s - \theta_s$, and therefore $M_s(2\theta_s - \theta_s - \theta_1) = 300VI$, where $s$ is the specific heat of brass (joule g$^{-1}$ deg C$^{-1}$), which now can be deduced. A similar calculation is carried out for the check experiment and the two results should agree. The answer should be given to two significant figures.

**Specific heat of a liquid**

**Method**

Connect up the apparatus as shown in Fig. 2. Pour in enough oil or paraffin just to cover the heating coil and the bulb of the thermometer in the vacuum flask. Adjust the current so that the temperature rise is approximately 1 deg C per minute. Switch off and allow the apparatus to cool and take the temperature when it is steady again.

Switch on the current and at the same time start the stopwatch. Allow the current to pass for 10 minutes, but 2 minutes before the end switch on the stirrer and allow it to continue to run for a full minute after switching off the current, recording the maximum temperature shown on the thermometer. Add a known mass $M$ of cooled oil (about 100 g) to the oil in the flask, switch on the stirrer for a few minutes and adjust the current to give a similar temperature rise of about 1 deg C per minute. Switch off and wait for steady conditions again, recording the temperature. Switch on again for 10 minutes and repeat the same stirring procedure as that of the first part of the experiment. It is important to remember that a certain amount of heat is generated by the stirrer and we hope to eliminate this by repeating the procedure exactly.

**Theory and Calculation**

The principle of this method is to eliminate heat losses by doing an experiment twice with different thermal capacities, under as nearly identical conditions as possible; also we hope to eliminate the measurement of the thermal capacity of the apparatus from our calculations. If $V_1$ volts is the p.d. across the coil in the first part of the experiment and $I_1$ the
corresponding current, then \( V_1 I_1 \times 600 \) is the number of joules supplied. If the corresponding temperature rise is \( \theta_1 \), then \( 600 V_1 I_1 / \theta_1 \) is the heat required to raise the temperature of the liquid and apparatus by one degree (thermal capacity + heat lost per degree rise). Similarly \( (V_2 I_2 \times 600) / \theta_2 \) is the increased thermal capacity in the second part of the experiment, where \( V_2, I_2, \) and \( \theta_2 \) have the same meaning as before. Thus,

\[
M \times s = \frac{V_2 I_2 \times 600}{\theta_2} - \frac{V_1 I_1 \times 600}{\theta_1}
\]

where \( M \) is the mass of the liquid added for the second part of the experiment and \( s \) the specific heat of the oil. Hence \( s \), which should be given to two significant figures.

**Comments**

The brass calorimeter you have just used is similar to Nernst’s calorimeter (brass is used because it is easy to turn on the lathe) except that the temperature is measured by a thermocouple and not by measuring the resistance of the heating coil (nickel-chrome has a low temperature coefficient of resistance, but a high resistivity, so that only a small coil is needed to produce the heating effect). Nernst used his calorimeter to measure the specific heats of metals at low temperatures. The results were found to be surprisingly low, but they were fully explained by the then newly discovered quantum theory, thus providing further confirmation of its validity.

**EXPERIMENT 33**

**To determine the latent heat of fusion of a solid and the latent heat of vaporization of a liquid**

**Apparatus**

Ice calorimeter (Fig. 1) consisting of a wide 3 in copper tube soldered to a copper funnel, 100 cm of 26 S.W.G. bare nickel-chrome wire wound on paxolin strip, and held centrally inside the tube for heating the ice electrically. The spout of the funnel passes through the cork of a wide necked vacuum flask. Three 100 ml beakers, (0–20 V) voltmeter, (0–2 A) ammeter, rheostat, switch, crushed ice, stopwatch, ‘Cooper’ latent heat of vaporization apparatus or any other modification of Henning’s apparatus.

**To determine the latent heat of fusion of ice**

**Method**

Connect up the circuit as shown in Fig. 1. Fill the copper tube with finely crushed ice, so that the level of the ice is at least 2 in above the top of the heating coil. Invert the vacuum flask and slip it over the tube and
firmly on to the cork. Rest the whole apparatus over a low stool with a hole drilled through the top of it, so that the funnel's spout passes through it. Letter or number two of the small beakers, which should be weighed accurately to the nearest milligram.

Allow the melted ice to drip away into your third beaker. When conditions are steady the water will drip very slowly, due to the heat conducted to the ice from the outside. Use one of the weighed beakers to collect some of the melted ice for 2 minutes, weigh again and deduce the mass of the melted ice $m_1$.

Switch on the current and adjust it to about 1 A and allow it to run for a few minutes till conditions are steady, recording the value of the current $I$ and $V$ the p.d. in volts, then using the other weighed beaker collect the melted ice for 4 minutes, again weigh the beaker, and find the mass $M$ of melted ice. Switch off the electric current, and wait till the melted ice drips slowly and steadily, then use the first beaker which has been dried after
use, and collect the water dripping for a further 2 minutes, again weigh the beaker, and deduce the mass $M_2$ of melted ice.

Lift up the flask and top up the ice level. Repeat the whole experiment using a higher current.

**Theory and Calculation**

The mass of melted ice $M$ collected in 4 minutes is partly due to the heat conducted to the ice from the outside, which though very small cannot be ignored altogether. Therefore the mass of melted ice due to the electrical heating is $M - (m_1 + m_2)$. In 4 minutes the number of joules electrically supplied is $240 \times VI$, hence

$$L[M - (m_1 + m_2)] = 240 \times VI$$

where $L$ is the latent heat of fusion of ice in joule g$^{-1}$, which can be deduced. Average both results and give the answer to the nearest whole joule g$^{-1}$.

**To determine the latent heat of vaporization of alcohol**

**Method**

Connect up the apparatus as shown in Fig. 2. (If the liquid used is water make sure that you use a sufficiently large water condenser. A higher voltage which should be alternating may be required.) Weigh two beakers as previously, but with the addition of a filter paper on top of each. Top up the liquid, though not so full as to splash, when boiling, into the condenser.

Switch on a comparatively high current and adjust it, so that when the liquid is boiling no uncondensed vapour comes out from the spout of the condenser. Record the current $I_1$ and the potential difference $V_1$, and adjust them if necessary so that they are kept constant throughout the experiment. When the condensed vapour drips down steadily collect the drips in one of the weighed beakers for 4 minutes; weigh again the beaker and contents with the filter paper covering the beaker. Deduce the mass of condensed vapour $M_1$.

Now alter the current to a lower value, so that when conditions are
steady only a small quantity of condensate issues from the spout of the condenser. Record the new values of current and voltage \(I_2\) and \(V_2\) respectively, and repeat the experiment as before, using the second weighed beaker and deducing the mass collected of the condensed vapour \(M_2\).

**Theory and Calculation**

The ingenuity of this apparatus ensures that the boiling liquid is surrounded by a jacket of its own vapour, so that heat loss is minimal. Nevertheless, some heat is bound to be lost to the outside, and when the experiment is repeated, and provided that vapour is still produced to jacket the boiling liquid, it is reasonable to assume that the same heat loss is incurred again.

Therefore \(M_1L = 240 \times V_1I_1 + h\) where \(L\) is the latent heat of vaporization of the liquid (joule g\(^{-1}\)) and \(h\) is the heat lost to the outside in the 4 minutes.

For the repeated experiment

\[
M_2L = 240 \times V_2I_2 + h, \\
(M_1 - M_2)L = 240(V_1I_1 - V_2I_2).
\]

From which \(L\) can be deduced correct to the nearest whole joule g\(^{-1}\).

**Comments**

The disparity between the latent heats of fusion of solids and the vaporization of liquids is clear from this experiment; in the case of water, for instance, the ratio of the two latent heats is 1:7. The breakdown of the long-range order among molecules when a solid melts requires much less energy than the overcoming of the total inter-molecular attraction (which increases very rapidly with decreasing distance) when the liquid evaporates. This inter-molecular attraction in the liquid is demonstrated in the phenomenon of surface tension.

**Experiment 34**

To compare the expansion of ether vapour with that of air (at constant pressure)

**Apparatus**

Pyrex capillary tube 40 cm long (internal diameter 1.5 to 2.0 mm), sealed at one end and blown into a bulb about 1-1.2 ml capacity, boiling flask or long stout test tube, clean mercury, small funnel, pump, drying tube containing calcium chloride, 0–100°C thermometer, Bunsen burner, clamp and stand, liquid ether, glass T-piece, fine eye-dropper, two screw clips, half-metre ruler, finely drawn hair-like capillaries.

**Method**

To get used to the technique it is best to start by filling the tube with dry air. Connect the tube to the glass T-piece as shown in Fig. 1. Pump
the air out, warming the bulb carefully at the same time to drive out any moisture, then isolate the pump by means of the screw clip and fill the tube very slowly with dry air. By means of the fine eye-dropper introduce a small index of mercury about 2 cm long which can be gently pushed down the tube with the help of a fine capillary, until it is about 4 cm from the bulb.

Immerse the tube in an iced water bath, stirring well, and record simultaneously the temperature and the distance \( l \) between the bottom of the index and the top of the tube, entering your observations as shown in table below. For determination of column 3, see Theory and Calculation. Repeat the experiment with water at room temperature and then at higher temperatures up to the boiling point of water. In each case allow the bath to cool and the temperature of the stirred water to become steady before taking any observations.

To find the volume of the bulb and the tube, introduce a larger index, measure its average length \( x \) cm at different parts of the capillary and find its mass \( m \) g by weighing the tube with the index, and the tube empty. Now fill the whole bulb with mercury up to the neck, by using the small funnel and a fine long capillary lowered into the bulb to let the air out. Measure the distance \( L \) cm of the top of the mercury from the top of the tube and find the mass of mercury \( M \) g by weighing the tube with the mercury.

<table>
<thead>
<tr>
<th>Temperature (^{\circ}\text{C} )</th>
<th>Distance from the top of the tube ( l ) cm</th>
<th>Volume of air in equivalent length of capillary ( y = (x + L - l) ) cm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

To fill the tube with ether vapour introduce a drop of liquid ether into the tube, immerse it in a warm water bath until all the liquid ether is vaporized and then introduce a short index of mercury while the tube is still in the hot bath, but leaving the index at the top this time. Gently lower the temperature of the bath and measure as before the temperature and the distance \( l \). Tabulate your results as before.
ADVANCED LEVEL EXPERIMENTS

THEORY AND CALCULATION

It makes the calculation easier if the total volume of the gas or vapour is given in terms of equivalent length of the capillary tube $y$ which is assumed to be of uniform bore (is it?). Thus the volume of the bulb up to the neck is $M/p$ ml where $p$ is the density of mercury and the average volume of 1 cm of the capillary is similarly $m/\rho x$ ml, hence the equivalent length of the bulb is $Mx/m = \lambda$ cm. Thus the advantage of having the bulb is effectively to double the length of the tube without increasing the depth of the bath. Complete the tables and plot $y$ against temperature in each case.

COMMENTS

As temperature is defined from the gas thermometer in terms of the pressure of an ideal gas at constant volume and as the ideal gas must obey Boyle's Law, then the volume of an ideal gas at constant pressure must vary proportionately with the absolute temperature.

This experiment is not intended to show whether dry air is an ideal gas, for if air does depart from the ideal state this method would certainly not show it, as it is hardly accurate enough. Consider the expansion of ether vapour from your graph. At a temperature only 30 deg C above its liquefaction point its expansion becomes very nearly similar to that of air, for you can produce a straight part of the curve to cut the $y$-axis at $y_0$. Find $y_{100}$ from the graph and deduce the cubical coefficient of expansion $a = \frac{y_{100} - y_0}{100y_0}$ which as we know is by definition about $\frac{1}{273}$ per deg C for an ideal gas. Deduce it for air too. Thus air at room temperature, which is some hundreds of degrees above its point of liquefaction, behaves very nearly like an ideal gas. Consider the effect of doing this experiment with a trace of water in the tube. In what way will the results be modified? Explain.

EXPERIMENT 35

To investigate the variation of the saturated vapour pressure with temperature, and also to compare the densities of liquid and saturated vapour

APPARATUS

Lengths (12 in) of capillary glass tubing, 1 mm internal and about 5 mm external diameters; different liquids (amyl alcohol, aniline, etc.); stout
boiling tubes, 9 in long and 1 ½ in internal diameter; 0–100°C thermometer, copper stirrer, some ice, ruler, Bunsen and tripod, stand and clamp, tap and rubber tube, small beakers. 'Sparklet' syphon cartridge, simple cartridge opener which enables release of gas to be controlled (Fig. 2), large toy balloon, large displacement vessel (a bucket full of water will do), cotton thread.

**Method**

Fix the tap and rubber tube to one end of a capillary tube and gently draw in a small index of water from a beaker, seal one end either by heating it or, better still, by using epoxy resin (this does not set quickly but can be warmed to hasten setting) so that the index traps an air column of about 2 cm. The same procedure is repeated with amyl alcohol and aniline but with longer air indices. Place the three tubes in a long boiling tube filled with crushed ice and measure the position of the bottom of each index from the top when conditions are steady. Record the observations as shown in table on facing page. Replace the ice by tap water and record the temperature and \( l \). Heat the water in the tube very gently and repeat the above observations, with the Bunsen removed each time to keep the temperature reasonably steady. Continue until the water boils.

Weigh a 'Sparklet' and also weigh a toy balloon with a length of cotton. Place the Sparklet inside the opener and tie the balloon on to the rubber bung fitted over the exit tube (squeeze all air out of the balloon of course).
Now screw in the plunger, forcing the Sparklet against the steel pin, thus rupturing the diaphragm and releasing the carbon dioxide gas which will begin to fill the balloon. When no more gas appears to come out, carefully

<table>
<thead>
<tr>
<th>Temperature °C</th>
<th>Distance between the top of the tube to the bottom of the index l cm</th>
<th>Length of air index ( x = L - l ) cm</th>
<th>Partial pressure of the trapped air ( h ) mm Hg</th>
<th>S.V.P. ( \pi - h ) mm Hg</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

tie up the neck of the balloon and untie it from the rubber bung. Weigh the balloon and record weight (note that the balloon descends to the ground rapidly when released). To find its volume, push it carefully into

![Diagram of carbon dioxide cartridge](image)

*Fig. 2*

the displacement vessel and measure the overflow (\( V \) ml). This should be repeated, as a check. Weigh the empty Sparklet and record its weight. Widen the hole pierced in the diaphragm and fill the cartridge with water and weigh again. Deduce the internal volume of the cartridge. Find the
pressure of carbon dioxide inside the balloon by connecting it to a mercury manometer.

**Theory and Calculation**

One can assume that the S.V.P. at 0°C is small enough to be equated to the pressure exerted by the index, so that the pressure at 0°C of the trapped air index of length is virtually atmospheric. At the higher temperature $T^\circ K$, the length $x$ of the air index can be measured and the new partial pressure of the enclosed air deduced from the gas laws:

$$\frac{x_0 \pi}{273} = \frac{xl}{T}$$

whence $h$ can be calculated, and then S.V.P. = $\pi - \frac{h}{T}$. This assumes the bore of the tube to be uniform, which, however, is often not the case. This can be demonstrated by inserting a mercury index and measuring its length in different parts of the tube. Plot S.V.P. against temperature.

The cartridge is filled with liquid carbon dioxide at room temperature; this can be deduced from the knowledge of the volume of carbon dioxide gas and the volume of the cartridge. If carbon dioxide obeys the gas laws, it is easy to show that the pressure developed when it is compressed to the volume of the cartridge would run into hundreds of atmospheres and this is most unlikely. Further, as the critical temperature is about 31°C, carbon dioxide at such pressures and at room temperature is certain to be partially, at least, in liquid form. When you shake an unopened cartridge at room temperature, you cannot hear any movement of liquid, so that it must be completely full of liquid carbon dioxide. Derive from your observations the density of liquid carbon dioxide. When the balloon filled with carbon dioxide is weighed an allowance must be made for the upthrust of the air (volume of balloon x density of air) and this must be added to the observed weight to give the true weight of balloon and carbon dioxide.

Deduce the weight of carbon dioxide gas, which should be a little less than the weight of liquid carbon dioxide. From the knowledge of the pressure of carbon dioxide in the balloon, the volume of the balloon and room temperature deduce the density of carbon dioxide at S.T.P.

**Comments**

The variations of S.V.P. with temperature would appear from the graphs to be very similar for all the liquids used (you should be able to tell which one has the highest boiling point). You should have noted the rapid expansion of the air trapped by the water index as the temperature approaches the boiling point of water; in fact the index should blow out of the tube, so that the volume of trapped air becomes infinite, indicating that the S.V.P. must be getting nearly equal to the atmospheric or applied pressure. As air is usually dissolved (or occluded) in liquids, these trapped air bubbles, like the trapped air index, expand very rapidly (i.e. explosively) as the S.V.P. equals the outside pressure and that in fact is what happens when boiling occurs.

This can be used to measure S.V.P. conveniently by finding the boiling points corresponding to different applied pressures. You can for instance
find the S.V.P. of water at temperatures in excess of 100°C by heating a pressure cooker half-filled with water, containing a maximum registering thermometer (this is a kind of clinical thermometer with stout walls not affected by the high pressure), the pressure being calculated from the weight of the steam regulator and its cross-sectional area. This would extend your curve to about 120°C.

The pressure of vapour when heated in equilibrium with its liquid thus increases in some cases much more rapidly than that of the pressure of a trapped gas as your graphs show, and on this account you must not heat the carbon dioxide cartridge in any way.

If the cartridge is full of liquid carbon dioxide at room temperature, what would happen if you cooled it to say —10°C?

Calculate the ratio of the density of carbon dioxide gas at room temperature and pressure to the density of liquid carbon dioxide.

**EXPERIMENT 36**

*To determine the ratio (γ) of the specific heats of air on compressing it adiabatically*

**Apparatus**

Veridia precision tube 65 cm long and 19·00 mm internal diameter, water manometer to read to 30 cm excess water pressure, 19·00 mm precision steel balls, stopwatch, rulers, 6 litre aspirator with outlet at the bottom for the recovery of the steel balls (Fig. 1), magnet, small piece of soft foam rubber (e.g. as used in packing delicate apparatus) attached to a long string, methylated spirit, 1 litre measuring cylinder.

**Method**

Find the volume of the aspirator by filling it with water using the litre measuring cylinder. Empty out the bottle and dry the inside thoroughly before assembling the apparatus shown in Fig. 1. Clean the inside of the precision tube thus: lower the long string down into the bottle, then gently pull it out from the bottom outlet O, causing the foam rubber at the other end to be pulled down the tube, the foam rubber having previously been lightly wetted with methylated spirit.
Arrange a piece of foam rubber on the bottom (inside) of the aspirator so that it catches the falling steel ball and prevents cracking the aspirator. (This precaution is not necessary if the apparatus is made of polythene, instead of glass.)

Connect the manometer to the stopper at the bottom outlet. Wipe clean one of the steel balls and gently let it drop down the tube. You and your partner have to take two simultaneous readings, the lowest point reached by the ball on its first downward drop and the highest reading of the manometer, using two vertical rulers. Dropping half a dozen balls in succession, you should be able to get consistent readings. Recover the balls with the help of the magnet, and repeat the observations with you and your partner changing places.

Time the period of oscillation of each of these balls as carefully as you can (this will be of the order of 0.8 sec and, since the oscillation is heavily damped, the timing is short, and therefore repeated checks are necessary). Find the mass of one ball and the barometric pressure.

**Theory and Calculation**

This method is a modification of Ruchardt’s method, and is interesting and simple.

When the ball is allowed to drop into the tube, it compresses the air adiabatically; let \( A \) cm\(^2\) be the internal area of cross-section of the tube, \( d \) cm the distance dropped by the ball, \( h \) (millimetres) the highest water pressure read on the manometer, \( \pi_0 \) the atmospheric pressure in millimetres of mercury, \( V \) ml the total volume of the bottle and the whole tube.

Before compression the pressure and volume are \( \pi_0 \) mm Hg and \( V \) ml respectively, after compression \( \pi_0 + (h/13.6) \) and \( V - Ad \), 13.6 g cm\(^{-3}\) being the density of mercury.

Therefore
\[
\pi_0 V^\gamma = (\pi_0 + \frac{h}{13.6}) (V - Ad)^\gamma.
\]
Dividing both sides by \( \pi_0 V^\gamma \) we get
\[
1 = \left(1 + \frac{h}{13.6\pi_0}\right)\left(1 - \frac{Ad}{V}\right)^\gamma
\]
as \( V \gg Ad \)
\[
1 + \frac{\gamma Ad}{V} \approx 1 + \frac{h}{13.6\pi_0}
\]
or
\[
\gamma = \frac{Vh}{13.6Ad\pi_0}
\]
The period of oscillation \( T \) can be easily deduced, remembering the assumption made \( (V \gg Ad) \), and you can show that \( T = 2\pi \sqrt{\frac{V_0 m}{\gamma A^2\pi_0}} \) where \( m \) is the mass of the ball and \( V_0 \) the volume of the bottle, hence \( \gamma \).

**Comments**

To ensure adiabatic compression the larger the vessel the better, as the surface area (assumed spherical) from which heat is lost increases as \( R^2 \)
while the thermal capacity increases as $R^2$. The compression takes place quickly (less than 0.4 sec), and the temperature rise $\delta T$ is not very great

\[
\frac{\pi_0^{\gamma-1}}{T^\gamma} = \left( \frac{\pi_0 + \frac{h}{13.6}}{T + \delta T} \right)^{\gamma-1}
\]

(calculate it for the above) where $T^0$ is the absolute temperature of the room); the overall heat transfer will be small.

Repeat the above experiment filling the vessel with moisture-free carbon dioxide gas and comment on the value of $\gamma$ for carbon dioxide. Calculate the height of the water manometer having the same period of oscillation as that of the steel ball. What is the advantage of having both periods the same, in the above experiment?

**EXPERIMENT 37**

**To determine the thermal and electrical conductivities of a wide range of substances**

There are three methods of obtaining results in this experiment.

**Apparatus**

(a) Two or three cylindrical bars of different metals (brass, steel, aluminium, etc.) 2 in diameter and 6 in long, each having two holes drilled 2 in apart, copper cooling coil to fit over one end of each bar, fibreglass insulating jacket to fit over the bar (Fig. 1), two thermometers (0–50°C), two thermometers (0–200°C), measuring cylinder, stopwatch, vernier callipers, micrometer screw gauge, electric iron, constant pressure head apparatus.

(b) Lee’s disc thermal conductivity apparatus (Fig. 2) with test specimens of poor conductors, 6 V battery, ammeter (0–1 A), glass beaker.

**Method (a)**

Place all the thermometers in water in the beaker, compare and record their readings. Measure the diameter of each bar and the distance between the two holes drilled in them.

Assemble the apparatus as shown in Fig. 1; at first set the electric iron to ‘high’ (to speed up the heating), then turn it down to ‘low’ and adjust the rate of flow of the cooling water to give an adequate temperature rise on its way through the cooling coil. When conditions are reasonably steady measure the rate of flow of water, using the measuring cylinder and stopwatch. Use a magnifying glass to read all the four thermometers almost simultaneously and accurately (the two thermometers inserted in the metal should be smeared with glycerine beforehand).

Repeat with each of the cylindrical bars provided and, if time is available, alter the setting of the iron and repeat for each metal as a check.
THEORY AND CALCULATION (a)

Deduce the corrected temperature difference $\theta$ between the readings of the two thermometers inserted into the cylinder, the corrected temperature rise $\phi$ of the cooling water, and the mass of cooling water $M$ flowing per second. Then the heat current in joules sec$^{-1}$ flowing along the cylinder is equal to $KA(\theta/d)$, where $d$ is the separation between the two thermometers, $A$ cm$^2$ the area of cross-section and $K$ the thermal conductivity.

If we assume the heat loss to the fibreglass is negligible then $Ms\phi = KA(\theta/d)$, hence $K$ joules cm$^{-1}$ sec$^{-1}$ deg C$^{-1}$ given to two significant figures, where $s$ is the specific heat of water in joules g$^{-1}$ deg C$^{-1}$.

METHOD (b)

Assemble Lee’s disc apparatus as shown in Fig. 2 after you have measured the diameter of the copper discs and of the test specimen, the thickness of the test specimen and the thickness of each of the copper discs used.

If the current taken from the battery is a small one, you can assume that the p.d. of the low tension supply remains constant and can either be assumed from the nominal p.d. of the battery or measured at the beginning and at the end of the experiment so that a voltmeter is not needed all the time.

Switch on the current and leave it for some time. When the conditions become steady, read all the thermometers as accurately as is justified by the method. Measure, using the fourth thermometer provided, the temperature of the room.

Repeat with different test specimens.
Theory and Calculation (b)

It is clear that the above method is not suitable for poor conductors (where $K$ is extremely small) as $\theta$ will be too small to measure. To get a bigger heat current, $A$ must be kept large but $d$ reduced considerably so that the test specimen will be in the form of a thin disc. The modified apparatus, first designed by Lee, is not cooled by water but air-cooled, and is based on the assumption that any heat transfer from the surface is dependent on the temperature difference between the disc and the surroundings, and is the same, for a given temperature difference, for all the surfaces having equal area.

When conditions are steady the input electrical power $VI$ is equal to the total rate of heat transfer from all the surfaces.

Therefore

$$VI = \alpha[(\theta_1 - \theta_0)A_1 + (\theta_2 - \theta_0)A_2 + (\theta_3 - \theta_0)A_3]$$

(1)

where $A_1$, $A_2$ and $A_3$ are the areas of the exposed surfaces of the discs.
respectively, and $a$ is a constant, being the same for all the surfaces (we assume that little heat is lost from the edge of the specimen, as it is sufficiently thin).

The heat current through the specimen $KA(\theta_2 - \theta_3)/d$ is clearly lost from the surface of disc No. 3, therefore

$$KA(\theta_2 - \theta_3)/d = a(\theta_3 - \theta_0)A_3 \quad \ldots \quad (2)$$

Combining equations (1) and (2) deduce $K$ (joule cm$^{-1}$ sec$^{-1}$ deg C$^{-1}$) for all the test specimens.

Method (c)

Electrical conductivities of the specimen cylinders can be measured using the circuit shown in Fig. 3. Pass a known current along the cylinder, pressing flat copper electrodes against the ends of the cylinder by means of a clamp, with insulating wooden blocks (see diagram). Measure the p.d. at two points, $l$ cm apart, along the cylinder. Since the p.d. is very small (of the order of a few microvolts), reverse the current flow, take the p.d. again and calculate the mean value. The electrical conductivity $\sigma = \frac{l}{VA}I$ where $A$ is the area of cross-section already calculated. (Note the similarity between the measurements of the two conductivities!)

Comments

What do you make of the relation between $K$ and $\sigma$ for the different metals? The electrical conductivities of the test specimens of poor thermal conductivity can also be found, but the method has to be modified considerably.

The relationship between $K$ and $\sigma$ is recognized by the Wiedeman–Franz Law and that the conduction in both cases must be due in large measure to the 'free' electrons in the pure metal, but the fact that there is no perfect heat insulator suggests that thermal transfer must also be due to propagation by increased thermal vibrations of the molecules of the solid.
ADVANCED LEVEL EXPERIMENTS

In non-metals at very low temperatures, the conduction of heat is largely due to such transfer which can be looked upon, from the wave theory point of view, as energy waves (called phonons). These can be scattered by imperfections in the atom lattice in much the same way as light (made up of photons) can be scattered by the imperfections in glass which render it less transparent.

EXPERIMENT 38

To investigate the variation in electrical potential with distance from: (a) a charged sphere and (b) a charged cylinder

(a) Charged sphere

Apparatus

Gold leaf electroscope with focusing screen and a circular scale (Fig. 1), series of H.T. batteries for calibrating the gold leaf electroscope, small spherical ball attached to a long rod, small proof plane with insulated handle, 5 μC radium source, long fine copper wires, 5 KV E.H.T. (rectified R.F.), 12 watt straight filament bulb, rulers.

Method

Calibrate the gold leaf electroscope using the several batteries provided. The gold leaf should be illuminated by the electric bulb, fixed with its straight filament horizontal and directed endwise to the electroscope, so that a clear shadow of the gold leaf and the electroscope is cast on the focusing screen. Tabulate your results and plot the calibration curve, which will also be needed for Experiment 39.
Arrange the apparatus as shown in Fig. 1 with the small sphere attached by a fine wire to the positive terminal of the E.H.T. supply. Before the E.H.T. is switched on, support the proof plane on a burette stand and measure the distance $x$ between the sphere and the proof plane with a ruler for different positions of the base of the stand, as measured by a different ruler clamped to the bench. Fix the small radioactive source close to the proof plane in order to ionize the air and to ensure that the proof plane acquires the potential at that point rapidly. Find the steady readings of the gold leaf electroscope for different values of $x$, tabulating your results as follows:

<table>
<thead>
<tr>
<th>Position of base of burette stand</th>
<th>Distance $x$ cm</th>
<th>Deflection of electroscope</th>
<th>Potential volts</th>
</tr>
</thead>
</table>

**Theoretical and Calculation**

Plot the potential against $x$ and you should get two rectangular hyperbola joined by a flat plateau of constant potential (in the position occupied by the sphere). By measuring the gradients at different points, verify the relationship between electric field intensity and distance from the centre of the sphere. Also plot the reciprocal of $x$ against $V$ and explain the graph obtained.

What happens to these graphs if a sphere of larger radius is connected to the E.H.T. supply?

**(b) Charged cylinder**

**Apparatus**

Shallow circular perspex dish, 6 in diameter (the top lid of some processed cheese packs would do), a thin copper band bent round to fit just inside the dish, a small copper calorimeter (1½ in diameter), accumulators, valve voltmeter with a pin strapped to the probe, a graph paper and a stiff copper wire.

**Method**

Set up the apparatus as shown in Fig. 2. The shallow perspex dish should be fixed with a bit of plasticine to the millimetre graph paper. Pour into it a small quantity of copper sulphate solution. Using the pin probe investigate the variation of potential with distance from the edge of the calorimeter, dipping the probe into the copper sulphate solution and using the graph paper to measure distances. Tabulate your results as follows:
Distance of probe from centre  
\( x \text{ cm} \)  
Potential in volts

(If a valve voltmeter is not available, an ordinary 2 volt potentiometer with centre zero galvanometer will do. See Experiment 48.)

**Fig. 2**

**Theory and Calculation**

It can be shown that the potential at a point \( P \) distance \( x \) from the centre of the inner cylinder (potential \( V_0 \)) is \( V_0 \ln(b/x) \). (This implies that the electric field intensity varies inversely with \( x \).) Though this applies to electrostatic potentials, it can be shown mathematically that the potential distribution is unaffected when the charged conductors are immersed in a uniformly conducting medium, provided that their potentials are maintained by means of batteries. This gives an elegant method of investigating potential distributions due to charged conductors. Plot \( V \) against \( \ln x \) and verify the relation \( V = V_0 \ln(b/x) \) completely.

**Comments**

Replacing the calorimeter at the centre by the copper wire, investigate the variation of the potential with distance. This is important as it will
show that the potential drops very rapidly from the wire, and this explains why the electrons from a hot tungsten wire in the axis of a cylindrical anode gain most of their energy directly they leave the wire (see Experiment 63).

Also investigate what happens if you surround the copper wire by several circllets of fine copper wire (grid) all connected to a variable potential. Note the effect on the potential at a point close to the central wire when (a) the potential of the grid is changed by a small amount, say 0.1 V, (b) the potential of the anode (the copper band) is changed by the same amount.

EXPERIMENT 39

To investigate the factors affecting the capacitance of a parallel plate capacitor

Apparatus

Calibrated and illuminated gold leaf electroscope (see Experiment 38), 2 flat metal discs (say 8 in and 5 in diameter respectively) with insulated handles, 12 circular cellulose acetate spacers 1 mm thick and 25 mm diameter, smooth circular cellulose acetate disc 5 in diameter and 1 mm thick, 4 9-V dry batteries, large flat metal sheet.

Introduction

In electrostatics one deals with high voltages and minute charges, so that leakage of charge is the most serious factor which can mar or even stop an electrostatic experiment altogether. In this experiment it is hoped that potentials are measured sufficiently quickly to avoid leakages. It is important that all conductors are well polished and flat.
**Method**

Set up the apparatus as shown in Fig. 1, using the larger disc. Touch the top of the electroscope with the terminal momentarily, and lifting off the disc quickly note the deflection in degrees of the gold leaf electroscope. Discharge electroscope and repeat. Repeat this experiment with 2, 3 and 4 spacers, tabulating results as follows:

<table>
<thead>
<tr>
<th>Charging potential $v$ volts</th>
<th>Number of spacers</th>
<th>Deflection $\theta_1$, $\theta_2$</th>
<th>$V_1$ volts</th>
<th>$V_2$ volts</th>
<th>Mean $V$ volts</th>
<th>$(V/v) - 1$</th>
</tr>
</thead>
</table>

Repeat the whole of the experiment with two 9 V batteries as a further check. Repeat the whole of the experiment with the smaller disc with four 9 V batteries and tabulate your results as before.

Placing the large cellulose acetate disc and one spacer as shown in Fig. 2, charge as before, lift up the disc and read the deflection. This should be done several times, removing the large cellulose disc before each observation and making sure that there is no residual charge left on it.

**Theory and Calculation**

Let the charging potential be $v$ volts, $V$ the final voltage, $C$ the capacitance between the plates, $C_0$ the capacitance of the electroscope (farads) and the insulated disc (the latter is assumed small).

Thus the initial capacitance is $C + C_0$ and the final capacitance is $C_0$ and if we assume the total charge to be unchanged then we have

$$(C + C_0)v = C_0V \quad \text{or} \quad \frac{C + C_0}{C_0} = \frac{V}{v}.$$  

therefore $C = C_0\left(\frac{V}{v} - 1\right)$.

Investigate the relations between $C$ and the separation between the plates; between $C$ and the area of the plates.

To investigate the effect of the presence of the dielectric, the above procedure was adopted, as it is found that without the spacer one tends to charge the acetate by the sudden lifting of the top disc. If the relative
permittivity* of the acetate is \( k \), and \( C \) is the capacitance of the air space capacitor with two spacers, then, as used in the above experiment, half of the air space is filled with the dielectric, so that we have in effect two capacitors in series, one air-filled capacitor of capacitance \( 2C \) and the other dielectric-filled capacitor of capacitance \( 2kC \).

If the total capacitance is \( C_1 \) then
\[
\frac{1}{C_1} = \frac{1}{2C} + \frac{1}{2kC} = \frac{k + 1}{2kC}
\]
and
\[
\frac{C}{C_1} = \frac{k + 1}{2k}.
\]

Find \( k \).

**Comments**

In the above experiment we assumed that \( C_0 \) remains the same whether the small insulated disc or the large disc is connected to the electroscope. An estimate can be made of the capacitance of each disc \( C_d \) (in relation to the surrounding) in terms of the capacitance of the electroscope \( C_e \). Charge up the electroscope and one of the discs (the latter being lifted to the height usually reached in the previous experiment), and note the voltage \( v_1 \). Disconnect the disc (without touching the electroscope), discharge it and connect it again to the electroscope and let the new potential be \( v_2 \).

Now
\[
v_1C_e = v_2(C_e + C_d)
\]
or
\[
\frac{v_1}{v_2} = 1 + \frac{C_d}{C_e}.
\]

Hence
\[
\frac{C_d}{C_e} = \frac{v_1}{v_2} - 1.
\]

Adapt the above method to find the capacitance of the electroscope in terms of that of a large sphere, hence calculate the capacitance of each of the parallel plate condensers used above.

This is an instructive but not an accurate method (subject to about 10% error).

**EXPERIMENT 40**

To find the capacitance of a large capacitor by charging it to a high voltage and calculating the charge stored in it

**Apparatus**

6 \( \mu \)F capacitor (250 V working), a board wired as shown in Fig. 1, microammeter, 90 V battery, stopwatch.

**Method**

Connect up the circuit as shown in Fig. 1. Select a suitable range for the microammeter (0–15 \( \mu \)A). Put \( K_3 \) to position (a) so that the capacitor is shorted out, leave \( K_4 \) open and close \( K_1 \). Record the value of the current.

* Relative Permittivity \( \equiv \) dielectric constant.
Turn $K_3$ quickly over from position (a) to (b) and start the stopwatch simultaneously. Record the value of the current at first every 15 seconds, later every half-minute, until the charging current is reduced almost to zero, recording your observations as follows:

<table>
<thead>
<tr>
<th>Time</th>
<th>$I$ $\mu\text{A}$</th>
<th>Mean $I$</th>
<th>Charge $\mu\text{coulomb}$</th>
<th>$90 - \overline{RI}$ mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$I_0$</td>
<td>$\frac{I_0 + I_1}{2}$</td>
<td>$15 \times \frac{I_0 + I_1}{2}$</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>$I_1$</td>
<td>$\frac{I_1 + I_2}{2}$</td>
<td>$15 \times \frac{I_1 + I_2}{2}$</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>$I_2$</td>
<td>$\frac{I_2 + I_3}{2}$</td>
<td>$30 \times \frac{I_2 + I_3}{2}$</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>$I_3$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Open $K_1$ and turn $K_3$ over back to position (a), halve the resistance $R$ by shorting out $K_2$, select a higher range of current for the microammeter (0–30 $\mu\text{A}$) and repeat the above experiment (but this time note that the initial current is larger and tends to fall off more rapidly).

**Theory and Calculation**

Calculate the mean current in each timing interval and complete column 3 in the above table. Work out the charge going into the capacitor every timing interval and add up to give the total charge $Q$ $\mu\text{coulomb}$. The capacitance of the capacitor is $Q/90$ $\mu\text{F}$. Repeat the above calculation with the smaller resistance $R$ in the circuit and you should get the same total charge. You will see that the effect of the resistance is to regulate the speed of charging.

This is clearly seen when the potential $V$ across the condenser is calculated at each instance as shown in column 5 in the above table.
Plot $V$ against $t$ for each set of readings, and you will notice that the initial part of each curve is almost a straight line. Find the ratio of the two initial gradients and comment on the result.

COMMENTS

This is an important experiment as it links both static and current electricity. It can be used not only to measure capacitance but also to measure an interval of time. $CR = (Q/V) \times R = Q/I$ has the dimension of time and this is used in time bases, where a capacitor with a high resistor is used in conjunction with a thyratron (a gas-filled valve which becomes highly conducting when the potential of the anode reaches a critical value—similar to a neon tube). If the initial part of the $V - t$ curve is used, sometimes suitably amplified, the voltage across the capacitor rises uniformly with time so that the spot of the cathode ray tube moves at constant velocity across the face of the screen, this velocity being varied continuously by altering $R$. To make the time base repetitive, the capacitor is discharged when its voltage reaches a predetermined value.

The idea of using series capacitor and resistor as a timing device is not unlike the old hour-glass where the capacity of the glass and the degree of constriction determine the length of time the top compartment empties or the bottom part fills up.

EXPERIMENT 41

To investigate the relation between the magnetic field and the strength of the current flowing in (a) a straight wire, (b) a circular coil, (c) a solenoid

APPARATUS

3-core power cable wound round a wooden frame ($\frac{3}{4}$ in square battens) one metre or more wide and twice the height of a movable table (Fig. 1), the free ends of the cable being joined together as shown so that the current flows through the loop three times; 8 ohm (rated 5 A) rheostat, reversing switch, avometer or multi-shunt ammeter, 6 V accumulator, millimetre graph paper, Sellotape, trough magnet, plotting compass needle, tangent galvanometer with two extra coils each having two turns of different diameters and mounted concentrically and vertically with the galvanometer coil.

Solenoid about 40 cm long, uniformly wound with three layers of cotton-covered copper wire of widely varying gauges, giving different numbers of turns (not known to the pupil, but each with the terminals marked in different colours); small plotting compass with periscope device.
(a) **Straight wire**

**Method**

Use the trough magnet and turn the table so that one side is parallel to the earth's field. Place the large wooden frame close to the middle of one of the sides of the table which is perpendicular to the earth’s field, so that the frame has its plane in the magnetic meridian. Sellotape a square of graph paper to the top of the table with its diagonal in the magnetic meridian (i.e. in the plane of the frame) as shown in Fig. 1.

![Diagram](image)

*Fig. 1*

Connect the circuit diagram shown in Fig. 1, keeping the rheostat and the ammeter (0–5 A) away from the table. Place the plotting compass in the plane of the coil at a distance $d$ of about 14 cm from the vertical wire and adjust the current so that the needle is parallel to one of the graph lines; reverse the current and adjust the current again. Decrease $d$ and repeat the operation several times. Rotate the plane of the wooden frame through 180° as shown dotted in Fig. 1 and repeat. Thus for every position of the compass there are four current readings; hence complete the following table:
Theoretical and Calculation

Do you notice any relation between $I$ and $d$? Plot $1/d$ against $1/I$ and deduce the gradient $m$. If the horizontal component of the earth’s magnetic field is $H_0$, and the field of the wire at distance $d$ is $H$, then $H = H_0$ (as the resultant field makes 45° with $H_0$).

But

$$H = \frac{6I}{10d} \quad \text{or} \quad \frac{1}{d} = \frac{5}{3} \frac{H_0}{I}$$

(remember the current goes round the frame three times) and $m = 5H_0/3$, hence $H_0$ (oersted) given to two significant figures.

M.K.S. $H = \frac{100 \times 3 \times I}{2\pi d} = \frac{150}{\pi} \times \frac{I}{d} = H_0$

therefore

$$\frac{1}{d} = \frac{\pi H_0}{150} \times \frac{1}{I}, \text{ so that the gradient } m = \frac{\pi}{150} \times H_0, \text{ hence } H_0$$

(A m$^{-1}$).

(b) Circular coil

Place the tangent galvanometer and coils on the same table as in the previous method and turn the coils so that their plane is in the magnetic meridian and the aluminium pointer set at 0-0. Use the same electrical circuit as before but with a single accumulator (or two if necessary). Connect each of the coils in turn, adjust the current to a suitable value, read both ends of the pointer, reverse the current and read the pointer positions again, tabulating your results as follows:

<table>
<thead>
<tr>
<th>Current A</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\theta_3$</th>
<th>$\theta_4$</th>
<th>Mean $\theta$</th>
<th>tan $\theta$</th>
<th>Diameter of coils, $D$ cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>coil 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>coil 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>coil 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Theory and Calculation

The field at the centre of a circular coil is \( \frac{2\pi n I}{10R} = \frac{2\pi n I}{5D} \) where \( n \) is the number of turns and \( R \) the radius of the coil, hence \( H_0 \tan \theta = \frac{2\pi n I}{5D} \).

Verify this relation and deduce \( H_0 \) in each case.

M.K.S. \( H_0 \tan \theta = \frac{100\pi n I}{D} \).

(c) A solenoid

The small periscope device consists of two small plane inclined mirrors, mounted above and below a small compass attached to a small ruler or stick. Usually the compass is placed centrally inside the solenoid and a line is drawn on the compass as shown in Fig. 3. When the axis of the solenoid is in the magnetic meridian, the tiny compass needle coincides with the marked line. Set the solenoid with its axis making 45° with the magnetic meridian and the compass needle will be as shown in Fig. 3(c). Connect up the circuit as shown in Fig. 3(b), using any one of the three pairs of terminals, and adjust the current in the solenoid until the needle sets at right angles to the marked line (Fig. 3(d)). Connect the other two coils in turn and record the adjusted current in each case. Turn the axis of the coil through 90° and repeat all observations.
PRACTICAL PHYSICS

THEORY AND CALCULATION

The field $H$ inside a long solenoid is $2\pi nI/5$ oersted ($n =$ the number of turns/cm), when the current was adjusted so that the needle was at right angles to the axis of the coil (Fig. 3(d)) then $H_0 = H/\cos 45^\circ = \sqrt{2}H$. Use the mean value of $H$ you have already found and deduce $n$ for each coil.

M.K.S. $H$ (long solenoid) = $nI$ ($n =$ turns/metre)

therefore $H_0 = \sqrt{2}nI$.

COMMENTS

Part (a). Why are the currents for the same distance $d$ not quite the same? Why plot $1/d$ against $1/I$ instead of $I$ against $d$? Does the line plotted on the graph pass through the origin? (Remember there are the fields produced by the horizontal parts of the frame but their effect is not affected seriously when $d$ is varied; work out the rest yourself.)

Part (b) If $n$ the number of turns per cm or metre of the coil is kept the same but the radius of the coil is increased, does this affect $H$? How do you reconcile your answer with your findings in part (b)?

Part (c) Use the available apparatus to investigate the variation of the field along the axis of the solenoid.

EXPERIMENT 42

To calibrate an A.C. ammeter using a dynamometer, and also to calibrate a dynamometer as a power meter

APPARATUS

The dynamometer consists of a large coil about 30 cm diameter and of 100 turns, and a small coil of a similar number of turns but of smaller diameter (about 5 cm); the small coil rests freely by means of two brass knife edges on two parallel brass rails. The deflection of the coil is observed by reflecting light from a small mirror, mounted on the coil, on to a vertical scale. The couple on the coil is balanced by a small rider which can be moved on a non-magnetic wire stretched parallel to the axis of the coil; the motion of the coil is air-damped by means of a stiff paper vane. A.C. ammeter 0–2 A, 10 ohm coil, rheostat 8 ohm (5 A), heating coil, insulated calorimeter, stopwatch, 8–12 V A.C. supply, measuring cylinder, thermometer.

Calibration of an A.C. ammeter using a dynamometer

METHOD

Connect up the circuit shown in Fig. 1. Record the zero on the vertical scale, and move the rider 5 mm along the wire from the axis, causing the
coil to tilt. Switch on the current and, if the tilt of the coil is increased further, change over the connections either at the terminals of the small coil or the large coil, whichever is the more convenient. Now adjust the current so that the spot returns to the zero position; record the current. Repeat, moving the rider farther along, and tabulate your results as follows:

<table>
<thead>
<tr>
<th>Weight of rider $W_g$</th>
<th>Distance moved $x$ cm</th>
<th>Ammeter reading $I_1$, A</th>
<th>$I$ (calculated), A</th>
</tr>
</thead>
</table>

Weigh the rider (or riders) and record.

**Theory and Calculation**

Let $N_1$ and $N_2$ be the number of turns in the small and large coil respectively, $A$ cm$^2$ be the area of cross-section of the small coil, $D$ cm the diameter of the large coil and $I$ the current in both coils, then:
C.G.S.
Field in the centre of the large coil
\[ \frac{N_1 A I}{10} = \frac{2\pi N_2}{5D} I \]
The couple on the small coil
\[ \frac{N_1 A I}{10} \times \frac{2\pi N_2}{5D} I = \frac{2\pi N_1 N_2 A I^2}{5D} \]
dyne cm
\[ = W \times 981 \times x \]

M.K.S.
Field in the centre of the large coil
\[ \frac{10^4 N_1 A I}{D} = \frac{4\pi \times 10^{-5} N_2 I}{D} \text{ Wb m}^{-2} \]
The couple on the small coil
\[ \frac{N_1 A I}{10^4} \times \frac{4\pi \times 10^{-5} N_2 I}{D} = \frac{W}{10^3} \times 9.81 \times \frac{x}{10^2} \]

Deduce \( I \) and plot it against \( I_1 \) as read by the ammeter and comment on your graph.

Calibration of a dynamometer as a power meter
Connect up the circuit shown in Fig. 2. Add, using the measuring cylinder, a sufficient amount of water so as to cover the heating coil completely, insulate the calorimeter well and cover it. Again record the zero position of the coil and move the slider a known distance \( x \) cm along the wire. Switch on the current and adjust it until the coil returns to its zero position. Find the time (\( t \) sec) taken by the heating coil to raise the temperature of the water by 1 deg C, stirring well and keeping the current adjusted for zero deflection of the coil. Repeat with different values of \( x \), using the same volume of water each time; tabulate your results as usual.

Theory and Calculation
If \( V \) is the p.d. of the heating coil which takes current \( I \), the power dissipated in the coil is \( VI \) W. But the current in the large coil is \( V/R \), where \( R \) is the total resistance of the large coil and the series resistor of 10 ohms. (The resistance of the small coil is negligible.)

C.G.S.
Couple exerted on the coil
\[ \frac{N_1 A I}{10} \times \frac{2\pi N_2}{5D} \times \frac{V}{R} \]
\[ = \frac{2\pi N_1 N_2 A}{50DR} \times VI \]
\[ = W \times x \]

M.K.S.
Couple exerted on the coil
\[ \frac{N_1 A I}{10^4} \times \frac{4\pi \times 10^{-5} N_2 V}{DR} \]
\[ = \frac{4\pi N_1 N_2 \times 10^{-9}}{DR} \times VI \]
\[ = \frac{W}{10^3} \times 9.81 \times \frac{x}{10^2} \]
Thus the power dissipated is proportional to \( x \), but the power is dissipated in the form of heat and this is proportional to \( 1/t \), so that \( 1/t \) should be proportional to \( x \). Verify this.

**Comments**

The couple exerted on the small coil is proportional to \( I^2 \), so that the reversing of the current does not affect the direction of the couple and this instrument can thus be used to measure A.C.; as the strength of the alternating current changes all the time, there must be very many ways of averaging it, the one commonly used being the square root of the mean of the sum of the squares of the current taken over the whole cycle (i.e. the steady current which will produce the same heating effect as the A.C., called R.M.S.), and it is clear that the dynamometer, in which the measurable quantity \( x \) is proportional to \( I^2 \), will in fact measure the R.M.S. value of current and will do so in absolute units (i.e. in terms of mass, distance and time).

With some A.C. circuits the power dissipated cannot be calculated from the product of \( V \) and \( I \) measured by voltmeter and ammeter respectively as these two quantities are not in phase (e.g. in a choking coil). The dynamometer will measure however actual dissipated power and it is therefore of importance.

**Experiments 43**

**The determination of the magnetic elements in the laboratory**

**Introduction**

The magnetic elements are (a) the horizontal components of the earth's field, (b) the angle of dip, (c) the declination, which is the angle between the geographical and the magnetic meridians.

**Apparatus**

Tangent galvanometer (two turns in the coil), ammeter (0–0.1 A), rheostat, reversing switch, one or two accumulators, mu-metal rod 1 metre long and about 1 cm diameter, the whole length of which is closely and uniformly wound with cotton covered wire (about 12 turns/cm), long cord and weight, trough magnet.

**Method**

(It is best to carry out this experiment on a separate table away from iron pipes and close to a suitable window so that it receives the sun at noon.)

Set up the circuit shown in Fig. 1, making sure that the 90°–90° line on the deflection magnetometer is in the plane of the coil, and that the
coil itself is turned so that its plane is in the magnetic meridian (the pointer should then be set at 0°–0°); also make sure that the rheostat and ammeter are as far away from the galvanometer as possible. Adjust the current until the pointer is deflected through 45° (if the pointer is not straight, adjust the current so that each end of the pointer in turn is set at 45°). Record the values of the current, which should then be reversed and the observations repeated and averaged.

Move the tangent galvanometer to another place on the table and repeat the whole experiment.

Fig. 1

Now connect up the circuit shown in Fig. 2 with the mu-metal rod vertical, using the deflection magnetometer from the tangent galvanometer, and recording the pointer readings with the rod removed (there is no need to adjust the magnetometer previously).

When the rod is in position the magnetometer is deflected. (Reverse the rod—do you get a similar deflection?) Now switch on the current so that the deflection is reduced and adjust the current for zero deflection. Record the current, reverse the rod and current and repeat.

Turn the rod over so that it is lying horizontally in the magnetic meridian and adjust the current for zero deflection as before. (When the current is so adjusted, does the displacement of the rod in the magnetic meridian produce any deflection?) Reverse the rod again and repeat.

To find the declination, hang the weight by the cord at the window so that its shadow, when the sun is shining, is cast on the table and determine its position precisely at noon (remember Summer Time). Use the trough magnet to locate the magnetic meridian accurately and measure the angle between the direction of the shadow (the geographical meridian) and the magnetic meridian (record whether East or West) to the nearest degree.

Theory and Calculation

If the mean current required to deflect the pointer through 45° is $I$ A then $H_0 \tan 45° = 2\pi NI/10R$ where $R$ is the radius of the coil in centimetres, $H_0$ is the horizontal component of the earth’s field and $N$ is the number of turns. Hence $H_0$ oersted.

\[
\text{(M.K.S. } \quad H_0 \tan 45° = \frac{NI}{2R}, \text{ R being in metres. Hence } H_0 (\text{A m}^{-1}).\text{)}
\]
ADVANCED LEVEL EXPERIMENTS

For the second experiment the vertical component of the earth’s magnetic field \( V_0 = \frac{4\pi n I_1}{10} \) and the horizontal component \( H_0 = \frac{4\pi n I_2}{10} \) where \( I_1 \) and \( I_2 \) are the mean currents required to neutralize the magnetic field when the rod is vertical and horizontal respectively, \( n \) being the number of turns/cm.

\[
\begin{align*}
(M.K.S.) & \quad V_0 = nI_1, \ n \text{ being the number of turns/m.} \\
& \quad H_0 = nI_2
\end{align*}
\]

How does \( H_0 \) compare with that obtained from the first part? If the angle of dip is \( \theta \) then \( \tan \theta = \frac{I_1}{I_2} = \frac{V_0}{H_0} \). Give \( \theta \) to the nearest degree.

COMMENTS

As the deflection of the magnetometer in the first part of the experiment can only be read to the nearest 0.5°, and if the calibration of the ammeter is correct to within ±1‰, what is the maximum percentage error in \( H_0 \)? Do you think higher accuracy in measuring \( I \) is justified? Test the hypothesis that the variation of the magnetic elements over the earth’s surface fits in roughly with that produced by a magnetic dipole in the centre of the earth by using the arrangement shown in Fig. 3. Draw a large quadrant of a circle representing half of the northern hemisphere and place a ticonal magnet at the centre with its axis along the magnetic axis of the earth (look up the latest geographical location of the magnetic north pole). Place a coil of 2000 turns of very fine wire containing a ferrite core connected to a ballistic galvanometer (Scalamp) at a point on the quadrant corresponding to the latitude of the laboratory. Lift the coil up quickly (without turning it) when placed radially and tangentially, and by comparing the throws of the ballistic galvanometer get a rough value of the angle of dip \( \theta \). If \( \lambda^\circ \) is the latitude investigate whether \( \tan \theta = 2 \tan \lambda \) for different points on the quadrant.

EXPERIMENT 44

The determination of the magnetic moment of a short magnet and the investigation of the variation in magnetic field along its axis

APPARATUS

A short ticonal magnet (about 4 cm long), large circular coil (about 60 cm in diameter and 100 turns), chemical balance with small
accessories (see text), 12 V D.C. supply, rheostat, ammeter (0–1 A), switch, tangent galvanometer, 50 cm ruler, wooden retort stand.

**Determination of the magnetic moment of a short magnet**

**Method**

Clip the ticonal magnet to the balance beam so that it rests horizontally just above the central knife-edge, as shown in Fig. 1. Place the large coil on top of the glass case of the balance so that its centre lies above the centre of the magnet.

![Diagram of magnetic moment setup](image)

**Fig. 1**

Connect up the circuit as shown. In the absence of any current in the large coil, counter-balance the two scale pans to within a milligramme. Switch on the current and adjust it to 0.1, counter-balance again and note the weight required to restore balance, to the nearest milligramme. Repeat five times with higher values of current, tabulating your results as shown in table on facing page.

**Theory and Calculation**

When a magnet is placed with its axis at right angles to a uniform magnetic field $H$, it will experience a couple $MH$, where $M$ is the magnetic moment. The magnetic field at the centre of a circular coil is
2πNi/r where N is the number of turns, i the current in absolute amperes and r the radius of the coil in cm (for M.K.S. \( H = NI/2R \), I in amps, R the radius in metres). Therefore \( \frac{2\pi NI}{10r} \cdot M = \frac{w}{1000} \cdot 981 \cdot l \) where l cm is the distance between the pivot of the beam of the balance and the point of suspension of the pan (for M.K.S. \( \frac{100NI}{2R} \cdot M = \frac{w}{10^6} \cdot 9.81 \times \frac{l}{100} \)).

In the theory given above, it is assumed that the magnetic field is not only uniform over the whole length of the magnet, but is the same as that at the centre of the coil. The value of \( M \) deduced from the above equation can therefore only be a close approximation.

If you plot \( I \) against \( w \), you get a straight line passing through the origin, from which you can read off the value of \( w \) when \( I \) equals 1 amp. Substitute in either of the above two equations and deduce \( M \) (M.K.S.: \( M \) is in weber metre).

**Investigation of the variation in the magnetic field along its axis**

**Method**

Set the tangent galvanometer with the plane of the coil in the magnetic meridian, the aluminium pointer reading 0–0. Fix the 50 cm ruler level with the magnetometer, along the axis of the coil, i.e. along the magnetic E–W direction. Place the ticonal magnet so that its axis coincides with that of the coil, the mid-point of the magnet being approximately 10 cm from the centre of the coil. Connect the tangent galvanometer to the circuit as in Fig. 2 and adjust the strength of the current until the aluminium pointer returns to zero position (you may find it necessary to interchange connections to the tangent galvanometer to ensure that the field of the coil is in opposition to that of the magnet; you should select a suitable number of turns on the coil, in order that
the balancing current should not be too small). Repeat the experiment after increasing the distance $x$ between the magnet and the coil. At greater distances (say over 30 cm) fewer turns of the coil might be used. Tabulate your results as follows:

<table>
<thead>
<tr>
<th>Distance $x$ cm or metre</th>
<th>Balancing current $IA$</th>
<th>Number of turns $N$</th>
<th>$1/\sqrt[3]{NI}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The distance $x$ need not be measured from the centre of the coil, but from any convenient fixed point.

**Theory and Calculation**

The field along the axis of the magnet varies inversely with the cube of the distance $d$ to the centre of the magnet, but in this experiment, the field of the magnet is balanced by the field of the coil which is $\pi NI/5r$ oersted, $r$ being the radius of the coil in cm (for M.K.S. $NI/2R$ ampere-turn metre$^{-1}$, $R$ being in metres). The deflection magnetometer is merely used as a null-point instrument. To verify the relationship, $NI \propto 1/d^3$ graphically, we cannot plot $NI$ against $1/x^2$ as $x$ is not necessarily equal to $d$. We have instead to plot $1/\sqrt[3]{NI}$ against $x$ and we should get a straight line of which the slope should be deduced. From this slope we can verify the expression for the field of a magnet along its axis, $2M/d^3$, for $\frac{\pi NI}{5r} = \frac{2M}{d^3}$

and hence $\frac{1}{\sqrt[3]{NI}} = \frac{3}{10} \frac{\pi}{RM} \cdot d$; as $M$ is known from part 1, $r$ can be found, and the formula verified (for M.K.S. as $\frac{NI}{2R} = \frac{M}{2\pi \mu_0 d^3}$ thus $\frac{1}{\sqrt[3]{NI}} = \frac{3}{R} \frac{\pi \mu_0}{\mu_0} \cdot d$ where $R$ is the radius in metres and $\mu_0$ the space-permeability $\mu_0 = 4\pi 10^{-7}$ Henry metre$^{-1}$).

**Comments**

Some free atoms and molecules behave like simple magnets and the above experiment, illustrating the behaviour of magnets in a magnetic field, could just as well be applied to such atoms and molecules. Another experiment which illustrates this behaviour is described below.

The apparatus consists of two large Helmholtz coils, each 100 turns, 60 cm diameter, placed coaxially with their planes parallel (as shown in Fig.3), separated by 30 cm, arranged so that the magnetic field produced by the
energized coils is parallel or anti-parallel to the earth’s field. A ticonal magnet in a small stirrup is suspended by 20 cm of copper wire (30 S.W.G.) from a circular brass disc (or torsion head) which can be rotated about a vertical axis, the rotation being measured on an annular protractor with its axis on the centre of rotation.

Mark the axis of the coil with a chalk line as in the diagram, and turn the torsion head so that the magnet comes to lie parallel with that axis. Read off the position of the torsion head after doing this. Now connect the coils in series to a 12 V D.C. supply, variable resistor, ammeter and switch, switch on the current and adjust it to a minimum. Turn the torsion head gently and you will note that the magnet turns also but to a much lesser degree, indicating that the suspension wire is being twisted, due to a couple on the magnet. When the magnet has set so that its axis is at right angles to the axis of the coils, it will be experiencing a maximum couple, and if the torsion head is twisted farther, the magnet quickly untwists. Measure the maximum angle of twist \( \theta \) (i.e. the angle turned by the torsion head less 90\(^\circ\)). Check your result again. Repeat with increasing values of current; tabulate your results.

Plot \( \theta \), the angle of twist, against \( I \), the current in the coils. The straight line which you get does not pass through the origin; find its slope.

\[ C, \text{ the torsional constant (dyne cm radian}^{-1}\text{), is given by} \]

\[ \frac{C \cdot \theta \pi}{180} = M(H \pm H_o) \]

where \( M \) is the magnetic moment of the magnet, \( H_o \) is the horizontal component of the earth’s magnetic field and \( H \), the magnetic field of the Helmholtz coils, is given by \( H = \frac{16\pi NI}{25\sqrt{5}r} \) (\( r \) is the radius of the coil, \( N \) is the number of turns of the coil, and \( I \) is the current in amperes).

In this case \( r = 30 \text{ cm}, N = 100 \).

To find \( C \), remove the magnet and replace it by a brass bar of known moment of inertia \( (a) \), as found from its mass and dimensions, and find the period of oscillation \( T \) as accurately as you can. We know that \( T \) equals \( 2\pi \sqrt{a/C} \), hence estimate a value for \( M \) (for M.K.S. \( H \), the field in ampere turns per metre, is \( 8NI/5\sqrt{5}r \), \( N = 100 \) turns, \( r = 0.3 \text{ m} \) and \( I \) is in amperes. The moment of inertia should be calculated in kg m\(^2\) and \( C \) is newton-metres per radian.
EXPERIMENT 45

The study of the conduction of electricity in (a) non-metallic conductors, (b) liquids and (c) flames

(a) Non-metallic conductors

APPARATUS

Metrosil disc (specification No. 96) with a fine copper wire soldered carefully to each face of the disc, 18 V D.C. supply, milliammeter (0–15 mA) or an Avometer, variable resistor (plug type) 0–10,000 (maximum heat dissipation less than 0.2 W), switch.

METHOD

Connect up the circuit shown in Fig. 1; starting with R set at maximum read the current and tabulate as follows:

<table>
<thead>
<tr>
<th>I mA</th>
<th>R ohm</th>
<th>RI/1000 V</th>
<th>— RI/ 0V</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Decrease R and repeat until R is zero.

(b) Liquids

APPARATUS

Two platinum electrodes set rigidly apart, immersed in dilute sulphuric acid. 12 V A.C. supply, 8 ohm rheostat used as a potentiometer divider, A.C. ammeter or an Avometer, ruler.

METHOD

Connect up the circuit as shown in Fig. 2, but before switching on the
current measure the distance \( L \) between the positions of the slider when no voltage is tapped off and when it is set at a maximum; then set the slider at a distance \( d \) along to give a small voltage and read off the current on the A.C. ammeter, tabulating your results as follows:

<table>
<thead>
<tr>
<th>( d ) cm</th>
<th>( 12 \times d/L ) V</th>
<th>( I ) mA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Repeat up to \( d = L/2 \) or 6 V.

(c) **Flames**

**Apparatus**

This is shown in Fig. 3 and consists of 24 short clay tubes fixed by Araldite to a long narrow box made of tinplate. When lit, this modified Bunsen burner must be held above the bench (to keep it cool), and the flame must be non-luminous and hot (this is done by adjusting the gas inlet
jet just before final assembly). Two iron electrodes, variable H.T. supply, microammeter, 1 megohm safety resistor.

**Method**

Connect up the circuit as shown in Fig. 3. The clay tubes are meant to insulate the flame, but make sure that there is no large leakage of current between one of the electrodes and the metal gas burner. Vary the H.T. supply and measure the current on the Scalamp, tabulating your results as shown, checking each observation at least once.

<table>
<thead>
<tr>
<th>V V</th>
<th>I µA</th>
<th>Check I</th>
<th>Average I</th>
</tr>
</thead>
</table>

**Theory and Calculation**

Plot in each case the p.d. in volts against the current and comment on each graph. You will note that the Metrosil, unlike metallic conductors, does not obey Ohm's Law; find by plotting \( \log V \) against \( \log I \) the relation between \( V \) and \( I \). Estimate the resistance between the iron electrodes in the last experiment.

**Comments**

Metrosil is a ceramic resistance and has many industrial applications. Can you think of any?

Alternating p.d. is used in part (b) to reduce the effect of polarization (some periodic shaking of the electrodes would give more consistent results) and this is usually the method for studying the conductivity of electrolytes.

For moderate voltages, gases are almost non-conducting, as there are very few ions produced continuously in them to act as carriers for current, but in flames or in the presence of other ionizing sources like ultra-violet, X-rays or radioactivity they become conducting. Investigate the relation between current and the distance between the electrodes at constant voltage, and also study the effect of lowering the temperature of the flame (you can measure the temperature of the flame using a calibrated chromel-alumel thermocouple).
EXPERIMENT 46

To use the Wheatstone bridge (a) to study the effect of the alloying of metals, (b) to compare the variation of resistance with temperature of metals, alloys and semi-conductors, and (c) to convert a galvanometer into a voltmeter.

The Wheatstone bridge has already been used in Experiment 13 and the purpose of this experiment is to use the bridge to investigate some problems connected with the conductivity of electricity in solids and also to extend its practical use.

APPARATUS

Wheatstone bridge, accumulator, slider, centre zero galvanometer, with a sensitivity switch, 1 ohm standard resistor, 3 bobbins consisting of 4 metres of 26 S.W.G. bare wire of copper, nickel and nickel-copper alloy (Ferry wire) respectively, switch, thermistor (preferably in glass capsule), liquid paraffin in a test tube, 2 metres of 32 S.W.G. bare copper wire wound on paxalin former to fit inside the test tube, thermometer, water bath, crushed ice and stirrer, 1 metre 26 S.W.G. bare eureka wire on paxalin former. P.O. box, 200 ohm resistance box, multi-range meter without shunt or series coil, 5 ohm standard resistor, a dry battery.

(a) The effect of the alloying of metals

METHOD

Connect up the circuit shown in Fig. 1, using one of the bobbins as the unknown resistor X. Shunt the galvanometer and test for opposite deflection by touching both ends of the slide wire with the slider. Locate the

![Fig. 1](image-url)

balance by 'bracketing' it, touching (not scraping) the wire 1 mm on either side of the balance with the shunt off (try not to warm the slider in your hand—hold it by the insulation). Record $l_1$ and $l_2$. Interchange resistors and repeat. Replace the bobbin and repeat until all the bobbins have been tested.
THEORY AND CALCULATION

It has been established in Experiment 13 that \( \frac{X}{R} = \frac{l_1}{l_2} \). Hence \( X = \frac{l_1}{l_2} R \).

Use the data from the experiment to calculate the resistance in ohms per metre for each bobbin.

(b) Variation of resistance with temperature of metals, alloys and semi-conductors

METHOD

Use the circuit shown in Fig. 1 to measure the resistance of the copper wire bound on paxalin and, similarly, the eureka wire. The wires should be immersed in liquid paraffin contained in a test tube surrounded by crushed ice and water. Wait until conditions are steady, take the temperature, find the balance point, interchange resistors and repeat. Tabulate your results as follows:

<table>
<thead>
<tr>
<th>Temperature (^\circ)C</th>
<th>( l_1 )</th>
<th>( l_2 )</th>
<th>Check ( \frac{l_1}{l_2} )</th>
<th>( \frac{l_1}{l_2} )</th>
<th>Check ( \frac{l_1}{l_2} )</th>
<th>Average ( \frac{l_1}{l_2} )</th>
<th>( X = \frac{l_1}{l_2} R ) ohm</th>
</tr>
</thead>
<tbody>
<tr>
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</table>

Now use water at room temperature and boiling water and repeat in each case. If you try to measure the resistance of the thermistor at \( 0^\circ\)C using the

![Figure 2](image-url)
metre bridge, you will find the bridge insensitive, so that the P.O. box will have to be used, connected up as shown in Fig. 2. Use ratio arm 1, \( R_1 = R_2 = 1000 \) ohm, find the balance point by adjusting \( R_3 \) and record it. As with copper and eureka, replace the crushed ice with tap water and boiling water and repeat the observation each time.

**Theory and Calculation**

Calculate \( X \) in each case for the three steady temperatures taken. Plot \( X \) against temperature and comment on the three results. Calculate the temperature coefficient of resistance \( a \) for the two metals from the relation

\[
a = \frac{X_{100} - X_0}{100X_0}.
\]

(c) **Conversion of a galvanometer into a voltmeter**

**Method**

Use the circuit shown in Fig. 1 again with \( R = 5 \) ohm standard resistor, \( X \) the universal indicator (the positive terminal connected to the left terminal of the gap) and \( S \) adjusted for 100 ohm. Find the balance point in the usual manner, interchange resistors and repeat.

![Fig. 3](image_url)

To find the current needed to give the galvanometer a full scale deflection (F.S.D.) the circuit in Fig. 3 will be found adequate. Before you close the switch see that the plugs of the box are all out and then replace them gradually until the galvanometer is giving its full deflection; record the corresponding resistance of the box. You may find the e.m.f. of the accumulator using an Avometer—otherwise assume it to be 2.0 V. Your task now is to calculate a suitable series resistor which would convert the indicator to read up to 1.5 V; use your resistance box for this purpose and find the p.d. of the dry battery provided.

**Theory and Calculation**

Calculate the resistance of the indicator \( r \) and if the F.S.D. is 1 A then

\[
I = 1.5/(R + r)
\]

where \( R \) is the value of the series resistance required. From (b) you have already found the resistance/m of 26 S.W.G. eureka wire; find the number of metres of this wire required to make a bobbin suitable to put in series with the indicator so as to read up to 6 V.
COMMENTS

The results from the parts (a) and (b) of the experiment are revealing. You have found that while the resistances of copper and nickel are comparable, the resistance of the alloy is between 15 to 30 times that of the constituent metals, also that while the resistance of copper varies roughly with the absolute temperature (i.e. \( \alpha = \frac{1}{375} \)), that of eureka hardly varies at all. In the case of the thermistor the resistance actually decreases in a marked and complicated way when it is heated.

You can deduce from the comments on Experiment 15 that the current \( I \) carried by one type of ion (say negative) is \( I = neA \cdot u \cdot \frac{V}{l} \), where \( n \) is the number of negative ions per ml, \( A \) the area of cross-section, \( l \) the length of the conductor, \( V \) the p.d., \( e \) the electronic charge in coulombs, \( u \) the velocity of the ion in a unit electric field (ionic mobility). Putting \( V = RI \) where \( R \) is the resistance in ohms, we get, cancelling \( I \),

\[
1 = neAu \cdot \frac{R}{l}.
\]

But \( R = \frac{sl}{A} = l/\alpha A \), where \( s \) is the specific resistance in ohm cm and \( \alpha \) is the electrical conductivity in mho cm\(^{-1}\).

Therefore

\[
1 = neAu \times \frac{l}{l} \times \frac{l}{\alpha A} \quad \text{or} \quad \alpha = neu.
\]

As both copper and nickel are good conductors, \( n \) must be comparable for both. In the alloy, one suspects \( u \) to be small. One also expects hardened wire to have a higher resistance than annealed wire, suggesting that \( u \) must depend on how perfect the crystalline structure is. Any disorganization such as alloying, defects through heat treatment, or just raising the temperature, causing the amplitude of oscillation of the atoms to increase, will decrease \( u \). The last fact can explain the value of \( \alpha \) for copper and also explain why \( \alpha \) is nearly zero for eureka, for the atoms in an alloy are sufficiently disorganized for the resistivity of the alloy to be unaffected by increased oscillations. By the wave theory one associates the conduction

![Fig. 4](image-url)

of electrons with the propagation of waves, which can be scattered by imperfections just like light in imperfect glass (see comments on Experiment 37).

The decrease in resistance of the thermistor with temperature can only be explained by the fact that \( n \) increases with temperature, as more electrons held captive in the valence band acquire sufficient energy to overcome the narrow barrier and escape into the conduction band. This narrow barrier \(< 1 \text{ eV}\) is the hallmark of a semi-conductor \((\alpha \text{ is negative})\) and when this barrier is wider the solid becomes an insulator. For a semi-
conductor $R \propto \varepsilon^{1/T}$, where $T$ is the absolute temperature (in contrast to metals where $R \propto T$). Verify.

The P.O. box as the term suggests is the portable version of the Wheatstone bridge used by Post Office engineers in testing for telephone line faults. If a 'short' occurs between one line and earth (Fig. 4) at distance $d$ from a testing point, the far point is shorted and the bridge is connected as shown, balancing by adjusting $R_s$. Verify that, if $\rho$ is the resistance per unit length, then $\frac{R_1}{R_2} = \frac{R_s + \rho d}{2L_\rho - \rho d}$. Hence $d$ may be calculated. This is called a Varley loop and in practice modifications of these circuits are adopted to suit the circumstances.

For greater accuracy in measuring a resistor the P.O. box is to be preferred, as the uniformity of resistance of the metre bridge wire cannot be taken for granted, its 'end' effects cannot be ignored and though the thermoelectric effect at the slider can be reduced (by reversing the current in the wire and repeating observations), it cannot be eliminated.

EXPERIMENT 47

To find how the current and power taken from a battery depend on the external resistance; also to investigate the relationship between the internal resistance of a Daniell cell and the current taken from it

**Apparatus**

Dry battery 1-5 V, dry battery 90 V, ammeter with two shunts (0-1 A, 0-0-3 A), resistance box 0-20 ohm, resistance box 0-20,000 ohm, Daniell cell (zinc sulphate used in place of sulphuric acid), centre zero galvanometer, potentiometer, accumulator, two switches.

![Fig. 1](image1.png)

![Fig. 2](image2.png)

**Method**

Connect up the circuit shown in Fig. 1. Check for any zero error in the ammeter. Vary the resistance $R$ and record the corresponding current, tabulating your results in the ordinary way. Connect up the circuit shown in Fig. 2, using the H.T. battery and the variable high resistance. Start with the maximum resistance of 20,000 ohm, reducing it by steps, but do
not use a series resistance lower than 500 ohm. Do not allow the current to flow for long; switch off as soon as you can. Allow the battery a little time to recover and repeat your observations as a check, tabulating your results as follows:

<table>
<thead>
<tr>
<th>$R$ ohm</th>
<th>$I$ mA</th>
<th>$I$ mA</th>
<th>Av. $I$ mA</th>
<th>Power $I^2R/10^9$ W</th>
</tr>
</thead>
</table>

Connect up the circuit shown in Fig. 3. Close $K_2$, with $R = 0$ for two minutes, then open $K_2$. As with all 'null-point' methods you must first check with 'opposite deflection' test by touching both ends of the potentiometer wire quickly with the slider, making sure that the deflections of the galvanometer are opposite. If not, check first whether the positive terminal (the copper) of the Daniell cell is connected to the positive of the accumulator, then touch the wire with the slider but with $K_1$ open (i.e. driver cell out). If there is no deflection, then the Daniell cell or its connections are faulty; if on the other hand there is deflection but in the same direction as when $K_1$ is closed then the potential drop along the wire is not enough to balance the potential of the Daniell cell. Check for faulty contacts, or replace the accumulator.

Find the balance point $X_1$ by 'bracketing' (moving the slider 1 mm either side of the balance point, noting opposite deflections). After noting $l_1$ make $R = 1$, close key $K_2$ and quickly find the new balance length $l_1$; open $K_2$, recording your observations as shown in the table on following page. This is repeated with different values of $R$. Replace the Daniell cell by the dry battery (1.5 V) and find the balance length $l$ or the wire with $K_2$ open.
\[ l_1 \text{ cm} \quad R \text{ ohm} \quad l_2 \text{ cm} \quad r = \frac{l_1 - l_2}{l_2} R \text{ ohm} \quad I = \frac{E l_2}{R l} \text{ A} \quad E = 1.5 \text{ V} \]

**Theory and Calculation**

A cursory examination of your results with the L.T. and the H.T. dry batteries, particularly the H.T., should have convinced you that when the external resistance has been doubled the current does not seem to be halved. A possible explanation is that there is an additional resistance in the circuit, located inside the battery and called the internal resistance, designated by \( r \). If the battery generates a potential \( E \) volts (called electromotive force, e.m.f., being the p.d. of the battery when not delivering a current) then, extending Ohm’s Law to the whole circuit, the current \( I = E/(R + r) \), \( R \) being the external resistance. This supposition can be put to the test either by solving simultaneous equations any pair of observations of \( I \) and \( R \) or, better still, by plotting a graph; for \( 1/I = (R + r)/E \) and by plotting \( 1/I \) against \( R \) you can verify whether you do in fact get a straight line. The intercept on the \( R \) axis (when \( 1/I = 0 \)) gives the internal resistance \( r \) and the gradient gives \( 1/E \), hence \( E \). Do the two batteries have equal or comparable internal resistances? Use the results from the second battery to calculate the power dissipated in the external circuit. Plot the power against the resistance, and deduce the value of \( R \) which gives the highest output power.

In the final experiment, the internal resistance of the Daniell cell can be calculated from the relation \( r = \frac{l_1 - l_2}{l_2} R \). The e.m.f. \( E \) of the dry battery having been already determined, this can be used to calibrate the potentiometer, thus if \( l_2 \) is the balance length when the external resistance of the Daniell cell is \( R \) then \( E/l = Rl/l_2 \) where \( I \) is the current from the Daniell cell. Hence \( I = El_2/Rl \). Now complete the last table and plot \( r \) against \( I \); comment on your graph.

**Comments**

The importance of this experiment is that it extends Ohm’s Law to a whole circuit and introduces two new ideas, those of e.m.f. and internal resistance. Powers delivered by different generators, having the same e.m.f., depend on the internal resistance, for the above experiment should have shown that the maximum power is obtained from the battery when the external resistance is equal to the internal resistance, so that the maximum power is \( E^2/4r \). The internal resistance also sets an upper limit to the current which can be taken from the battery. Accumulators have
usually very low internal resistances (see Experiment 86) so that to short an accumulator can be quite serious. Calculate the current which each of the two batteries gives when shorted accidentally.

Show that the power $P$ dissipated in an external resistance $R$ connected to a generator of e.m.f. $E$ and internal resistance $r$ (both assumed to be constant when $R$ is varied) is $RE^2/(R + r)^2$ and hence, by differentiating $P$ with respect to $R$, deduce the condition for maximum output power. How would you propose to find the e.m.f. of a 5 kV generator using one of the above methods?

**EXPERIMENT 48**

**To use a D.C. potentiometer (a) to calibrate a voltmeter, (b) to calibrate an ammeter, and (c) to compare low resistances**

**(a) Calibration of a voltmeter**

**Apparatus**

One metre slide wire potentiometer, sensitive centre zero galvanometer with shunt, 2 sliders, lead-acid accumulator (neither freshly charged nor run down), key, Weston cadmium standard cell with 1000 ohm series resistor and key, throw-over switch, 0–1·5 V voltmeter (for calibration).

**Method**

Connect up the circuit as shown in Fig. 1. Because of the uncertainty of

![Fig. 1](image-url)
the zero on the slide wire (so-called ‘end’ error) an artificial zero is made by clamping one of the sliders at a point 1·0 cm on the ruler (see inset in Fig. 1). Make sure that the correct polarities are joined together or else you will get a large current passing through the galvanometer as well as the standard cell when the latter is connected, to the serious detriment of both.

Short out the centre zero galvanometer and check, with the voltmeter in the test circuit, that the potentiometer is functioning correctly. Put the standard cell with the 1000 ohm resistor in series, in the test circuit, and locate approximately the balance point, but with the galvanometer shunted. Now short out the 1000 ohm resistor, cut out the galvanometer shunt and ‘bracket’ accurately the balance point and record the balance length \( l \). Switch over to the voltmeter in the test circuit, shorting out the galvanometer, record the positions of the tapped points corresponding to the voltmeter deflections of 0·1, 0·3, \ldots, up to 1·5, and complete the following table:

<table>
<thead>
<tr>
<th>Voltmeter readings</th>
<th>Tapped points</th>
<th>Tapped length ( x )</th>
<th>Calibrated voltage ( V = \frac{1.0183}{l} \times x ) V</th>
</tr>
</thead>
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</tbody>
</table>

Check with the standard cell again.

(b) Calibration of an ammeter

Apparatus

1 ohm standard resistor, 0–1·5 ammeter (to be calibrated—preferably a universal indicator with shunt), rheostat, accumulator, key.

Method

Connect up the circuit shown in Fig. 2, having previously checked on the resistance of the standard resistor using a P.O. box (see Experiment 46). As with the previous method calibrate the potentiometer using the standard cell and record the balance length \( l \). Switch on the side circuit and adjust the current to 0·3, 0·5 \ldots, 1·5 A, balancing the p.d. between the standard resistor and the potentiometer wire each time, recording your observations in a table similar to the one above. Again check back with the standard cell.
(c) The comparison of low resistances

**Apparatus**

0.1 ohm standard resistor (four terminals), a 1.5 A shunt for a universal and holder (see second experiment), another rheostat.

**Method**

Connect up the circuit shown in Fig. 3, join the standard 0.1 ohm resistor to the side circuit by the two outer terminals and the two inner terminals of it to the throw-over switch and potentiometer. Adjust the current so that the balance point when the 0.1 ohm is in the test circuit is nearly at the other end of the potentiometer wire. Find the balance length
$l_1$ when 0.1 ohm is inserted in the test circuit and $l_2$ when the shunt is in the circuit. Repeat with a different current in the side circuit. In what way does the inclusion of 1000 ohm in series with the galvanometer affect the balance point? Investigate.

**Theory and Calculation**

It is no exaggeration to say that most ammeters are used merely to indicate the approximate magnitude of the electrical quantities; to measure them properly you need a calibrated potentiometer. In the first method you are using the potentiometer as a calibrated potentiometer divider. Remember in recording the tapped length $x$ to deduct 1.0 due to the 'artificial' zero. If the slide wire is of uniform resistance then, if the accumulator current is kept constant, the calibrated voltage $V$ is proportional to $x$ and $V/x = 1.0183/l$, hence $V = (1.0183 \times x)/l$. Plot indicated voltages against the corresponding calibrated voltages.

In the second method, if $I$ is the current in the side circuit and $R$ is the resistance of the standard resistor, then $RI/x = 1.0183/l$. Hence $I$. Complete the table and plot a similar graph as before.

In the third method, as the same current passes through both the 0.1 ohm and the shunt (resistance $X$ ohm) then $0.1/x = l_1/l_2$. Hence $X$.

**Comments**

The slide wire used above gives rather limited accuracy (discuss sources of error); for higher accuracy the slide wire is only used for fine adjustments to the balance, as several series resistors, each equal to the whole slide wire, are used with it; the number of resistors being selected depends on the p.d. under test. For higher accuracy still, no slide wire is used at all but only banks of resistors.

In the first method it is assumed that the resistance of the voltmeter is very high compared with that of the slide wire, otherwise the p.d. measured is not the same as that calibrated by the standard cell. This point must be borne in mind whenever a voltmeter is used to measure p.d., for when the p.d. across a high resistance is to be measured such a voltmeter as you have just used would be useless. By measuring the resistance of the slide wire and the resistance of the voltmeter, estimate the percentage error likely by the shunting action of the voltmeter.

With low resistances the P.O. box becomes unreliable, as the sum of the resistance of the leads and the contact resistance becomes comparable with the resistance under test. In the third method the leads resistance does not affect the balance point, as you must have found, for, when a resistance as high as 1000 ohm was joined in series with the centre zero galvanometer, it merely made the balancing less sensitive.
EXPERIMENT 49

To measure the thermoelectric e.m.f., to use a thermocouple to measure the temperature of a Bunsen flame and to use a telluride alloy thermocouple to study 'heat pumping'

APPARATUS

A slide wire potentiometer, 1000 ohm resistance box, accumulator, Pye Scalamp microammeter, eureka-copper thermocouple, manganin-copper thermocouple, chromel-alumel thermocouple, oil bath, sand tray, tripod, 0–300°C thermometer, Bunsen burner, crushed ice in vacuum flask; a telluride alloy thermocouple bolted to thick copper strips as heat sinks, two D.C. ammeters (ranges, 0–5 A and 0–0·5 A), two rheostats (one heavy duty), special heating coil to fit over the top of the thermocouple (made of eureka wire embedded in Araldite polished and shellacked, resistance about 3 ohm), a throw-over and an ordinary switch, 20 ohm resistor, two accumulators.

METHOD

Find the resistance of the slide wire using a P.O. box and adjust the series resistance $R$ (Fig. 1) so that the p.d. between the ends of the slide wire is about 5 mV (assume that the e.m.f. of the accumulator is 2·0 V). Place one junction of the eureka-copper thermocouple in ice and the other in warm water and test that the thermocouple is connected with the correct polarity to the potentiometer by noting the opposite deflection of the microammeter when the far end of the wire is touched with switch K open and closed (this would also give an idea of whether the voltage sensitivity of the galvanometer is adequate, as this can be altered). Find the length of wire $l$ which will balance the thermoelectric e.m.f. when the hot junction is in boiling water, alter $R$ and repeat as a check.

Repeat the experiment using the manganin-copper thermocouple. The arrangement for the chromel-alumel thermocouple is shown in Fig. 2. There are two cold junctions where the copper leads join the two alloys and these two junctions should each be well wrapped with cotton wool and kept together. $R$ should then be reduced to give a larger p.d. across the slide wire (about 40 mV). Besides the steam point, the oil bath should be used to provide further calibration points. The oil bath should be heated carefully on a sand tray and the temperature kept steady while the
thermoelectric e.m.f. is being measured. Place the hot junction in the hot part of a Bunsen flame and balance the steady e.m.f. developed against the slide wire. Record the balance length.

The use of the Peltier effect to produce cooling, which will be utilized in Experiment 61, is next investigated and the circuit in Fig. 3 should be connected up. Note the reading on the Scalamp when the thermocouple is connected to it as this is a measure of the temperature of the top junction of the thermocouple. The thermocouple is next connected to the bottom circuit and a large current is switched on to make sure that heat is pumped out of the top of the thermocouple, or else the current in the thermocouple is reversed. Strap the heating coil with a small rubber band to the top of the thermocouple, which is smeared with glycerine. Switch on for a few minutes both the heating current and the thermocouple current, set at maximum (depending on the thermocouple used), then open the switch K₁ and simultaneously connect the thermocouple to the Scalamp microammeter. From the Scalamp reading and your previous reading find whether the heating current is adequate to supply as much heat as that removed from the thermocouple. It is possible to adjust the heating current so that no change in the deflection of the microammeter is detected; record both heating current $I$ and 'heat-pumping' current $C$ in the following table:

<table>
<thead>
<tr>
<th>Heating current $I_A$</th>
<th>Current in thermocouple $C_A$</th>
<th>Power in heating coil $P_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Repeat with different values of $C$. 
THEORY AND CALCULATION

If the resistance of the slide wire (taken to be 100 cm long) be \( r \), then the p.d. between its ends is \((2 \cdot 0 \times r)/(R + r)\) and the p.d. per cm is \(20r/(R + r)\) mV cm\(^{-1}\). Deduce the e.m.f. per degree C difference in temperature between hot and cold junctions for the three thermocouples used (take room temperature as the cold junction temperature for the chromel alunel couple). Comment. Plot the power developed in the heating coil against the ‘pumping’ current \( C \) and you should get a straight line.

COMMENTS

The thermocouple, because it is simple, small, and has a low thermal capacity, is used extensively. A third wire of different material does not affect the e.m.f. provided that the temperature at the two points where it joins the thermocouple is kept the same, thus extending the leads without increasing appreciably either the resistance or the cost.

When a current is sent through a thermocouple one junction gets warmer and the other cooler. This reverse effect, called the Peltier effect, became a practical proposition, for refrigeration, with the advent of semi-conducting materials which can combine high thermoelectric e.m.f. with low thermal conductivities.

EXPERIMENT 50

To calibrate an A.C. ammeter

APPARATUS

Small squat vacuum flask half filled with water, heating coil made of eureka (about 4 ohm) fixed to a cork to fit the vacuum flask, 8 ohm rheostat, 12 V A.C. and D.C. supplies, standard 1 ohm resistor, potentiometer and accumulator, A.C. ammeter (0–1 A), sensitive centre zero galvanometer, switch, stopwatch, 50°C thermometer (reading to 0.1 deg C). Magnifying glass. Standard Weston cadmium cell.

METHOD

The circuit is connected as shown in Fig. 1, using, first, the 12 V A.C. supply. Adjust the current by means of the rheostat to give a steady 1 A as shown by the ammeter. Record the temperature every minute (using the magnifying glass), care being taken to stir the water well by shaking the flask at regular intervals, the results being tabulated in the usual way (a total temperature rise of 5 deg C is adequate).

Now change the electrical supply to D.C., making sure that the A.C. ammeter is either disconnected or shorted out by the key \( K_1 \). Allow the water to cool a little before starting the experiment again (to hasten this process, lift the cork up, without splashing out any water).
Switch on the D.C. supply and take similar temperature-time observations as before, but in addition balance the p.d. across the standard resistor against the length of the potentiometer wire $l_1$. Use the throw-over switch and balance the e.m.f. of the Weston cadmium cell, making sure that sufficiently high resistance is in series with it (remember that the maximum current allowed to pass through a standard cell is measured in $\mu$A; some cells have a high resistor already incorporated in series with them, so that the series resistance you include in your circuit is related to the particular cell you use). When a balance point is reached, short out the series resistor and find a more accurate balance length $l_2$. Repeat this at least once as a check. If time is available repeat the whole experiment with a lower alternating current.

**Theory and Calculation**

Plot on the same graph paper the two temperature-time results, draw the lines of best fit for each and deduce their gradients $a_1$ and $a_2$, for alternating and direct currents respectively (we shall assume that little heat is lost through the outside—this you can check anyway).
If \( I_1 \) A is the alternating current and \( I_2 \) A is the direct current, then
\[
\frac{a_1}{a_2} = \frac{I_1^2}{I_2^2} \quad (1)
\]
(the resistance of the heating coil is not likely to change).
If the resistance of the standard resistor is \( R \) then
\[
\frac{I_2 R}{1.0183} = \frac{l_1}{l_2} \quad \text{or} \quad I_2 = \frac{1.0183 l_1}{l_2 R} \quad (2)
\]
1.0183 V being the e.m.f. of the Weston cadmium cell.
Eliminating \( I_2 \) from equations (1) and (2) we get
\[
I_1 = I_2 \sqrt{\frac{a_1}{a_2}} = \sqrt{\frac{a_1}{a_2}} \cdot \frac{1.0183 l_1}{l_2 R} \quad \text{A}.
\]

**Comments**

The magnitude of an alternating current, like any average, can be defined in many ways. The R.M.S. value of alternating current is compared with that of direct current producing the same power when both are passed in turn through the same resistor.

**EXPERIMENT 51**

**To study the variation of the power of a tungsten lamp with its resistance**

**Apparatus**

A potentiometer consisting of two metres of 26 S.W.G. nickel-chrome wire, doubled up twice, 75 mV multi-range meter with two bobbins converting it into 0–1.5 V and 0–15 V, 7 ohm standard resistor (open type) 6.5 V, 0.3 A pea bulb in holder, 200 ohm (1 A) rheostat, sensitive centre zero galvanometer with shunt to render it less sensitive when required, tapper and 8 V D.C. supply.

**Method**

Connect up the circuit as shown in Fig. 1, using a single accumulator and the rheostat set at maximum resistance. Use the meter without its bobbins, so that it measures 75 mV on full-scale deflection. Using the centre zero galvanometer at its insensitive position test for opposite deflection by touching with the tapper either end of the two metre wire, then locate the balance-point roughly, change over the centre zero galvanometer to its sensitive position and 'bracket' accurately the balance-point (as you have learnt from Experiments 13 and 46) to the nearest millimetre. Measure the distance \( x \) from the balance-point to one end of the
wire A and the potential difference $V$ (which should only be a few millivolts) with the voltmeter. Record your observations in the following table:

<table>
<thead>
<tr>
<th>$V$</th>
<th>$x$ cm</th>
<th>$y$ cm</th>
<th>$R = 7x/y$</th>
<th>$I = \frac{V}{R+7}$</th>
<th>Power</th>
<th>$R_{293}$</th>
<th>$R/R_{293}$</th>
<th>Temperature $^\circ$K</th>
<th>$T^4$</th>
</tr>
</thead>
<tbody>
<tr>
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</tbody>
</table>

Increase the p.d. across the bridge in small steps and repeat and record the observation each time. Use the 1.5 V bobbin, and again increase the p.d. in small steps recording each observation in turn, and noting when the bulb begins to glow. Now replace the 1.5 V bobbin by the 15 V bobbin and use all the accumulators provided, increasing the voltage as before until the voltmeter registers the maximum p.d. of about 8 V when the rheostat resistance is cut out and the lamp is burning brightly. You should have about 20 observations in all.

**Theory and Calculation**

The basic principle of the circuit used is that of a 2 metre Wheatstone bridge. If $R$ is the resistance of the lamp in any particular observation, $x$ is the distance from the balance-point to the end A and $y$ the distance from the balance-point to the end B ($y = 200 - x$) then the condition for balance is $R/7 = x/y$ and $R = 7x/y$.

To find the power of the bulb we need to know the current $I$ passing through the bulb which by Ohm’s Law is $V/(R + 7)$ and hence the power of the bulb $PR$.

Next we need to know the resistance of the filament of the bulb at room temperature, but unfortunately any current passing through the bulb is
bound to heat up the filament, so we deduce the resistance at room temperature by plotting the power of the bulb against resistance (up to the value when the bulb begins to glow), extending the curve to cut the resistance axis (whence the power dissipated is zero) and recording its value as $R_{293}$ in the above table. (We shall assume that the room temperature is 20°C and the absolute temperature is therefore 293°K.)

![Graph](image)

Now complete the table shown, deducing the ratio $R/R_{293}$ for all observations in which the bulb appears to glow. Use the graph provided (Fig. 2), taken from the classical paper of Jones and Langmuir (General Electric Review, vol. 30, no. 6, page 312), to deduce the absolute temperature $T$. Complete column 9.

At high temperatures, radiation plays an important part in dissipating the heat and, although tungsten is not a black body, the heat radiated is proportional to $T^4$. Plot the power $P$ against $T^4$ and comment on the graph.

**Comments**

The importance of this experiment is historical. It was in 1879 that Stefan deduced empirically the fourth-power temperature law of radiation from Tyndal’s results on heat lost from hot wires, and confirmed earlier
work by Dulong and Petit on the heat lost from a blackened thermometer in vacuo. Five years later Boltzmann gave a theoretical proof to this law when applied to black bodies, now called the Stefan-Boltzmann Law.

The main difficulty with the above experiment, as with Tyndal's results, lies in estimating the temperature. The dependence of resistance on temperature is not sufficiently sensitive, nor is the determination of resistance at room temperature sufficiently accurate, as it is seriously affected by contact resistance. Other parameters than resistance can be used to evaluate temperature, but unfortunately they require knowledge of the dimensions of the tungsten filament.

**EXPERIMENT 52**

*To use the law of electromagnetic induction for the comparison of different magnetic materials which are in the form of rods*

**APPARATUS**

A long rotating copper tube, with a brush, of flexible copper strip, contacting each end. The tube is rotated by an electric motor (with a revolution counter) connected to it by a long shaft made of a glass rod with a small piece of rubber tubing at either end. A glass tube on which cotton-covered copper wire is closely wound to give 15 turns, the bore of this tube being wide enough to admit the rod specimens, which are of equal diameter and made of iron, hard steel and mu-metal. Millivoltmeter (Scalamp), L.T. supply, reversing switch, 0–200 ohm rheostat, switch.

![Fig. 1](image)

**METHOD**

Assemble the apparatus as shown in Fig. 1, insert the steel rod, which should be magnetized, into the glass solenoid, switch on the motor, find
the speed of the motor (number of revolutions per minute) and the induced e.m.f. across the brushes by pressing down the switch. Vary the speed of the motor and repeat the observation as many times as convenient. Tabulate your results in the usual way.

Now set the motor at a convenient speed (say 600 rev/min) and demagnetize the specimen by constantly reversing the current in the solenoid, starting with maximum current and then slowly reducing it to zero. Start with a small current in the solenoid, find the corresponding induced e.m.f., increase the current by stages to maximum and repeat the observations. Decrease the current; does the e.m.f. check with the previous value for the same current in the solenoid? Repeat the observations until the current is zero again, tabulating your results.

Repeat the experiment using the mu-metal and iron rods. Check throughout the experiment that the speed of the motor is constant. Remove the specimens and note any deflection of the voltmeter when the maximum current is switched on in the solenoid.

Theory and Calculation

At first it may seem odd that the rotating copper tube is cutting any magnetic lines of force at all, but remember that the magnetic flux emerging from the solenoid has both an axial and transverse component, and it is the latter which will be constantly cut by the rotating tube.

The first set of results provides further proof of the law of electromagnetic induction; here the flux is kept constant but the speed of rotation is altered and the graph of the e.m.f. against rev/min should give a straight line passing through the origin. In the magnetization of the specimens the speed is constant, so that the e.m.f. is a measure of the induced magnetic flux or of the magnetic flux density $B$, as all the specimens have the same cross-sectional area. A graph of the e.m.f. against the magnetizing current gives a revealing picture of how $B$ is related to the magnetizing force or field $H$. Comment on the similarities and dissimilarities of the various magnetization curves and their respective residual magnetism.

Comments

You will have noted the effect on the induced e.m.f. when the specimens are inserted (particularly the mu-metal) with the maximum current in the solenoid. The flux is increased a hundred times or sometimes a thousandfold and that is the reason why iron or other suitable ferromagnetic cores are used in electromagnets, relays, transformers and inductances. For inductances we require a high rate of change of flux for a given change of magnetizing current. Can you explain, with reference to your results, why the inductance goes down when high currents are used?
EXPERIMENT 53

The use of a ballistic galvanometer to (a) compare capacitances, (b) measure magnetic flux density $B$ with the help of a search coil

INTRODUCTION

Any sensitive galvanometer can be used ballistically provided the motion of the coil is undamped mechanically and electromagnetically. It follows therefore that once motion is started it will take the coil a long time to come to rest if it is in an open circuit, but once the coil is shorted, electromagnetic damping results and it is brought quickly to rest.

APPARATUS

A ballistic galvanometer, shorting key, two throw-over switches, 200 ohm resistance box, a mutual inductance (long solenoid with a secondary coil wound over its centre, both of known area of cross-section and number of turns), a large permanent magnet, search coil consisting of about 20 turns of cotton-covered copper wire wound over a suitable short brass or copper tube with a handle with which to jerk it out of the field, an accumulator, an ammeter (0–1 A), two condensers (about 1 $\mu$F each, one being unknown), rheostat.

METHOD

Connect up the circuit shown in Fig. 1, the rheostat being used as a potentiometer divider to provide a variable charging voltage. Try a test run first with a small charging voltage, using each condenser in turn by charging it first and then discharging it through the ballistic galvanometer. This should give an idea of the charging voltages required and which condenser has the bigger capacitance.

Start the experiment proper using the larger condenser (say $C_1$) first, with a suitable charging voltage. Charge the condenser and then rapidly discharge it through the ballistic galvanometer, noting the throw of the galvanometer. Without disturbing the rheostat, repeat the experiment with $C_2$, then with $C_1$ again and finally with $C_2$ again, completing the table overleaf. Before repeating the experiment with different charging voltages, check the zero. Repeat as many times as you can. To calibrate the ballistic galvanometer, measure the
voltage and the corresponding throw when the condenser with the known capacitance is used.

To measure the magnetic flux density, use the circuit in Fig. 2. Adjust $R$ to a suitable value (this should be in excess of a critical value, usually given by the instrument makers, for a lower resistance renders the galvanometer aperiodic and useless as a ballistic instrument) and test for the throw, lifting the search coil quickly out of the field. Now adjust the current $I$ in the primary of the mutual inductance and test for a suitable ballistic throw when the current is switched off (or reversed).

To start the experiment proper, note the zero reading and the deflection $a$ when the search coil is jerked out of the field, check the zero reading and repeat several times, and let the mean deflection be $a_0$. (Why are so many checks necessary?) Now find the ballistic throw when $I$ is switched off or reversed, check once and repeat with different values of $I$. Record in each case the value of $I$, the zero reading, the deflection of the galvanometer, its check and the average throw $a$ in a table similar to the one shown above.

**Theory and Calculation**

As both condensers are charged to the same potential, $a_1/a_2 = C_1/C_2$, or $C_1 = (a_1/a_2) \cdot C_2$, or $C_2 = (a_2/a_1) \cdot C_1$. Average $a_1/a_2$ from your table and deduce either $C_1$ or $C_2$ as the case may be. If $V$ is the p.d., $a$ is the throw in millimetres when the known condenser, capacitance $C$ (in $\mu$F), is used, then $a$ corresponds to a charge $VC \mu$coulomb and $VC/a$ is the charge sensitivity of the galvanometer in $\mu$coulomb mm$^{-1}$.
To calculate the flux density $B$, first plot $a$ against the current $I$ which should give you a straight line passing through the origin. Deduce the gradient $m (= a/I)$, let $N_1$ be the number of turns in the search coil, $N_2$ the number of turns in the secondary of the mutual inductance, then

**C.G.S.**

Let $A_1$ cm$^2$ be the area of the search coil, $A_2$ cm$^2$ be the area of the secondary coil, $n$ be the number of turns per cm of the primary coil, then

$$\frac{\alpha_0}{N_1 A_1 B} = \frac{\alpha}{4\pi n I A_2 N_2} = \frac{10m}{4\pi n A_2 N_2}$$

Hence $B$ (gauss).

**M.K.S.**

Let $A_1$ m$^2$ be the area of the search coil, $A_2$ m$^2$ be the area of the secondary coil, $n$ be the number of turns per m of the primary coil, $\mu_0$ the space permeability, then

$$\frac{\alpha_0}{N_1 A_1 B} = \frac{\alpha}{\mu_0 n I A_2 N_2} = \frac{M}{\mu_0 n A_2 N_2}$$

Hence $B$ (weber m$^{-2}$).

**Comments**

There are other practical problems which will arise in the rest of the book in which the ballistic galvanometer will be used. These problems can be tackled using the ideas already discussed above, but they are listed below in case the interested student might find time to do them beforehand:

1. Variation of the field along the axis of a long solenoid.
2. Variation of magnetic field with energizing current in an electromagnet.
3. Variation of the mutual inductance of two coils with the distance between them, or (for two fixed coils) with the position of a ferrite core.

A ballistic galvanometer is often used as a sensitive aperiodic galvanometer so that both current and charge sensitivity will be required for any given instrument. If $k$ is the current sensitivity (say $\mu A$ mm$^{-1}$) then it can be shown that the charge sensitivity is $kT/2\pi$ $\mu$coulomb mm$^{-1}$ where $T$ is the period of oscillation of the coil.

Fig. 3 shows a simple circuit which enables you to find the current sensitivity $k$ of the galvanometer. As the current through the galvanometer must be of the order of a few $\mu A$, $R$ must be about 200 ohm and the p.d. across the 1 ohm resistor will have to be of the order of 1 mV. Choose a fixed value for $S$, say 2000 ohm; vary $R$ to give you different deflections $\theta$ of the galvanometer. By plotting a suitable graph deduce the resistance of the galvanometer and the current sensitivity $k$. Find the period $T$ and hence deduce the charge sensitivity.

Modern suspended coil galvanometers have extremely light coils.
These are robust, because an accidental jerk to the galvanometer will not damage the suspension if the inertia of the coil is very small. Unfortunately there is a lower limit to this because a small moment of inertia would mean for a given suspension a short period $T$ which is undesirable in a ballistic galvanometer, as it is essential that the galvanometer should receive all the charge before the coil begins to move.

EXPERIMENT 54

A simple study of practical electric motors and generators

Apparatus

An increasing number of manufacturers are producing reasonably priced motor-generators; the one used in the author's laboratory will be described, see frontispiece.

One kilowatt 110 V D.C. motor. This is trunnion mounted, which means that the cage of the motor carrying the field coils is freely pivoted; thus the torque provided, when the motor is running, can be directly measured by means of a spring balance (this can be calibrated in ft lb wt

![Diagram of motor and generator circuits](image-url)
or m kg wt). 110 V generator (A.C. as well as D.C., using either slip rings or commutator) with maximum output current of about 5 A. A flexible coupling of motor and generator. Connection to the armature, shunt field coils, series field coils for the motor are made through clearly marked terminals on a panel. Similarly the two armatures of the generator and field coils are available on another panel.

One kilowatt transformer rectifier with 110 V D.C. output (provision can be made to obtain an A.C. output of about 110 V from this unit, which should prove useful in the laboratory for use with many American mains-operated appliances). A suitable starter for the D.C. motor, variable resisters in series with the field coils, 5 load resistance mats, suitably ventilated, to take the output current from the generator, each with its own separate switch.

Measuring instruments consisting of 0–150 V D.C. voltmeter, 0–150 V A.C. voltmeter, 0–10 A D.C. ammeter, 0–10 A A.C. ammeter and two 0–1 A D.C. ammeters.

A neon lamp and holder.

Method

It is not practicable in the space available to describe all the experiments which can be performed with this apparatus. Only one typical circuit will be discussed fully and suggestions will be made for further experiments.

A start can be made by measuring the resistance of the armature of the motor. Using the Wheatstone bridge (see Experiment 46) estimate this resistance to the nearest 0.01 ohm.

There are three ways of connecting up the motor: (a) as a shunt wound, (b) series wound, (c) compound wound motor. In this experiment it is connected as a shunt wound motor so that the armature is connected in parallel with the shunt field coils. Disengage the motor from the generator and connect up the motor circuit shown. Note that the shunt field coils (Z, ZZ) are connected directly to the output of the transformer rectifier unit marked 1 and 2 while the armature (A, AA) is connected to 3 and 4 in series with the starter. Adjust the torque balance to read zero and switch on the A.C. mains. Reduce the resistor F, in series with the shunt field coils to zero, so that the ammeter A, is reading the maximum current possible. Now very slowly turn the starter handle and note the large initial current in the armature read on the ammeter A, as the first stud is engaged, also note the large initial torque measured by the balance. As the armature speeds up you will note that both the armature current and the torque decrease. This operation takes nearly 30 seconds before the maximum speed of 3000 rev/min is reached. Now record the rectifier voltage, the field current, the armature current and the torque and tabulate your results as shown in the table overleaf.

If you divide the rectifier voltage by the armature resistance you get a value for the armature current X differing greatly from the one you recorded. Why? You appreciate the need for the starter resistance—in its absence the initial current would have been equal to X. The decrease in the armature current, as the armature speeds up, suggests that there must
be an opposing generated electromotive force (dynamo effect), which builds up to a value almost equal to the voltage applied to the motor. This ‘back’ e.m.f. multiplied by the armature current provides the useful power of the motor.

The large initial torque (due to the large current) is much greater than the torque needed to maintain the armature at a steady speed, thus providing the necessary acceleration. To prevent the overheating of the armature, the motor should be speeded up quickly and therefore it should not be started ‘on load’.

Now predict what happens to the speed of the motor as the field current is reduced. Check by observation.

Couple the generator to the motor and connect up the circuit as shown in the diagram. The generator is used as an alternator (i.e. generating A.C. —hence slip rings are used) and its field coils are separately excited by connecting them directly to the rectifier with a variable resistor in series set at maximum. Now adjust the motor field resistor $F_1$ until the motor rotates at 3000 rev/min. There is no need for a special stroboscope to do that as an ordinary neon lamp can be used to illuminate the commutator segments until they appear steady. Read the output generator voltage $e_\theta$ and the generator field current $i_\theta$, also record the rest of the observations shown in the above table.

Next switch on the first resistance mat across the generator; you will note that the armature current immediately rises, so does the torque, but what happens to the speed of the motor? To avoid using a stroboscope, adjust the resistor $F_2$ until the field current in the generator is $i_\theta$ again and therefore the output voltage of the generator is a measure of the speed of the motor.

This is repeated with increasing load current, recording all the observations in the appropriate columns of the table.

Now repeat the whole experiment using a higher value of field current in the generator, provided that the rated output power is not exceeded.

To switch off the motor-generator, you must first switch off the generator load, then give a sharp tap to the starter handle to cause it to spring back, switching off the A.C. mains supply at the same time.
THEORY AND CALCULATION

If \( V \) is the rectified input voltage to the motor, \( E \) is the back e.m.f., \( I \) the armature current, \( r \) the armature resistance, then

\[
V = E + rI.
\]

Multiplying both sides by \( I \), we have

\[
VI = EI + rI^2
\]

or

\[
EI = VI - rI^2 \quad \quad \quad \quad \quad \quad \quad (1)
\]

It is easy to show that \( EI \) is the useful power developed in the motor. The mechanical output power of the motor is easily calculated from the torque \( \Gamma \) m kg wt, for in one revolution \( 2\pi \Gamma \times 9.81 \) joules is done by the motor, and if the number of rev/sec is \( n \) the power generated is

\[
2\pi \Gamma \times 9.81 \times n \text{ W} \quad \quad \quad \quad \quad \quad \quad (2)
\]

If the voltage output of the generator is \( e \), \( n = \left( e/e_0 \right) \times 50 \). Plot \( EI \) as calculated from equation (1) against the mechanical power developed given in equation (2) and you should get a straight line passing through the origin. [Where \( \Gamma \) is in ft lb wt then the horse-power is \( \frac{2\pi \Gamma \times n}{550} \) or \( \frac{2\pi \Gamma \times n \times 746}{550} \text{ W} \) where 1 h.p. = 746 W.]

Next plot the relation between the speed of the motor \( n = \left( e/e_0 \right) \times 50 \) and \( \Gamma \) and comment on it. Unfortunately, if the transformer rectifier has a poor regulation, its output voltage may decrease markedly with increasing load current, hence the speed of the shunt wound motor would vary more markedly with the load than if the supply voltage were kept constant.

Finally, plot the efficiency of the motor against the load \( \Gamma \) m kg wt. The efficiency of the motor is

\[
\text{output power} = \frac{2\pi \Gamma \times 9.8 \times n}{V(I + i)}.
\]

If \( \Gamma \) is in ft lb wt then the efficiency \( = \frac{2\pi \Gamma n \times 746}{550V(I + i)} \) where \( I \) and \( i \) are the armature and field current of the motor. As one would expect, the efficiency increases with load to reach its maximum at the highest rating of the motor. What is surprising is that even with a low power motor such as this one, the efficiency of converting electrical power into mechanical power can be more than 75%. How does this compare with the best heat-engine, say a steam turbine?

Now examine the performance of the generator. The output power is clearly \( (e_{out} - V_i) \) and the input power is \( 2\pi \Gamma \times 9.8 \times n \), so you can plot the efficiency against \( i_{out} \). Here again a fairly high efficiency is achieved compared with other methods of producing electrical power, for example, from a fuel cell, or from solar batteries (thermo-electric power). The overall efficiency of the motor-generator is not very impressive, and is roughly the square of that of the motor or generator alone. How does this compare with the efficiency of the transformer-rectifier itself, where A.C. is transformed to D.C.? Investigate.
COMMENTS

It is possible to break down the losses in either the motor or generator. There are the frictional losses at the bearings and due to air resistance, the copper losses in the form of heat and the iron losses due to hysteresis and magnetic reaction. One hopes that the student would appreciate that all practical machines are more or less inefficient until superconductivity and superfluidity cease to be laboratory wonders and become practical realities.

The above experiment can be repeated, using the generator as a self-excited D.C. machine (i.e. use the commutator segments). When using the motor as a series wound motor, the generator must always be coupled to the motor and should always be 'on load'; if not, the armature will 'race' because of the enormous torque, causing damage to the machine; also some sort of overload release is essential. It is worth remembering that this initial large torque of a D.C. series wound motor makes it more preferable for traction than any other motor, D.C. or A.C.

The use of the motor and generator as compound wound machines offers also an interesting exercise.

The regulation of the transformer-rectifier unit can be studied by using the load resistance mats already provided.

Finally, a complete hysteresis loop for the material of the field magnet could be determined by keeping the motor at constant speed (using the neon lamp) and taking the output e.m.f. of the generator as a measure of $B$ and $i_f$ as a measure of $H$. How does one reverse the direction of the motor? (The torque spring balance should be clamped for this part of the experiment as it is designed to work only for one direction of rotation of the motor.)

EXPERIMENT 55

To make an elementary study of A.C. circuits

APPARATUS

Large coil of about 4000 turns (see Appendix 2) with removable iron core, solid or laminated. Auto-transformer. A.C. voltmeter (0–50 V). Multi-range A.C. ammeter (0–1, 0–0.1 A). 6 μF condenser. 500 ohm non-inductive resistor (5 W rating). P.O. box and accessories.

Self-inductance

METHOD AND CALCULATION

The resistance of the coil is found to the nearest ohm, using the P.O. box, and checked.

To find the self-inductance of the coil connect it (without the iron core) to the output of the auto-transformer in series with the ammeter (using
the 0–1 A range), as shown in the circuit (Fig. 1). (Check both meters for zero error before use and record.)

Vary the output p.d., recording both the voltage and the current. Five sets of observations should suffice. Tabulate the results as shown in table below. Plot $V$ against $I$ (include the origin) and find the gradient, which is the impedance of the coil in ohms. From your theory, the impedance of the circuit,

$$Z = \frac{V}{I} = \sqrt{R^2 + 4\pi^2 L^2 f^2}$$

where $R$ is the resistance of the coil (already determined), $f$ the frequency of the supply, and $L$ the self-inductance of the coil in henries. Calculate $L$.

Measure the length of the coil $l$ cm and calculate the mean cross-sectional area $A$ cm$^2$, record the total number of turns $N$ and calculate the self-inductance, which is equal to $4\pi N^2 A / 10^9$. (For M.K.S. $L = \mu_0 N^2 A / l$)

<table>
<thead>
<tr>
<th>p.d. (to one decimal place)</th>
<th>Current (to two decimal places)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$</td>
<td>$I$ A</td>
</tr>
</tbody>
</table>

$\mu_0 = 4\pi \times 10^{-7}$, $A$ in sq metres, $l$ in metres.) How does the calculated value compare with the experimental value? Comment.

To find the self-inductance of the coil with the solid iron core, we have the added complication that the self-inductance depends on the current. The following circuit is used (Fig. 2). With the 500 ohm resistor shorted out, adjust the voltage to give a suitable current (say 0.05 A). Read the voltage and record. Now include the 500 ohm resistor and adjust the output of the transformer to send the same current as before into the circuit. Read the voltage again and record.

Calculate the impedance of the circuit in each case and equate it to $\sqrt{R^2 + 4\pi^2 L^2 f^2}$ and $\sqrt{(500 + R)^2 + 4\pi^2 L^2 f^2}$ respectively.

Do you get the same value of $L$ from the two equations?

The reason for the discrepancy is the generation of eddy currents in the solid core because of the changing magnetic flux. The power dissipated is
in the form of heat so that a greater voltage is required to drive the same current through the coil and thus there is an additional resistance $X$ ohm due to this power loss. The expressions for the impedances have to be modified as follows:

Impedance without 500 ohm $Z_1 = \sqrt{(R + X)^2 + 4\pi^2L^2f^2}$

Impedance with the 500 ohm $Z_2 = \sqrt{(500 + R + X)^2 + 4\pi^2L^2f^2}$

This is simple algebra; you should easily be able to show that

$$Z_2^2 - Z_1^2 = 1000(R + X) + 500^2.$$

Hence $X$ and $L$ can be calculated.

Now repeat the same experiment using the laminated core and keeping to the same current. Find the new values of $X$ and $L$. Repeat the whole experiment with a different value for the current.

**Capacitor**

**Method and Calculation**

Use the same circuit as in Fig. 1, but with the capacitor replacing the coil and the range of the ammeter set to 0–0·1 A. Using the same tabulation draw a similar graph, the gradient this time being the reactance of the capacitor, which is $1/2\pi fC$. Deduc $C$ (farads).

Next put the 500 ohm resistor in series with the capacitor, but this time record the voltage of the circuit, the p.d. across the resistor ($V_R$) and the p.d. across the capacitor ($V_C$) and the current ($I$) in the circuit as shown in the table below. You will soon discover that $V$ is not equal to $V_R + V_C$ but that $V$ is the vector sum of $\vec{V}_R + \vec{V}_C$ as shown in Fig. 3. Complete columns 5 and 6 and check the value of $C$. \( Z = \sqrt{R^2 + \frac{1}{4\pi^2f^2C^2}} \)

The voltmeter used should have a very high resistance and preferably be a valve voltmeter.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>p.d. across</td>
<td>IA</td>
<td>$V_R$ V</td>
<td>$V_C$ V</td>
<td>$V = \sqrt{V_R^2 + V_C^2}$ V</td>
<td>$Z = V/I$ ohm</td>
</tr>
<tr>
<td>transformer output $V$</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>
Capacitor and inductance in series

Method and Calculation

Join the condenser, the coil (with laminated core), the ammeter (set to 0–1 A) to the output of the transformer, which should be set at 10 V, as shown in Fig. 4. Draw out the core slowly and you will note that the current will rise until it reaches a maximum (at this point the core is felt to be vibrating); note this maximum current. Draw out the core farther and the current will drop again. Clearly the maximum current corresponds to a point of resonance where the oscillating frequency of the circuit corresponds to that of the mains \((i.e., f = \frac{1}{2\pi(\text{LC})^{1/2}})\).

Set the core again at resonance and without disturbing it find its inductance at the resonating current (using circuit shown in Fig. 2). Calculate the self-inductance \(L\) and the resistive load \(X\) due to eddy currents. (You may require a voltmeter with higher range for this part of the experiment.) Verify*

(a) \(f = \frac{1}{2\pi(\text{LC})^{1/2}}\)

(b) \(I = \frac{V}{X + R}\)

Comments

The reason for the laminations of the cores of transformers, rotors, etc., should have been clear from the first experiment. With solid core, \(X\) can be as high as \(2R\), so that the effect is to treble the total heat dissipation, while with the lamination, \(X\) is only a fraction of \(R\).

The study of phase relationship between current and voltage in the inductance can best be followed using the vector diagram. Referring again to Fig. 2, if \(V\) is the output p.d. of the transformer, \(V_1\) the p.d. across the 500 ohm resistor and \(V_2\) the p.d. across the inductance, then \(V\) is the resultant vector of \(V_1\) and \(V_2\) as shown in Fig. 5. Using the cosine law:

\[ V = \sqrt{V_1^2 + V_2^2 + 2V_1V_2 \cos \theta} \]

where \(\tan \theta = \frac{2\pi Lf}{R + X}\).

* This is not a complete verification as \(L\) is deduced by assuming \(f\), though \(C\) can be found by other methods without assuming \(f\).
If the voltages are recorded, then the above vector treatment can be verified.

The last part of the experiment illustrates another example of resonance showing that the current at resonance is limited by the resistance of the circuit only. The whole experiment shows that, though in inductances and capacitors the voltages and currents are in quadrature (i.e. related by a phase difference of $\pi/2$), their reactances must be opposite in sign, so that at resonance their total reactance is zero, i.e. $2\pi f L = \frac{1}{2\pi f C}$ or $f = \frac{1}{2\pi (LC)^{1/2}}$
a point which should have been noted.

**EXPERIMENT 56**

The study of the efficiency and voltage regulation of a step-down power transformer, and an investigation of some of the properties of a ferrite transformer

**APPARATUS**

Step-down mains power transformer (say 12 V output, 60 VA rating), Avometer, 0–5 A A.C. ammeter, two 8 ohm (5 A) rheostats, ferrite rectangular core on which are wound three separate windings of 400 turns of cotton-covered copper wire, accumulator, 0–5 A D.C. ammeter, switches.

**METHOD**

Connect up the circuit shown in Fig. 1; great care should be taken to see that there are no bare terminals or wires in the mains circuit containing the Avometer $A_1$ set as a 0–1 A A.C. ammeter. To study the efficiency of the transformer, vary the output current in the secondary by altering the rheostat, starting with zero current (the key open) and recording the corresponding primary current. Tabulate your results as follows:

![Fig. 1](image)

<table>
<thead>
<tr>
<th>Primary current $I_1$</th>
<th>Secondary current $I_2$</th>
<th>Secondary voltage $V$</th>
<th>Input power $EI_1$</th>
<th>Output power $VI_2$</th>
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</table>


Measure the voltage of the mains supply $E$, also the D.C. resistance of the primary winding, using a P.O. box as in Experiment 46.

To study the voltage regulation, repeat the experiment after removing the Avometer from the primary circuit and using it instead as a voltmeter (shown dotted in Fig. 1) across the secondary winding. Use the same values of output current and record in the above table the corresponding output voltages.

Use the output of the transformer to supply the A.C. current to the primary of the ferrite transformer. This is not a power transformer and we are concerned here with investigating the relation between the output voltage of the secondary $V$ and the current $I_1$ in the primary. Use the circuit shown in Fig. 2, increasing the current in the primary $I_1$ in steps to 5 A and recording the corresponding voltage, again using the Avometer as a voltmeter. Set the current in the primary at 2 A and investigate the variation of the direct current in the auxiliary windings (shown in Fig. 2) against the output A.C. voltage, increasing the current up to 5 A.

**Theory and Calculation**

How does one explain the increase in the primary current when a current is taken from the secondary? If one assumes an ideal transformer then the current in the primary at zero current load ($I_2 = 0$) is not in phase with the mains voltage $E$; it is maintaining the changing magnetic flux so that an induced e.m.f., $E_{1i}$, is set up, equal and opposite to $E$ (i.e. a purely inductive circuit).

When a current is drawn from a secondary, then by Lenz’s Law the magnetic flux it produces opposes that of the primary, so that there is a temporary reduction in the back induced e.m.f., $E_{1i}$, causing an increase in current in the primary until the reduction in magnetic flux is made good. As the magnetic flux depends on the ampere-turns then, if $N_1$ is the number of turns in the primary, $I_0$ is the primary current on zero current load in the secondary and $N_2$ is the number of turns in the secondary then
$N_1(I_1 - I_0) = N_2I_2$. This can be verified approximately from your observations (remembering that $\frac{N_1}{N_2} = \frac{\text{primary e.m.f.}}{\text{secondary e.m.f.}}$), as the transformer is not an ideal one, because of the heating in the windings (copper losses), and heating in the core (eddy current and hysteresis losses). One can estimate the efficiency of the transformer from the ratio $\frac{\text{output power}}{\text{input power}}$ (we are being a little unfair to the transformer by putting input power $= EI_1$ as part of this power is virtual, but one is usually made to pay for $EI_1$ whether it is all working power or not). Plot the efficiency against current load $I_2$ and the output voltage $V$ against $I_2$ and comment on both graphs. The ferrite transformer should have shown you the effect of magnetic saturation on the output voltage of the secondary and that can best be understood pictorially by plotting $V$ against $I$. The reason why a ferrite core is chosen is because it reaches saturation with comparatively low magnetizing fields.

**Comments**

The complete theory of the transformer is complicated and is omitted from these comments, but a striking point is the very high efficiencies (85–90%) with which a transformer transforms electrical energy from one potential to another, provided it is made to work under optimum load conditions, as your results should have shown.

Magnetic ferrites consist of complex powdered oxides which are sintered into compact homogeneous material of definite shapes. It is easy to check with an Avometer that their electrical resistance is extremely high so that some ferrites have not only high permeability but extremely low eddy
losses (as well as hysteresis losses) so that they can be used at very high
frequencies where no conventional magnetic materials would have done.
Thus air cored inductances and transformers can be reduced to extremely
small sizes. Ferrites have special application in H.F. aerials.

The last part of the experiment is meant to illustrate the principle of the
magnetic amplifier, which utilizes special ferrites possessing square hys-
teresis loops (used also in computer memory storage). Can you explain the
reduction in the output voltage when a direct current is switched on in the
auxiliary windings? Connect the circuit shown in Fig. 3 and you will note
that on reversing the connections of one of the windings you get different
currents depending on whether the windings are opposing each other or
helping each other. Arrange the reversing switch for minimum current
and switch on the direct current in the auxiliary circuit; note the increase
in the alternating current. This is the principle of the magnetic amplifier.

EXPERIMENT 57

To investigate how the speed of travel of a vibration in a tensioned
string depends on the tension and to use this fact in determining
the frequency of the alternating mains supply

Apparatus

15 ft elastic rubber (square in cross-section) \( \frac{3}{4} \) in thick with small loops
at each end, 10 lb spring balance, measuring tape, stopwatch, stands and
G-clamps. Long fine bare copper wire, clean metal pulley which can be
clamped on to a stand, light scale pan, box of weights, large permanent
magnet, rheostat, L.T. A.C. supply, crocodile clip attached to a long lead.

![Diagram of apparatus](image)

Fig. 1

Method

First weigh the elastic, then hook one loop on to a clamped stand and
fix the other loop to the spring balance which is attached to another stand
clamped at a suitable position in the laboratory. Stretch the elastic and
measure the tension \( T \) recorded by the spring balance and the stretched
length \( L \) of the elastic. Record \( T \) and \( L \) in a table in the usual manner,
checking your observations by reducing the tension until the string is almost slack again.

Now remove the spring balance but hook on the two loops to the two stands as before, give the rubber a sharp rap at one end and watch the disturbance travelling to and fro, timing as many times as you can, recording both the distance between the loops \( L \), the number of journeys done \( n \) and the time taken \( t \), tabulating your results as follows:

<table>
<thead>
<tr>
<th>Distance ( L ) ft</th>
<th>Number of journeys ( n )</th>
<th>Time taken ( t ) sec</th>
<th>Speed ( = nL/t )</th>
<th>Corresponding ( T ) lb wt read off from graph</th>
<th>Mass of band per unit length ( W/L )</th>
<th>( \sqrt{TL/W} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Repeat as many times as you can, and with a thinner elastic if available.

Weigh accurately a known length of the copper wire and also the scale pan (if its weight is not known), then assemble the apparatus shown in Fig. 2, measuring the total length \( L \). Switch on the alternating current and adjust the weight on the scale pan until the wire is set vibrating in resonance with the frequency of the supply. Count the number of stationary loops formed and record it in the following table:
The weight $W$ should be altered to give altogether six observations.

**Theory and Calculation**

You will have noted that with increasing tension, the velocity of propagation of a disturbance increases; because with increasing tension one needs more force to pluck the string, hence there is bigger acceleration when it is released and speedier travel. But acceleration is \( \frac{\text{unbalanced force}}{\text{mass}} \) so that the velocity depends not only on the tension but on the mass of the string. Actually \( v = \sqrt{\frac{\text{tension}}{\text{mass/unit length}}} \) and this can be verified by plotting the speed \( nL/t \) against \( \sqrt{TL/W} \).

If \( f \) is the frequency of the mains supply, then \( f \lambda = v \), the velocity of propagation, and \( v = \sqrt{Wg/\mu} \) where \( \mu \) is the mass per cm of the wire. Thus \( \lambda = \frac{1}{f} \sqrt{\frac{g}{\mu}} \cdot \sqrt{W} \). Plot \( \lambda \) against \( \sqrt{W} \) and you should get a straight line passing through the origin, the gradient of which is \( \frac{1}{f} \sqrt{\frac{g}{\mu}} \), hence \( f \).

**Comments**

If one were to take an instantaneous picture of the vibration of the wire in the second half of the experiment, one would see a wavy line, part of which is shown in Fig. 3 (try it with a stroboscope). Each part of the wire would be moving with velocity \( v \), and as the wire is curved, each part of it will have to be accelerated towards the centre of curvature. Because of the tension \( T \), an element of the wire AB would experience an unbalanced force of \( 2T \sin (\alpha/2) \) directed towards the centre of curvature O. Therefore

\[
2T \sin (\alpha/2) = \text{mass of AB} \times \frac{v^2}{R} \quad \text{(see Experiment 18)}
\]

\[
2T \sin (\alpha/2) = \alpha \mu R \times \frac{v^2}{R}
\]

and as \( \alpha \) can be made as small as you like, \( 2 \sin (\alpha/2) \to \alpha \). Therefore

\[
v^2 = \frac{T}{\mu} \quad \text{or} \quad v = \sqrt{\frac{T}{\mu}}
\]
PRACTICAL PHYSICS

The velocity of sound in a medium will therefore depend on the force resisting the disturbance (or the elasticity, which measures the force resisting deformation of the medium) and the inertia of the medium.

EXPERIMENT 58

To find the velocity of sound in gases and liquids

Apparatus

Variable audio-frequency oscillator and vibrator, loudspeaker with large baffle, large drawing board, simple stethoscope, clamp and stand, rulers, thermometer. Long cardboard tube (diameter about 2½ in) with lid to fit over one end. Long glass tube 1½ m long and 2½ in internal diameter; a small earphone insert glued to a cork fits into one end of the tube and a similar cork is fixed at the other end and contains a short brass tube joined to another brass tube by a short piece of rubber tubing. A long thin-walled glass tube probe slides into the brass tubes freely but is held firmly by the rubber tubing to make a gas-tight joint. Gas can be admitted to the large tube by means of small inlet and outlet tubes pushed into the corks (Fig. 3), CO₂ cylinder. A steel tube 3½ ft long, 2 in external and 1½ in internal diameter, closed at the bottom end with a strong rubber membrane. A plunger made of glass tubing (¾ in internal diameter) fitted with a cork at one end, the bottom end of the tubing being closed by a taut rubber (toy balloon) membrane.

Method

(a) Place the drawing board about 1 m from the loudspeaker (Fig. 1) and place either the stethoscope or preferably a hand microphone connected to an oscilloscope with the plane of the microphone facing the reflector halfway between the loudspeaker and the reflector. Set the oscillator at about 3000 c/s and tune carefully for maximum sound (if a stethoscope is used make sure that the sound you hear is coming from the funnel).

Record successive positions of minima due to the stationary waves set up between loudspeaker and reflector, and check in the reverse direction, tabulating your results in the usual manner. Record the temperature of the room and the frequency of the oscillations.
The next experiment is to find the modes of vibration in an open and a closed tube, using the method of resonance, the tube being set up as shown in Fig. 2. Attach a small circular disc of cardboard or cork to the vibrator which is placed just below the end of the tube. Next connect the vibrator to the oscillator and gradually increase the frequency. Resonant frequencies can be easily picked up, though when uncertain the stethoscope should be used. Check these as the frequency is decreased. Tabulate your results as follows:

<table>
<thead>
<tr>
<th>Length of pipe l cm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td><strong>Open pipe</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Overtone ratio n</td>
</tr>
<tr>
<td>---------------------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>etc.</td>
</tr>
<tr>
<td><strong>Closed pipe</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Overtone ratio n</td>
</tr>
<tr>
<td>---------------------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>etc.</td>
</tr>
</tbody>
</table>

Close the top of the tube with the lid and repeat the experiment.

To find the velocity of sound in various gases the apparatus shown in Fig. 3 is used. Connect the oscillator to the leads of the earphone insert and set the frequency at 2000 c/s. A gas, for example carbon dioxide, can be admitted into the tube through the inlet tube, air being forced out through the outlet, and both tubes are then closed. Now move one of the corks in or out slightly so that the gas column resonates with the applied frequency. Pull out the probe slowly and note the maxima positions by connecting the end of the probe tube to the stethoscope (without the funnel). Check these positions by pushing in the probe, filling the tube up again with carbon dioxide, in case some air has diffused in. Record your observations as follows:
The experiment can be repeated with air as the gas. The frequency can be checked against a standard frequency of 2 kc/s from test records.

---

(d) The velocity of sound in water can be found using the arrangement shown in Fig. 4. Fill the vertical steel tube with water and place the vibrator in contact with the rubber diaphragm at the bottom. Employ
frequencies between 5000 and 8000 c/s and, for each frequency, note a set of positions of the plunger giving maxima, as heard through the stethoscope. Record observations in a table similar to that in the previous experiment.

**Theory and Calculation**

The distance between successive nodes or antinodes when stationary waves are set up is \( \lambda/2 \) where \( \lambda \) is the wavelength of sound in that medium, so that the velocity of sound is \( \frac{\lambda}{\lambda} f \) where \( f \) is the frequency of the source. This idea is used in all the experiments discussed above.

In (a) the accuracy with which the stationary points are located does not seem high enough to justify any elaborate checking of the frequency of the source. It is usual however to express the velocity of sound at \( 0^\circ \text{C} \), therefore if \( c \) is the velocity determined at temperature \( t^\circ \text{C} \) then

\[
c_0 = c \sqrt{\frac{273}{273 + t}}.
\]

In (b) the fundamental resonant frequency is not the same for open and closed pipes, nor do the overtones bear the same ratios to their corresponding fundamentals.

Comment on this. For the open pipe the resonant frequencies are in the ratio of 1, 2, 3, ..., to the fundamental, while for the closed tube they are 1, 3, 5, ..., to the fundamental. Divide each resonant frequency by the overtone ratio \( n(f/n \) or \( F/n \) and average, giving \( f_0 \) and \( F_0 \) the fundamental frequencies for the two pipes respectively. If \( a \) is the end correction for the pipe (see Experiment 11) then if \( L \) is the length of the pipe:

\[
2 \times f_0(L + 2a) = 4 \times F_0(L + a).
\]

Hence \( a \) and the velocity of sound in air at the temperature recorded.

In (c) the stationary points can be determined with sufficient accuracy to justify calibrating the oscillator. The turn-table on which the test record is played should be checked with an accurate stopwatch (33\( \frac{1}{3} \) rev/min).

The standard error \( \sigma_{\text{error}} = \frac{\text{standard deviation}}{\sqrt{n - 1}} \) where \( n \) is the size of the small sample. The velocity of sound at \( 0^\circ \text{C} \) should then be given to within limits corresponding to \( \pm \sigma_{\text{error}} \).

In (d), though a stout steel tube has been used, the walls of the tube 'give' at the pressure nodes so that the velocity calculated from the results should be low and it should be multiplied by \( \sqrt{1 + \frac{2ak}{hE}} \) where \( k \) is the bulk modulus of the liquid, \( h \) cm is the thickness of the walls of the tube,
a cm the mean radius of the tube, and \( E \), Young’s modulus of the material of the tube.

**Comments**

Two sound detectors have been used, in the above experiment, which utilize different properties of wave motion. The stethoscope detects sound by the displacement of the air while a crystal microphone detects changes in pressure—verify this experimentally (deduce the power transmitted to a detector in terms of pressure and velocity—can any power be detected by pressure or displacement velocity alone?).

In the third method it is worth noting that the microphone insert is at a pressure node when stationary waves are set up in the pipe. Though this may appear paradoxical, as the insert must be vibrating if any energy is to be transmitted to the tube, this paradox can be cleared up if one watches the hand holding the end of the taut elastic of Experiment 57. When the elastic is set vibrating into 2 or 3 stationary loops one sees the hand moving up and down, yet the end of the elastic is a node.

The experiments above should have convinced you that the velocity of sound is (within limits) independent of the frequency and amplitude, a fact expected on theoretical grounds, but depends on the physical properties of the medium. It can also be reasoned out that these physical properties must be elasticity and inertia. In the case of a fluid the only elasticity it can possess is the bulk modulus \( k_a \) (adiabatic, why?) and the inertia, being the density \( \rho \), \( c = \sqrt{\frac{k_a}{\rho}} \). For a gas \( k_a = \gamma p \), \( \gamma \) being the ratio of the specific heats \( c_p/c_v \) and \( p \) the pressure. Deduce \( \gamma \) for air and CO\(_2\). Deduce \( k_a \) for water and compare it with that of air.

**EXPERIMENT 59**

**The study of the phenomena of interference using Young’s double slit and Lloyd’s mirror**

**Apparatus**

An adjustable slit (with further adjustment to keep it vertical), a small exposed photographic plate (with holder) on which two pairs of very fine lines, 0·30 and 0·40 mm apart respectively, are ruled, a micrometer eyepiece, sodium lamp, test tube filled with water, Lloyd’s mirror, 10·0 cm converging lens with a stop, travelling microscope.

**Method**

It is best to carry out Young’s double slit experiment in darkness or subdued light, particularly if the slits are ruled with a sharp razor blade. Set up the apparatus as shown in Fig. 1, and use the wider pair of slits first. \( d \) is about 20 cm and \( D \) can be varied, though about 30 cm is a convenient
distance. Narrow the slit O and adjust its tilt until a clear contrasting set of fringes are visible in the micrometer eye-piece. Measure the distance between 20 bright fringes, repeat as a check and recheck again if necessary. Measure distance D. Repeat the experiment using the other pair of slits; note that the fringes appear closer together.

Open out slit O slowly and you will note that the fringes begin to disappear; if you open the slit farther out the fringes suddenly reappear, but look faint. Measure distance d (Fig. 1). Start again with slit O narrowed and open it out until the fringes disappear altogether. Remove the slit and measure its width with the travelling microscope. Reinsert the slit and narrow it further until the fringes reappear faintly. Measure the width of the slit again. Remove the photographic plate and find the mean slit separations for each pair, using the travelling microscope.

The practical arrangement for Lloyd's mirror is shown in Fig. 2. (This experiment can be done in daylight.) With the eye-piece removed, look in the direction of the adjustable slit and see its reflection in the mirror (the mirror surface should not be touched, as it is silvered on the front). Adjust the angle of the mirror as well as the tilt of the slit until it appears close and parallel to its own image in the mirror. Now insert the eye-piece and make further adjustments, if necessary, to get clear very contrasting fringes in the field of view. Find as before the distance between 20 bright fringes.

To measure the distance between the slit and its image insert the converging lens with a stop as shown in Fig. 2, and find the best position giving a sharp image of the slits. Measure the separation, OO', between them and the distances u and v as shown in the figure. Repeat at least once, as a check.
**Theoretical and Calculation**

It can be shown that the distance $x$ between two bright fringes in Young's famous experiment is $D\lambda/s$ where $s$ is the separation between the two slits, $D$ the distance between the double slits and the eye-piece (you will have noticed that the larger $D$ is the bigger is $x$ for the same pair of slits), and $\lambda$ is the wavelength of the sodium light. Calculate $\lambda$ for each pair of slits ($\lambda = xs/D$). Values should agree to within 5%.

For Lloyd's mirror the same formula applies and $s$ becomes the separation between the slit and its image. For a lens, the linear magnification $a/\text{(image width)} = \frac{v}{u}$ or $s = \frac{ua}{v}$. Deduce $\lambda$ as before.

**Comments**

Do the fringes in Young's double slit experiment all look the same; if not, how do you account for it? You will have noted that the fringes with Lloyd's mirror are much brighter, and that the first fringe is dark because of the change of phase of $\pi$ as light is reflected from a dense medium (this change can also be shown convincingly with microwaves).

To understand the disappearance of the fringes in the double slit experiment when the first slit O is widened out, you should refer to Fig. 3. We shall treat the two slits $S_1$ and $S_2$ as points and the wide slit O as made up of many tiny point slits. The middle point P of slit O gives rise to a bright fringe at $P_1$ ($PS_1 + S_1P_1 = PS_2 + S_2P_1$) and dark fringes at $p_1$ and $\pi_1$. Let the tiny slit at $Q$ give rise to a bright fringe at $q_1$ ($QS_1 + S_1q_1 = QS_2 + S_2q_1$) and similarly the slit at $R$ give a bright fringe at $r_1$. If $q_1$ coincides with $\pi_1$...
and \( r_1 \) coincides with \( p_1 \) then the whole field will appear uniformly illuminated and the fringes disappear. This means that the angular separation between two consecutive dark (or bright) fringes \( (x/D) \) is equal to the angular size of the slit \( O \) at the middle of \( S_1S_2 \) \( (b/d) \), where \( b \) is the width of the slit. Further, when the slit \( O \) is widened again, the points \( r_1 \) and \( q_1 \) move out to coincide with the next maxima on either side of \( p_1 \) and the fringes reappear but with reduced contrast. You should be able to verify quantitatively these facts from your observations to within 10%. This idea has been utilized by Michelson in an elegant method to measure the angular diameter of distant stars which otherwise would appear as point images through a telescope.

**Experiment 60**

The study of the diffraction of a single slit and the use of a coarse diffraction grating to estimate the wavelength of light

**Apparatus**

Adjustable vertical slit, vernier callipers, sodium lamp, water-filled test tube, straight filament lamp and battery, cylindrical lens 7.5 cm focal length (masked so that only the centre of the lens transmits the light), micrometer eye-piece, coarse grating (200 lines per cm), travelling microscope, rulers.

**Method**

Set up the apparatus as shown in Fig. 1. With the jaws of the vernier callipers about 4 mm apart (acting as a variable slit), align accurately the
slits and note the diffraction pattern, which resembles two straight edge
diffraction patterns placed opposite one another with no fringes in the
geometric shadow.

Now slowly narrow the vernier slit and note how the number of fringes
in the centre decreases until, when only one broad bright fringe remains,
several faint fringes appear in the geometric shadow. Narrow the slit
further and make sketches of the appearance of the field of view. You will

![Diagram of a sodium lamp, water condenser, vernier caliper, and micrometer eyepiece.](Fig. 1)

note that when the slit is very narrow and almost closed, no fringes appear
at all, but just a faint diverging pencil as if the slit is acting as a point
source, with the intensity of the light falling off at large angles of diffraction.

Widen the slit again until you see seven dark fringes in the centre of the
field of view, measure the width of the slit (by reading the vernier, having
previously checked for any zero error). Now narrow the slit very slowly
until the number of dark fringes, \( n \), is reduced to \( 6 \) and again read the
vernier. Continue to do this until you are left with one dark fringe. Widen
the slit again and check your observations, tabulating them as follows:

<table>
<thead>
<tr>
<th>Number of dark fringes ( n )</th>
<th>Slit width ( a_1 )</th>
<th>Slit width ( a_2 )</th>
<th>Average slit width ( a )</th>
<th>( \sqrt{4n+7} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Measure the distances \( u \) and \( v \) as shown in Fig. 1.

Now remove the vernier callipers and replace them by the ‘stopped’
cylindrical lens. Move the lens close to the micrometer eye-piece until a
sharp image is formed of the first slit. Insert the coarse grating in front of
the lens (Fig. 2) and note the large number of diffracted images of the slit;
measure the mean distance \( d \) between, say, the fourth image (or fourth order) on either side of the central undiffracted image, also measure \( x \).

Repeat as many times as you can using different values of \( x \). Replace
the sodium light by the electric lamp with the straight filament set up
parallel to the first slit. Note that the diffracted images are all coloured, each giving the spectrum of white light.

Observe also that the undiffracted image is white, that the red is diffracted more than the violet (compare this with the prism; see Experiment 31), that the widths of the spectra increase with the order, finally that

![Diagram showing the setup of a diffraction grating experiment](image)

the intensities of the spectra get weaker with the higher orders. Use the travelling microscope to measure the mean distance, \( e \), between the centres of two adjacent slits or clear spaces of the grating.

**Theory and Calculation**

It can be shown that for a narrow slit of width \( a \) the number of dark fringes \( n \) is related to \( a \) by the following formula:

\[
a^2 \left( \frac{1}{u} + \frac{1}{v} \right) = (7 + 4n) \lambda
\]

where \( n = 0, 1, \text{etc.} \).

By plotting \( a \) against \( \sqrt{7 + 4n} \) you should get a straight line the gradient of which is \( \sqrt{\frac{\lambda}{(1 + \frac{1}{u})}} \); calculate \( \lambda \). Considering the make-shift vernier slit used, one can only treat the result as an estimate of the wavelength.

It can be shown that with a plane diffraction grating, the diffracted angle \( \theta \) is related to the element of the grating \( e \), the wavelength \( \lambda \) and the order \( n \) of the spectrum by \( e \sin \theta = n \lambda \). In the above case \( \theta \) is small and \( \sin \theta \approx \tan \theta \), then \( (d/2x)e = n \lambda \) and \( n = 4 \).

If you plot \( d \) against \( x \), you should get a straight line and the gradient will be \( 2n \lambda/e \). Calculate \( \lambda \) to two significant figures.

**Comments**

With the coarse grating, the width of each slit element is of the order of \( 4 \times 10^{-5} \) cm or \( 2.5 \times 10^{-8} \) cm, which is much narrower than anything you could hope to measure with the vernier callipers, so that the diffraction pattern of each slit can be treated as a single diverging pencil and we can neglect any other fringes. This is usually done in the simple theory of fine gratings where the slit element width is of the order of \( 2 \times 10^{-4} \) cm (about 3 wavelengths across).
The theory of equation (1) can be approximately deduced from Fig. 3 which shows the position of the slit giving one single dark fringe at O so that (SP + PO) − SO is \( \lambda /2 \), and for a central bright fringe at O, the optical path difference is increased to \( \lambda \) and so on.

![Diagram of light passing through a slit](image)

*Fig. 3*

Therefore

\[
\sqrt{u^2 + \left(\frac{a}{2}\right)^2} + \sqrt{u^2 + \left(\frac{a}{2}\right)^2} - (u + v) = \frac{n\lambda}{2}, \quad a \ll u \ll v
\]

This gives

\[
a^2 \left(\frac{1}{u} + \frac{1}{v}\right) = 4n\lambda
\]

but a more detailed theory gives equation (1) above.

**EXPERIMENT 61**

To estimate a value of the Stefan-Boltzmann constant and to study the distribution of radiated energy in the spectrum of a hot body

**Estimation of the Stefan-Boltzmann constant**

**Apparatus**

A telluride alloy thermocouple (see Experiment 49), two D.C. ammeters (ranges, 0–5 A and 0–0.5 A), special heating coil to fit the top of the thermocouple, using eureka wire embedded in Araldite, polished and shellacked, resistance of about 3 ohm (the accurate value of this resistance should be marked on the coil), microammeter Scalamp (range 0–50 mA), a throw-over switch, an ordinary switch, 20 ohm resistor, two rheostats, special steam radiator and asbestos-covered stool (see Fig. 1), steam generator, Bunsen burner, 0–100°C thermometer, thin circular copper disc blackened on one side, low tension D.C. supply (2–6 V).
Method

Connect up the electrical circuit as shown in Fig. 1, making sure that the thermocouple is correctly connected, so that when the current flows in the thermocouple circuit, heat is being 'pumped' away from the top of the thermocouple. Place the copper disc with the blackened side up on top of the thermocouple, with a small drop of glycerine in between the two
surfaces. Pass steam through the radiator and then switch on the supply so that a current of 1A flows in the thermocouple circuit.

After the temperature of the radiator has become steady, wait about five minutes until the copper disc has reached a steady temperature, quickly turn over the switch and test the temperature of the thermocouple by connecting it to the galvanometer circuit. The spot of the Scalamp should have been adjusted to the middle of the scale and the galvanometer used as a ‘centre-zero’ instrument, having also checked beforehand the direction of the deflection when the top of the thermocouple is cooler than the bottom junction. (We shall assume the bottom junction to remain at a constant temperature throughout the experiment.) If the temperature is too high, increase the ‘pumping’ current, wait a few minutes and check again quickly for any deflection of the galvanometer (remembering that while you are testing for the temperature of the thermocouple, the current which is ‘pumping’ the heat away is cut off and therefore what you try to find is the instantaneous deflection of the galvanometer, if any). You should be able to determine fairly accurately the current \( I \) required to ‘pump’ away an amount of heat equal to the heat absorbed by the disc per second.

Now it is required to find the rate of cooling of the top of the thermocouple when a current \( I \) passes through it. Place the heating coil on top of the thermocouple with a small drop of glycerine in between, strapping it to the thermocouple by means of small rubber bands. Connect the heating coil to the low tension supply, a rheostat, an ammeter (0–0.5 A) and a switch. Switch on and adjust the current \( I \) in the thermocouple circuit and also switch on the current in the heating coil. Wait for a few minutes, then switch over the thermocouple to the galvanometer circuit, and simultaneously switch off the current in the heating coil with the other hand. From the direction and magnitude of the deflection of the galvanometer you should be able to tell whether the current in the heating coil is too much or too little. Adjust the current in the heating coil accordingly, so that on switching over to the galvanometer circuit, no appreciable current is detected, showing that the electrical power in the heating coil is equal to the ‘pumping’ power of the current in the thermocouple, which in turn is equal to the rate of absorption of radiated energy by the disc. Record the magnitude of this current \( I_1 \) and the mean temperature of the room during the experiment.

**Theory and Calculation**

You have already learned from Experiment 51 that the total heat radiated from a perfect radiator (black body) is proportional to the fourth power of the absolute temperature. This experimental law is of the utmost importance in measuring very high temperatures, when it is impossible or inconvenient to place a thermometer in contact with the hot body. In astronomy the only way of measuring the temperature of stars is by studying the intensity and composition of their radiations.

For a black body the radiated heat energy per square centimetre per second is equal to \( \sigma T^4 \) where \( \sigma \) is a universal constant known as the Stefan-Boltzmann constant. In the case of a black body at a temperature \( T \text{ K} \)
placed in a constant temperature enclosure \( T_0^\circ K \ (T > T_0) \), the resultant heat energy it receives in ergs per second = \( A\sigma(T^4 - T_0^4) \) where \( A \) cm\(^2\) is the effective area of the body. In the above experiment the steam radiator forms a black body enclosure surrounding the disc, \( T = 273 + \) temperature of steam, \( T_0 = 273 + \) temperature of the room, \( A \) = area of the circular hole in the water-cooled diaphragm immediately above the disc. The resultant radiated heat energy received by the disc is \( I_1^2 \times R \times 10^7 \) ergs where \( I_1 \) is the current in the heating coil required to balance the rate of cooling in the thermocouple and \( R \) is the resistance of the heating coil. Therefore \( I_1^2 \times R \times 10^7 = \sigma A(T^4 - T_0^4) \) from which \( \sigma \) can be deduced. As the blackened disc is not a perfect absorber (and therefore not quite a black body) you cannot expect the value for \( \sigma \) to agree with the accepted universal constant, but to be rather lower. The units of \( \sigma \) are ergs cm\(^{-2}\) deg C\(^{-4}\) sec\(^{-1}\).

**The distribution of radiated energy in the spectrum of a hot body**

**Apparatus**

60 W straight filament, small bayonet, 12 V bulb in holder, two Mullard Varite thermistors, 2000 VA, suitably mounted, short focus \((f = 10 \text{ cm})\)

![Diagram](image)

**Fig. 2**

single lens, P.O. box, 2000 ohm resistor, 1.5 V dry battery, microammeter centre-zero galvanometer, 200 ohm resistor, spectrometer, hollow glass 60° prism filled with carbon disulphide, 2 ohm heavy duty rheostat, voltmeter
Method

Use the P.O. box to measure the resistance of the lamp, keeping the current through the lamp as small as possible by connecting a 2000 ohm resistor in series with a dry 1.5 V battery to the P.O. box and using the microammeter centre-zero galvanometer. Record the temperature of the room. Strap the first thermistor on top of the telescope of the spectrometer so that the thermistor is about 25 cm from the centre of the turn-table. Connect the lamp in series with the heavy duty resistor and the ammeter and adjust the current so that the p.d. of the lamp, as measured by a voltmeter connected across the lamp, is about 6 V. Place the lamp about 28 cm from the centre of the turn-table but level with it, as shown in the diagram (Fig. 2). Insert the short focus lens close to the lamp and focus the image of the filament on a screen placed 25 cm from the centre of the turn-table on the opposite side to the lamp. Turn the telescope and the thermistor round until the thermistor is at the centre of the image of the filament (the lamp should now be on the axis of the telescope). Read and record the position of the telescope on the circular scale to at least a tenth of a degree. Now place the prism on the turn-table, raising it to the height of the lamp and turning it round to a position of minimum deviation for the red part of the spectrum, using a red Wratten filter.

Connect up the circuit diagram as shown in Fig. 3, leaving the battery key out so that the current from the battery flows all the time. Switch off the lamp, wait a few minutes and balance the P.O. box in the usual way by adjusting $R_3$ (you may find it necessary to use a higher resistance than 200 ohm in series with the first thermistor, so that $R_3$ is at least 200 ohm). Record the value of $R_3$ (say $X$). Now switch on the lamp again and adjust it for full brightness (actually there is no harm in over-running the lamp to 13 V to give a higher temperature) and record both the p.d. of the lamp and the current it is taking. Turn the telescope so that the deviation is about 40° and the thermistor is well in the infra-red region of the spectrum, balance again and record $R_3$, repeating at intervals of 1° until the thermistor is well in the ultra-violet region of the spectrum. You should be careful that the second thermistor is shielded from direct radiation. Tabulate your results as shown in table opposite. Column 3 is a measure of the radiation detected by the thermistor, and you will soon note that a sharply defined maximum occurs in the infra-red region. Now try to locate the angle of deviation of this maximum by taking other observations close to it so that this deviation is determined to a tenth of a degree.
Repeat this experiment with a lower current in the lamp, and you will note that the maximum becomes less clearly defined as the temperature of the filament is reduced, and shifts slightly further into the infra-red region. For lower values of the current try only to locate the position of maximum radiation and this should be done by repeating observations close to the suspected maximum. However, continue to record $V$ and $I$ as before. It is important that draught in the room is kept to a minimum during the experiment, as this affects the equilibrium temperature of the thermistors. Replace the lamp by a sodium lamp with a slit, placed at the position formerly occupied by the filament and adjusted to have roughly the same width as the diameter of the filament. Now rotate the telescope so that the thermistor occupies the centre of the image and read off the angle of deviation.

**Theory and Calculation**

Calculate the resistance $R$ of the filament in each set of observations recorded above, and using the value of $R$ you found at room temperature, calculate $R/R_{293}$ for each set of observations, and using the graph and Fig. 2 of Experiment 5 deduce the corresponding temperature of the tungsten filament. Arrange your results as in table overleaf.
<table>
<thead>
<tr>
<th>Resistance of lamp $R$</th>
<th>$R/R_{293}$</th>
<th>Temperature of filament $^\circ\text{K}$</th>
<th>Angle of deviation $D$ corresponding to maximum radiation</th>
<th>Calculated refractive index</th>
<th>Wavelength $\lambda_{\text{max}}$ cm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Assuming that the prism is also set for minimum deviation in the infra-red region, use the relation

$$\mu = \frac{\sin \frac{A + D}{2}}{\sin \frac{A}{2}}$$

(discussed already in Experiment 31) to find the refractive index, and with the help of Fig. 4, deduce the wavelength $\lambda_{\text{max}}$ corresponding to the maximum radiated energy and complete column 6.

You will note that $T\lambda_{\text{max}}$ is very nearly a constant, the relation being known as Wien’s displacement law, and also that this constant is related to Planck’s constant $h$, for $T\lambda_{\text{max}} = \frac{chN_0}{5 \cdot 0R}$ where $c$ is the velocity of light ($\approx 3 \cdot 0 \times 10^{10}$ cm sec$^{-1}$), $N_0$ is Avogadro’s number ($6 \cdot 03 \times 10^{23}$, which is the number of molecules in 32 g of oxygen), and $R$ is the universal gas constant ($8 \cdot 3 \times 10^7$ erg deg K$^{-1}$). Average the value of $T\lambda_{\text{max}}$ and deduce $h$ (units: erg sec). No great accuracy is expected, but you should get agreement within 20%.

**Comments**

The importance of this second method is both historical and practical. It was in the attempt to account for the spectral energy distribution which you plotted above, giving a peaked curve sloping more steeply towards the short wavelengths, that Planck discovered that radiant energy is exchanged only in certain fixed amounts (quanta) and not in any variable quantity as it was originally thought. It is also fitting that this experiment should enable us to deduce, though approximately, a value of Planck’s constant, $h$, so important in quantum theory. From the practical point of view, this second method and the verification of Wien’s displacement law give a quantitative relation between colour and temperature, so important in astronomy, and preferable to Stefan-Boltzmann’s law in determining the temperature of stars.
**EXPERIMENT 62**

**To estimate the quantity of charge carried by an electron**

**Apparatus**

The apparatus used in this experiment consists of a thick brass disc with a millimetre hole in the centre, attached to a short brass tube (this can be turned on the lathe in one piece). The disc (Fig. 1) rests on top of an ebonite ring which in turn is fixed to a circular metal plate. The ring has two small holes drilled into it horizontally for illumination and viewing, as shown in Fig. 2. The Millikan cell rests on a rigid framework and is assembled as shown in Fig. 3, the height of the microscope (with micrometer eyepiece and preferably 2 in objective) being adjustable to a convenient level.

A small (2.2 V) pea-bulb with a built-in focusing lens is used for illumination and care is taken to use a very small diverging pencil of light, cutting down any reflection from the inner sides of the ebonite ring, so that the oil drops are seen against a dark background. Besides the above, you need a power supply for the bulb, a variable H.T. (this can be made up of several dry batteries in series, the e.m.f. of each battery having been found with a valve voltmeter),
a toggle switch, two stopwatches, polythene oil-drop sprayer, vacuum oil, long pin, density bottle and vernier calliper gauge.

**Method**

It is best for two people to carry out this experiment, in darkness or in subdued light. Focus the microscope on the pin, which is inserted through the tube into the hole in the top disc (with the toggle switch off!). Remove the pin and squeeze the oil sprayer once so that the spray is directed into the tube, close the tube and wait for a few seconds till the oil drops appear as tiny spots of light moving upwards in the field of view (the microscope inverts the image). Switch on the H.T. and you will note that the majority of the oil drops will be unaffected, some will appear to accelerate upwards (these are positively charged and should be ignored), and a few will appear to reverse direction and move downwards (in fact, of course, they will be moving upwards). Select a small one and note its motion when the field is switched off; if this is slow then the drop is certain to be small and will usually carry one or two electrons. Find the time $t_1$ it takes to cover, say, 3 divisions (the reticule usually contains 10 divisions). Switch on the field and time its passage in the reverse direction $t_2$. This should be checked at least once, with your partner recording the times and setting the watch for you for the second run.

The result should be recorded as follows:

<table>
<thead>
<tr>
<th>No. of divs. $X$</th>
<th>$t_1$ sec</th>
<th>No. of divs.</th>
<th>$t_2$ sec</th>
<th>$v_1 = X_2/t_1$ cm sec$^{-1}$</th>
<th>$v_2 = X_2/t_2$ cm sec$^{-1}$</th>
<th>Voltage $V$</th>
<th>$ne$</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>Average</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1 Division = $\alpha$ cm

$\Sigma ne \Sigma n$

A dozen observations should be made, with the partners changing places to ease the eye strain. Watch for any blockage by lifting the top disc (H.T. disconnected) and blowing any oil out, also wiping away oil that may accumulate on the floor of the cell.

Use the vernier callipers to measure the air gap between the top disc and the bottom plate. Remove the microscope (without altering the focus), clamp it vertically and adjust its height above the jaws of the vernier callipers which should be opened out by say 2.0 mm. When the image is in focus, read the number of divisions corresponding to 2.0 mm and hence calculate the distance $\alpha$ cm corresponding to one division and insert in the table above. Use the density bottle to find the density of the oil $\rho$. 
**Theory and Calculation**

Small droplets falling freely under gravity reach a terminal velocity \( v_1 \) given by

\[
6\pi\eta v_1 = \frac{(4/3)\pi a^3 \rho g}{2\rho g} \text{ (Stoke's Law)}
\]

where \( \eta \) = viscosity of air (poises) at the temperature of the room, \( a \) = radius of drop. Hence \( a = \sqrt{\frac{9\eta v_1}{2\rho g}} \) cm (neglecting the upthrust of the air). When the drop is moving steadily upwards with velocity \( v_2 \) under the action of the electric field \( E \) statvolt cm\(^{-1} \) (= \( V/300d \)) where \( V \) is the p.d. between the plates in volts, \( d \) is the air gap in cm), then

\[
nEe - \frac{(4/3)\pi a^3 \rho g}{2\rho g} = 6\pi\eta v_2
\]

where \( n \) is the number of electrons on the drop, \( e \) the charge on the electron (e.s.u.). Substituting for the weight of the drop from equation (1) and rearranging, we get

\[
ne = \frac{6\pi\eta (v_1 + v_2)}{E} = \frac{6\pi\eta \sqrt{\frac{9\eta v_1}{2\rho g}} (v_1 + v_2)}{V} = \frac{5400\pi\eta d^3}{300d} \cdot \frac{\sqrt{v_1(v_1 + v_2)}}{V}.
\]

For M.K.S. equation (2) above is written

\[
6\pi\eta v_2 = 10^8nEe - \frac{(4/3)\pi a^3 \rho g}{2\rho g}
\]

where \( e \) is the charge on the electron in coulombs, \( E \) is the field intensity in volts per metre = \( 100V/d \), \( d \) cm being the separation between the two plates.

Thus

\[
ze = \frac{18 \times 10^{-7}\pi d\eta}{\sqrt{2\rho g}} \cdot \frac{\sqrt{v_1(v_1 + v_2)}}{V}.
\]

Calculate the charge on each drop using equation (3) and complete the table above. The smallest charge in the table is probably the electronic charge (in practice you will find two or more smallish charges which differ by less than factor 2 and the average of which can then be taken to be an estimate for \( e \)). Now deduce \( n \) (the nearest whole number) in each case. The final estimate of \( e \) is therefore the sum of the total charges (\( \Sigma ne \)) divided by the probable total number of electrons (\( \Sigma n \)). Therefore

\[
e = \frac{\Sigma ne}{\Sigma n}
\]
e.s.u. (or coulombs). Give answer to 2 significant figures only.

**Comments**

This experiment is of great importance as it gives us an estimate of the fundamental unit of charge (i.e. whenever a charge is removed or transferred, it can only be a multiple of this fundamental unit). Use equation (1) to calculate the radius of a drop. How does it compare with the mean wavelength of the light used to view the drops? With the small drops in particular did you notice any random sideways motion? How do you explain it? Einstein, by assuming that these droplets share the kinetic energy of the surrounding (and invisible) air molecules, deduced a relation
between the mean random displacement and Avogadro's number, giving a more direct proof to the kinetic theory of gases. By inspecting the table of results do you find any relation between the number of electrons and the size of the drops?

**EXPERIMENT 63**

**To obtain an estimate for the mass of an electron**

**Introduction**

There are several ways of ejecting electrons from metals and the method employed in this experiment is that of thermionic emission from a hot tungsten wire, in which electrons 'boil' out of the metal and are collected by a surrounding anode (*in vacuo*).

**Apparatus**

Ferranti GRD7 tube and holder, 6-3 A.C. supply, D.C./A.C. ammeter (0–3 A), rheostat (8 ohm), rheostat (12 ohm), 12 V D.C. supply, D.C. milliammeter (0–30 mA), two switches, variable H.T. supply up to 250 V, D.C. voltmeter (0–250 V), coil of 1000 turns and about 5 cm high, to fit over the diode, made by winding a suitable gauge of enamelled copper so that the total resistance is not greater than 6 ohm.

**Method**

Connect up the circuit shown in Fig. 1 in which the anode current proper is measured by the milliammeter (looking at the tube, you will note the main central anode cylinder, capped top and bottom by two short guard rings so that electrons emitted from the centre of the wire, which is probably more uniformly heated, are collected by the anode proper). Adjust the filament current to maximum and note that with no applied H.T. the voltmeter gives a small negative reading. Why? Take a series of anode current readings, using different H.T. potentials but keeping the filament current constant throughout. Record your observations as in table on facing page.

Repeat five times decreasing the filament current progressively. You will notice that as the H.T. is increased, the anode current reaches a maximum, called the saturation current, which varies in a marked way with the filament current and the temperature of the filament.

To find the mass of the electrons we must apply a force on them and
this is conveniently done in this experiment by applying a uniform magnetic field perpendicular to the plane of their motion. The coil is now placed over the tube, the ammeter in the filament circuit is taken out and placed in the auxiliary circuit of the coil shown in Fig. 2, the H.T. is raised until the saturation current is reached. Close $K_2$ and note the reduction in the anode current as the magnetic field is applied, adjust the current $I$ in the auxiliary circuit until on closing $K_2$ the anode current is reduced almost to zero. Record $I$ and the anode voltage $V_A$ (remember that your H.T. supply may not have good regulation, $V_A$ should be read when $K_2$ is closed). Increase $V_A$ and repeat; also vary the filament current and repeat—get as many observations as possible and tabulate your results in the usual manner.

Record the internal diameter of the anode (supplied by the manufacturer) and the total number of turns of the coil $N$ and its length $l$.

**Theory and Calculation**

The results from the first part of the experiment should be plotted $(Ia-Va)$ for different filament currents and the shapes of the family of curves commented upon.

In the second part of the experiment the electrons are forced by the magnetic field to follow curved paths and if we can assume that the presence of the electrons does not affect the potential distribution, and that the filament wire is extremely fine so that the electrons acquire their maximum velocities the instant they leave the wire (see Experiment 38), then the paths of the electrons will be circular. If the magnetic field is critically adjusted for anode current cut-off, then these circles just touch the anode
PRACTICAL PHYSICS

and the radius $R$ of these circles will be $D/4$, where $D$ is the diameter of the anode.

If $m$ is the mass of the electron, $v$ is the velocity acquired when the potential is $V$ volts, $B$ the magnetic flux density, then

$$V_e = \frac{1}{2}mv^2$$

$$Bev = \frac{mv^2}{R} \quad \text{and} \quad R = \frac{D}{4}$$

C.G.S.

$V$ (volts), $e$ (coulombs), $v$ (cm sec$^{-1}$)

then

$$Ve = \frac{mv^2}{2 \times 10^7} \quad \ldots \quad (1)$$

$$B = 4\pi \frac{NI}{10l}$$

Therefore

$$\frac{4\pi NI}{10^2l} ev = \frac{mv^2}{R}$$

$$= \frac{4mv^2}{D} \quad \ldots \quad (2)$$

Eliminating $v$ we get

$$\frac{e}{m} = \frac{2 \times 10^{11} \times l^2}{D^2\pi^2N^2} \times \frac{V}{I^2} \quad \ldots \quad (3)$$

M.K.S.

$v$ (m sec$^{-1}$), $D$ (m), $l$ (m)

$$B = \frac{\mu_0 NI}{l} = 4\pi \times 10^{-7} \frac{NI}{l}$$

$$: \quad Ve = \frac{1}{2}mv^2 \quad \ldots \quad (1)$$

$$Bev = \frac{4mv^2}{D} \quad \ldots \quad (2)$$

Eliminating $v$ we get

$$\frac{e}{m} = \frac{2 \times l^2}{\pi^2 \times 10^{-14}N^2D^2} \times \frac{V}{I^2} \quad \ldots \quad (3)$$

Plot $V$ against $I$ and you should get a straight line passing through the origin the slope of which is $V/I^2$. Substitute in equation (3) above and deduce $e/m$ (2 significant figures). Use the value of $e$ you already found in Experiment 62 and deduce $m$.

COMMENTS

You will have noted from the first part of the experiment that when the voltage of the anode is zero or negative hardly any electrons get across to the anode, illustrating the rectifying action of such a valve. Carrying out the latter part of the experiment in the absence of space charge (i.e. when saturated currents only are used) is important, because the space charge (you have already noticed how this affects the $I_A-V_A$ curves) can seriously affect the motion of electrons in the tube. The effect of the finite thickness of the filament can be allowed for but it involves an interesting mathematical exercise which you may like to tackle. The assumptions that the electrons leave the filament without any appreciable energy and that they all move in planes at right-angles to the axis of the filament are clearly serious sources of error, which can account for the difficulty of deciding on the cut-off current in the coil for a given $V_A$.

Calculate the time it takes an electron to travel from the filament to the anode (say, when $V_A$ is 100 V) and explain why this tube is not suitable for rectifying very high frequency alternating voltages.
EXPERIMENT 64

To study the phenomenon of photoelectricity and to estimate the order of magnitude of Planck’s constant $h$

INTRODUCTION

In this experiment, like Experiment 63, we are concerned with ejecting electrons, but in this case the energy is provided by the light incident on a metallic cathode.

APPARATUS

G.E.C. photocell CAV35 with base, inserted in a lightproof cylinder with a window the size of the cathode which can be opened by lifting the hood. A sensitive microammeter (a Scalamp was found satisfactory, but it is desirable to use with it a magnifier permanently focused on the spot and the scale) or a D.C. amplifier. 2 V accumulator and rheostat used as a potentiometer divider. H.T. variable D.C. supply. Voltmeters (0–3 V) and (0–40 V). 25 W, 12 V straight filament lamp. Mercury vapour lamp and 3 Wratten filters Nos. 22, 74 and 50.

METHOD

Assemble the apparatus shown by the continuous line in Fig. 1 and carry out the experiment with a partner in a dark room, leaving the photocell covered, except when taking a reading. Place the incandescent
lamp close to the cell and measure with the microammeter the anode current $I_A$ when there is no voltage applied to the anode. Repeat with varying potentials $V_A$ applied to the anode and record your observations in a suitable table. Repeat the experiment after increasing the distance between the lamp and the cathode.

Now replace the H.T. supply with the potentiometer arrangements shown by the dotted lines in Fig. 1, and replace the incandescent lamp by the mercury vapour lamp placed close to the photocell. A small variable but negative potential is now applied to the anode, and the purpose of this experiment is to find the required negative potential which will deter the photoelectron from reaching the anode.

The magnifier must now be used with the microammeter and the yellow filter (No. 22) is placed in front of the lamp. Increase the potential until, on lifting the hood, no change is detected in the position of the spot of the microammeter. This should be repeated with a higher potential which should be reduced until the spot just appears to move when the hood is lifted off. Tabulate your results in the usual way and average the values of the two potentials. Repeat the experiment using the remaining two filters in turn.

Theory and Calculation

For the first part of the experiment plot $I_A$ as ordinate and $V_A$ as abscissa for the two distances between the lamp and the photocell. The graphs look similar, in both the current increases slowly with the potential and then levels off. The saturation current clearly depends on the intensity of the light, that is on the square of the amplitude of the electromagnetic waves, and it can easily be shown that the saturation photoelectric current is, in fact, proportional to the intensity.

Each filter used in the second part of the experiment allows only one wavelength of light to pass through. By using a diffraction grating spectrometer (see Experiment 60) one can actually determine the wavelength of light passed through by each filter: for the yellow filter (No. 22) it is 5760 Å, green filter (No. 74) 5460 Å and the violet filter (No. 50) 4358 Å. If the retarding voltage for the electrons is $V_A$ volts then their kinetic energy must be $Ve$ joules, where $e$ coulombs is the electronic charge (see Experiment 63).

Does this kinetic energy depend on the intensity? One can show with the above arrangement that it does not, though it is better for this investigation to use a D.C. amplifier instead of the microammeter. This is a remarkable result, for when an electromagnetic wave consisting of alternating magnetic and electric fields falls on a cathode, it should accelerate all the electrons in its path. The more intense the light is, the stronger the fields, the greater the acceleration produced, and hence the higher the kinetic energy of the ejected electrons. It can also be shown that if the energy carried by the wave is shared by all the electrons, it would take several hundred days before they gain the necessary energy to escape. Yet a photoelectric current flows immediately one lifts off the hood!

The results from the second part of the experiment suggest that the kinetic energy of the electrons depends on the wavelength $\lambda$ of the light,
and increases when $\lambda$ decreases. Put in another way, $V_Ae$ increases with increasing frequency $v$ of the light used ($v = c/\lambda$ where $c = 3.0 \times 10^{10}$ cm sec$^{-1}$, the velocity of light). Thus the number of electrons ejected depends on the intensity, but the energy of the ejected electrons depends on the frequency.

It must be increasingly clear that in this phenomenon it is not helpful to consider the energy transferred from the light to the electrons as that between waves and particles; the facts are more consistent with the result of an encounter between particles of light (photons) and electrons. The probability of such an encounter cannot be high and that is why only a few electrons escape. Also the photons must possess energy proportional to their frequency, a fact already asserted by the quantum theory. Einstein was able to show that the kinetic energy of the photoelectrons was $hv - E_0e$ or $V_Ae = hv - E_0e$ where $h$ is Planck's constant and $E_0$ is the work-function of the metallic surface (see also Experiment 90). The work-function is analogous to the latent heat of evaporation of liquids, otherwise electrons escape spontaneously; furthermore one can show, using an infra-red radiator, that there is no photoelectric emission from this photocell. Why?

Plot $V_A$ against $v$ and you should find that the three points do in fact lie on a line, the gradient of which is $h/e$, hence $h$ (one significant figure only), the units being joules sec. Further points can be found using a sodium lamp, and the incandescent lamp with red Wratten filter.

Comments

It is ironical that Hertz, who first demonstrated the existence of electromagnetic waves, predicted theoretically by Maxwell, should also be the one to discover the photoelectric effect for which the electromagnetic theory failed to account. The advent of the quantum theory (see Experiment 61) enabled Einstein to give a satisfactory account of the photoelectric effect and also revived the duality of the nature of light, as waves and corpuscles.

Interaction between photons of high energy (due to X-rays) and electrons was actually observed in a cloud chamber (the Compton effect) where the wavelength of the X-rays appeared to increase as a result of such an encounter, denoting a transfer of energy to the electrons.

EXPERIMENT 65

The study of the characteristics of a junction diode, a transistor and a thermionic triode

Junction diode

Apparatus

Junction diode, sensitive microammeter (0–10 $\mu$A), slide wire potentiometer, accumulator, resistance box (0–20 ohm), slider.
**Method**

Find the resistance of the potentiometer wire using a P.O. box (Experiment 46), then connect up the circuit shown in Fig. 1, adjusting the resistance of $R$ so that the total p.d. across the potentiometer is about 50 mV. Record the current passing through the diode and the corresponding length of wire $l$ tapped off, tabulating your results as follows:

<table>
<thead>
<tr>
<th>Zero reading</th>
<th>Galvanometer reading</th>
<th>Current $\mu A$</th>
<th>$l$ cm</th>
<th>mV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tr>
</tbody>
</table>

Reverse the connections of the diode and repeat the experiment.

**Theory and Calculation**

Taking the e.m.f. of the accumulator to be 2.0 V, the resistance of the potentiometer wire $r$, series resistance $R$, then the p.d. across the wire is $2000r/(R + r)$ mV and from this the p.d. per cm can be deduced. The table shown above can be completed but it is important to remember that the p.d. across the galvanometer itself is not always negligible and should be subtracted from the last column to give the true p.d. across the diode (if the resistance of the galvanometer is 20 ohm, the p.d. with maximum current of 10 $\mu A$ is only 0.2 mV which is clearly negligible in view of the limited accuracy of the method).

Plot the current in the forward direction as ordinate against the p.d. in mV and find the gradients of the tangents at a dozen points on the graph, tabulating them as follows:

<table>
<thead>
<tr>
<th>Gradient ($= dI/dV$)</th>
<th>log gradient</th>
<th>p.d. mV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>
ADVANCED LEVEL EXPERIMENTS

Plot log gradient as ordinate against the p.d. as abscissa and you should get a straight line. Deduce the gradient of the slope $m$.

COMMENTS

In a crystal the various energy levels of the electrons associated with each atom merge into bands and mention has already been made (Experiment 46) that in a semi-conductor the conduction band is separated only by a narrow energy gap from the valence band. If a minute but controlled amount of impurity is added to the crystal, the energy levels (not bands) associated with the impurity atoms, if they are of the right kind, occupy positions in the gap either just below the conduction band or just above the valence band. In the first case this would give rise to an $n$-type semi-conductor where the majority carriers will be electrons (i.e. negative carriers) which jump the still narrower gap into the conduction band. In the second case we get a $p$-type semi-conductor where the majority carriers will be holes (or positive carriers) produced when electrons jump from the valence band into a vacant impurity energy level.

When a $p$-type and an $n$-type semi-conductor are joined together, some of the positive and negative carriers move by mutual attraction towards the common boundary, leaving most of the $p$-type crystal with negative potential and the $n$-type with positive potential. The current flowing through a diode is dependent on how the p.d. is applied, for when a positive potential is applied to the $p$-type end, it will raise its potential and a negative potential at the $n$-type end will lower the potential so that the potential 'hill' is much reduced and both positive and negative carriers can diffuse across, thanks to their thermal energies. When the connections of the diode are reversed the potential 'hill' is made much steeper and the minute reverse current flowing is due to minority carriers and is almost constant, provided the p.d. is not high.

It can be shown that the forward current $I = I_0 \left( \exp \frac{eV}{kT} - 1 \right)$ where $I_0$ depends on the concentration of ions, temperature, etc., of the semi-conductors, $e$ is the electronic charge ($\approx 1.6 \times 10^{-19}$ coulombs), $V$ the voltage, $k$ the Boltzmann constant ($\approx 1.38 \times 10^{-23}$ joule/°K) and $T$ is the absolute temperature of the diode (say 300°K) then

$$\log \frac{dI}{dV} = \frac{eV}{kT} \log (2.718) + \log \frac{I_0 e}{kT} = 0.4343 \frac{eV}{kT} + \log \frac{I_0 e}{kT}$$

thus the gradient $m$ is $\frac{0.4343 \cdot e}{kT}$.

The transistor and the thermionic triode

APPARATUS

Pegboard, OC71 transistor mounted on a small piece of board with two pegs to fit the pegboard, the three terminals, emitter (e), collector (c) and base (b), clearly marked. Variable low tension supply (10 V D.C. with rheostat used as a potentiometer divider), milliammeter (0–15 mA),
voltmeter (0–10 V), microammeter (0–100 \( \mu \)A), slide wire potentiometer, 2 V accumulator, resistance box (0–20 ohm), slider.

Miniature valve ECC82 suitably mounted like the transistor, variable 90 V supply (D.C.) with suitable D.C. voltmeter (or a series of 9 V batteries), 6.3 V supply for the filament.

**Method**

Connect up the circuit shown in Fig. 2, taking great care that the electrical supplies are connected with the correct polarities, as shown in the figure. \( R \) is adjusted so that the drop in potential across the potentiometer wire is 250 mV (the procedure is the same as in the first method). Set the base current to 20 \( \mu \)A and vary the collector voltage \( V_c \) (0–10 V), recording the corresponding collector current \( I_c \), tabulating your results as follows:

<table>
<thead>
<tr>
<th>Base voltage ( mV )</th>
<th>Base current ( I_b ) ( \mu )A</th>
<th>Collector voltage ( V_c )</th>
<th>Collector current ( I_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Repeat with increased base current from zero to \( I_b = 100 \mu \)A in steps of 20 \( \mu \)A. Now connect up the circuit shown in Fig. 3, start with zero voltage on the grid, vary the anode voltage in fixed steps, and record the
corresponding anode current. This is repeated with $V_g = -4, -3, -2, -1, +1, +2$ V. Tabulate your results as follows:

<table>
<thead>
<tr>
<th>Grid voltage</th>
<th>Anode voltage</th>
<th>Anode current mA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Theory and Calculation**

It is not surprising that with zero base current there is minute collector current (called leakage current) since the transistor can be treated as two diodes connected back to back. There are many useful ways of presenting the above experimental data: first plot the collector current $I_c$ against the base current $I_b$, which should give you a straight line the gradient of which is the current amplification factor; next plot the base current $I_b$ against the base voltage $V_b$ for a fixed collector voltage, the curve being similar to the forward current-voltage diode characteristics of the first method; finally plot $V_g$ against $I_c$ for a fixed base current $I_b$.

From the second set of results, plot three curves, each connecting the anode current and grid voltage for a fixed anode voltage. Because the grid is closer to the filament than the anode, an increase in grid voltage produces a bigger change in the anode current than the same increase in anode voltage. From the curves plotted (Fig. 4), if PQ represents the change in anode current for a change in anode voltage $V_A - V_A1 = \delta V_A$, then when PQ is extended to $R$ so that $PQ = QR$, $SR (= \delta V_g)$ gives the corresponding change in grid voltage producing the same change in anode current. Deduce the ratio $\left(\frac{\delta V_A}{\delta V_g}\right)I_A$ constant called the voltage amplification factor.

**Comments**

The transistor considered as two diodes, say $p-n-p$, will have, as a result of the migration of the majority carriers to the two boundaries, a low potential at the emitter which rises to a plateau at the base
and drops down again to the collector, a drop which becomes steeper when the collector voltage is applied. With no base current the potential barrier at the base is effective in stopping all but a few energetic carriers from reaching the collector. When a current is drawn from the base (in effect by applying a small negative voltage to the base) the barrier is lowered and a substantial number of carriers manage to diffuse across. Thus a small current flow from emitter to base produces a large flow from emitter to collector, and hence the transistor is essentially a current amplifying device.

With a thermionic valve, it is the small change in the voltage of the grid that produces a large change in the current flowing to the collector anode. Thus the valve is essentially a voltage amplifying device, and if a current is to be amplified it is first made to produce a potential difference.

The agreement between theory and experiment in the first method is not only that \( \log (dI/dV) \) is found to be related linearly with \( V \), but that the gradient should be so close to that predicted by theory. This is a proof that the electronic charge is the carrier of electricity in semi-conductors.

EXPERIMENT 66

The study of the cathode ray tube and its use to determine the ratio of the electronic charge to mass

APPARATUS

Cathode tube—Mullard DG4-2 and base, with ancillary E.H.T. (0–800 V) and L.T. (6.3 V), focusing potentiometer control and brilliance potentiometer control, 20 V alternating supply, Avometer, 45 ohm rheostat, Helmholtz coils (see Experiment 44), a pair of dividers, switches. Travelling microscope and supports with 3 in objective.

METHOD

Connect, electrically, the cathode ray tube as shown in Fig. 1 and place it along the axis of the Helmholtz coils as shown in Fig. 2 (the axis of the coils should pass half-way up the tube). Connect the two coils in series so that their magnetic fields reinforce one another and attach them to an A.C. supply as shown in Fig. 2, the Avometer being set as an A.C. ammeter 0–1 A.

Switch on the electrical supplies for the cathode ray tube (which should be placed away from the tube), adjust focusing and brilliance controls to give a reasonably bright but small spot. Switch on the supply in the Helmholtz coils. The resulting magnetic field will cause the electron beam to trace a straight line on the screen (at right-angles to the axis of the coils) which should, if the tube is placed correctly, be along the Y-axis of the screen (this can be checked by applying 20 V A.C. supply to the Y-plates).
Use the pair of dividers to measure the length of the trace $y$ to the nearest millimetre and record it against the corresponding current in the coils, tabulating your results in the usual manner. Repeat for five other values of current.

Disconnect the supply from the Helmholtz coils and connect the $Y$-plates to the potentiometer divider using the 45 ohm rheostat, the 20 V A.C. supply and the Avometer set as an A.C. voltmeter, with a suitable range. The alternating voltage applied to the plates will deflect the electron beam to give a line trace as before. Tabulate the length of the trace and the corresponding p.d. for six different values of the p.d.

Switch off all the supplies, very carefully remove the cathode ray tube from its base, mount it vertically close to a window and measure the various distances marked in Fig. 1 and Fig. 3 using the travelling microscope. You should find no difficulty in recognizing the $Y$-plates which are the ones nearer to the final anode $A_2$, a disc with a central hole. A section of the $Y$-plates is shown in Fig. 4; the ends facing the screen are bent outwards or splayed (why?). Measure the separation $a$ cm between the plates from both sides and average, also the distances $c, b, D, d$ and $D_0$. 

---

**Fig. 1**

**Fig. 2**
The action of the brilliance control is quite simple, for as you have learnt from Experiment 65, the grid of a valve, being very close to the cathode, controls the electron flow with very small applied voltages. Similarly, the grid in the cathode ray tube, which is negative, can be made to alter the electron flow or the brilliance of the spot on the screen by adjusting the output of a potentiometer divider connected across it and the cathode. The action of the focusing control can be followed by examining Fig. 1. The potential of the hollow cylinder (or anode A₁) is always less than that of the final anode, in fact it is negative with respect to it, though still positive with respect to the cathode. The electrons which are moving off the axis (thus possessing transverse velocities) will be decelerated radially as they pass inside A₁, thus emerging parallel and close to the axial stream. An analogy is the way water trickling down the mountainside is channelled into a narrow stream by placing a trough at a suitable point in its path. The raised sides of the trough, like the negative potential of the focusing cylinder, deflect the meandering drops into the main stream. It is also clear from this analogy that if the channelling action of the trough is to be effective it has to be less steep than the surrounding incline.

The electron beam, constituting a moving charge (i.e. a current), is deflected by a magnetic field (the motor effect) and the purpose of the first experiment is to show that the deflection \( y \) is proportional to \( H \) the magnetic field; this can be shown by plotting \( y \) against \( I \) (clearly \( H = kI \) where \( k \) is a constant depending on the dimensions of the coil). Deduce the gradient \( s₁ \) of the resulting line, which should pass through the origin.

The relation between \( H \) and \( y \) can be deduced with reference to Fig. 3. If \( m \) is the mass of the electron, \( v \) its velocity, \( e \) its charge, \( H \) the field and \( R \) the radius of the circle taken by the electrons:
C.G.S.

\[ B_{eV} = H_{eV} = \frac{mv^2}{R} \]

\( H \) oersted; \( e \) coulomb e.m.u.; \( v \) cm sec\(^{-1}\); \( R \) cm; \( m \) g.

Using the usual approximations we get

\[ D_0^2 = 2 \times \frac{y}{2} R = yR \]

Hence

\[ y = \frac{D_0^2He}{vm} = \frac{D_0^2ke}{vm} \]

or

\[ \frac{y}{I} = \frac{D_0^2ke}{vm} = s_1 \] (1)

If the radius of the coil is \( r \) cm, then

\[ H = \frac{16\pi NI_{\text{max}}}{25\sqrt{5}r} \]

where \( N \) is the number of turns in each coil, \( I_{\text{max}} \) is the peak value of the current.

Thus

\[ H = \frac{16\pi \sqrt{2}NI_{\text{RMS}}}{25\sqrt{5}r} \]

Hence \( k = \frac{8\sqrt{2}N}{5\sqrt{5}r} \). ... (2)

M.K.S.

\[ B_{eV} = \mu_0 H_{eV} = \frac{mv^2}{R} \]

\( H \) A m\(^{-1}\); \( \mu_0 \) space permeability, Henry m\(^{-1}\); \( e \) coulomb; \( v \) m sec\(^{-1}\); \( R \) m; \( m \) kg.

\[ D_0^2 = 2 \times \frac{y}{2} R = yR \]

or

\[ \frac{y}{I} = \frac{D_0^2\mu_0 ke}{vm} = s_1 \] (1)

If the radius of the coil is \( r \) m,

\[ H = \frac{8NI_{\text{max}}}{5\sqrt{5}r} \]

\[ I_{\text{max}} = \sqrt{2}I_{\text{RMS}} \]

\[ H = \frac{8\sqrt{2}NI_{\text{RMS}}}{5\sqrt{5}r} \]

Hence \( k = \frac{8\sqrt{2}N}{5\sqrt{5}r} \). ... (2)

When an alternating potential \( V \) is applied to the \( y \)-plates (separated by distance \( a \)), the electrons are attracted to the positive plate with a force \( Ve/a \) and an acceleration \( f = Ve/ma \) (Fig. 4) as a result of which the electrons acquire a total transverse velocity

\[ u = ft + f't' = \frac{Ve}{ma} \cdot \frac{d}{v} + \frac{2Ve}{m(a+b)v} \]

the second term is due to the splayed end of the plates, the separation being the mean distance between the plates \((a+b)/2\). But

\[ \tan \theta = \frac{u}{v} = \frac{y}{2D} \approx \frac{\sqrt{2}V_{\text{RMS}}}{m} \left( \frac{d}{a} + \frac{2c}{a+b} \right) \]

\[ \therefore \frac{y}{V_{\text{RMS}}} = \frac{1}{v^2} \cdot \frac{2\sqrt{2}eD}{m} \left( \frac{d}{a} + \frac{2c}{a+b} \right) \] ... (3)

Thus \( y \) is proportional to \( V \), so that on plotting \( y \) as ordinate and \( V \) as
abscissa, one should get a straight line passing through the origin, from which the slope $s_2$ can be deduced.

C.G.S.

$V_{RMS}$ is in e.m.u. and there are $10^8$ volt e.m.u. to one practical volt. Thus

$$\frac{y}{V_{RMS}} = s_2 = \frac{10^8}{v^2} \times \frac{2\sqrt{2}\varepsilon D}{m} \times \left( \frac{d}{a} + \frac{2c}{a + b} \right). \quad (4)$$

Eliminating $v^2$ from (1) and (4)

$$e = \frac{s_1^2 \cdot 2\sqrt{2}D}{s_2 \cdot k^2D_0^4 \left( \frac{d}{a} + \frac{2c}{a + b} \right)} \times 10^8 \quad . \quad (5)$$

coulomb e.m.u./g

M.K.S.

$$\frac{y}{V_{RMS}} = s_2 = \frac{1}{v^2} \frac{2\sqrt{2}\varepsilon D}{m} \times \left( \frac{d}{a} + \frac{2c}{a + b} \right). \quad (4)$$

Eliminating $v^2$ from (1) and (4) we get

$$\frac{e}{m} = \frac{s_1^2 \cdot 2\sqrt{2}D}{s_2 \cdot \mu_0^2D_0^4k^2} \times \left( \frac{d}{a} + \frac{2c}{a + b} \right). \quad (5)$$
coulomb/Kg

Give $e/m$ to two significant figures.

Comments

The cathode ray tube is essentially a voltmeter and its sensitivity is expressed in mm/volt when the p.d. is applied to the Y-plates (which are more sensitive than the X-plates—why?). It should be clear from your results that signals of the order of millivolts (say those from a crystal pick-up or microphone) cannot deflect the spot unless the signal is amplified by a factor of 10,000, so that a cathode ray tube is often used with an amplifier of high gain.

As the mass of the electron is extremely small, an electron beam can follow very rapid oscillations, making it invaluable when used in an oscilloscope.

EXPERIMENT 67

An attempt to obtain a rough estimate of the diameter of a molecule and to determine the mass of a single hydrogen atom

Introduction

It is fairly obvious that in spite of the continuity of the physical appearance of matter, for example, a polished glass surface or a pool of liquid, matter consists of discrete particles which we call molecules.

We know that when a soap film gets thinner by attenuation a black spot appears, indicating the thickness of the film must be less than a quarter
of the wavelength of light. Thus the diameter of a molecule must certainly be less than $10^{-6}$ cm.

**Rough estimate of the diameter of a molecule**

**Apparatus**

Two 1 ml graduated pipettes, one 20 ml pipette, one well-stoppered test-tube, one large black photographic dish 10 in. $\times$ 8 in, a large sheet of glass to cover the dish, lycopodium powder, 'Flashdry' marker, paper, absolute alcohol, small beakers, oleic acid, scissors and burette stand.

**Method**

Use the large pipette to transfer 20 ml of absolute alcohol to the test-tube, which should then be stoppered. Fill one of the 1 ml pipettes with oleic acid and allow it to drip vertically into the beaker. Find the volume of one drop by collecting a known number in a small measuring vessel and finding their total volume. Carefully, without disturbing the vertical pipette, allow three or four drops to fall into the test-tube containing the measured volume of alcohol. Stopper the test-tube again and shake vigorously to dissolve the oleic acid in the alcohol. Pour some water into the black photographic dish to the depth of one centimetre, sprinkle some lycopodium powder over the whole surface and cover the dish with the glass sheet. Fill the second graduated 1 ml pipette with the mixture of oleic acid and alcohol from the test-tube and again determine the volume of each drop. Place the pipette centrally over the photographic dish; with your finger on top of the pipette lift with your other hand the glass cover, and allow one single drop to fall on to the surface of the water. Replace the glass cover, but without disturbing the surface of the water.

You will note that the fallen drop will spread very rapidly in all directions, the lycopodium powder withdrawing ahead of the advancing, circular alcoholic patch. The alcohol however soon evaporates and lycopodium returns to the area, but a central patch of surface is left clear. This patch of oil appears dark and does not show any interference colour indicating that it is very thin and is probably a uni-molecular layer. Draw an outline of the clear patch on the glass. Take a square sheet of paper, larger than the size of the patch, measure the length of its side and hence find its area. Weigh it on a chemical balance and record its mass. Now trace the outline of the patch on the piece of paper by viewing the traced outline against the light of a window. Cut out the outline in paper and weigh it on a balance. Check this experiment for the area of the patch at least once, washing the dish carefully between experiments.

**Theory and Calculation**

You have already studied the 'drop weight' method in Experiment 25 for comparing surface tensions. Having found the volume of one drop of oleic acid $v_1$ ml you can calculate the total volume of the acid added to 20 ml of alcohol ($nv_1$), where $n$ is the number of drops added. Therefore 1 ml of the mixture contains $nv_1/20$ ml of oleic acid. If the volume of each
drop of the mixture is \( v_a \) then the volume of oleic acid in each drop is \( nv_a v_x/20 \text{ ml} \).

The area of the patch \( A \text{ cm}^2 \) can be calculated from the weight of the paper 'patch', the total area of the sheet of the paper and its mass. Then the thickness of the oil film will be \( nv_a v_x/20A \text{ cm} \). Give the answer correct to two significant figures.

**Determination of the mass of a single hydrogen atom**

**Apparatus**
Gas voltameter filled with dilute sulphuric acid (Fig. 1), 4 V supply, ammeter (0-0.05 A), rheostat, switch, stopwatch and thermometer.

**Method**
Set up the circuit shown in Fig. 1. Adjust the current to 0.045 A and allow it to flow for 30 minutes exactly, keeping the current constant by means of the rheostat. Record the volume of hydrogen generated and the pressure head \( h \text{ cm} \), the temperature of the room and the barometric pressure.

**Theory and Calculation**
In this method the total charge passing through the voltameter is \( It \) coulombs. The mass of hydrogen liberated can be calculated from its volume and pressure. If the barometric pressure is \( p \text{ cm of Hg} \) and the saturated V.P. at the room temperature is \( h \text{ cm} \), then the pressure of the hydrogen is \( \pi - p - [hp/13.6] \text{ cm} \) where \( \rho \) is the density of the dilute sulphuric acid (nearly 1 g/ml), 13.6 g/ml being the density of mercury. Taking the volume occupied by 2 g of hydrogen to be 22.4 litres at S.T.P., calculate the mass \( \mu \text{ g} \) of the hydrogen involved. Now, as the hydrogen ion carries only one unit charge, then \( It/\mu = e/M \) where \( e \) is the electronic charge found from Experiment 62 (about 1.6 \( \times \) 10\(^{-19}\) coulomb). Calculate the mass of the hydrogen atom \( M \).

**Comments**
The mono-molecular layer is a fascinating phenomenon and has several useful applications. In the mono-molecular layer of oleic acid, each molecule is up-ended with its hydrophilic end (containing COOH) pointing towards the water and its hydrocarbon end (the hydrophobic end) uppermost. Yet a drop of oleic acid, placed on water, does not spread but forms globules. It seems probable that the properties of the mono-molecular layer are not the same as that of the liquid in bulk. It can be shown that if the film is compressed, causing the molecules to pile up, then suddenly, when
the film reaches a critical thickness, its surface tension undergoes a change and reverts to that of oleic acid in bulk.

Calculate the Avogadro number \( N \) from the knowledge of \( M \), the mass of the hydrogen atom \( (N = 1/M) \). This gives one of the most accurate methods of determining \( N \). Several attempts have been made to determine \( N \) using Brownian movement and molecular mean free paths (see Experiment 71), and the fact that all these values agree constitutes one of the most striking proofs of the kinetic theory of matter.

**EXPERIMENT 68**

**The elementary study of radioactivity**

*Note* All the radioactive sources recommended in this experiment are approved by the Department of Education and Science; it is assumed that the student will only handle them with forceps and keep them as far away from himself and others as possible.

**Apparatus**

0.1 \( \mu \)c (Pu\(_{239}\)) \( \alpha \)-source, 1 \( \mu \)c Sr\(_{90}\) \( \beta \)-source, 5 \( \mu \)c Co\(_{60}\) (it is recommended that this source should be left in its thick lead container with its lead cover, the lid to be removed during the experiment so that the opening is away from the operator), \( \alpha \) source holder (Fig. 1), 5 kV E.H.T. variable supply, pair of dividers, a spark detector.

Stephenson’s dosemeter, variable 200 V H.T. supply, circular aluminium discs 1 mm thick, and 1 in diameter, stopwatch.

Geiger tube with scaler or ratemeter, several copper sheets 1\( \frac{1}{2} \) mm thick.

Special ionization chamber, used with Stephenson’s dosemeter for measuring the half-life of thoron.

This experiment consists of four parts:

**To find the range in air of \( \alpha \)-particles from Pu\(_{239}\)**

**Method**

Connect up the spark detector to the E.H.T. unit without the source and adjust the voltage until the counter just fails to spark over. Attach, with the forceps, the \( \alpha \)-source to the holder. When the source is lowered to within 3 cm from the counter, you will observe random sparks appearing between the insulated platinum wire and the plate. Note the effect of
placing a single sheet of paper between the source and the counter. Find
the range of the \( \alpha \)-particles by raising the source until all the sparks cease.
Switch off the E.H.T. unit and measure the distance between the estimated position of the source and the counter. Repeat as a check.

**Theory and Calculation**

It must be clear that the energies of \( \alpha \)-particles must have well defined values, for a 1 mm increase in the distance between the source and the detector can stop the sparking altogether. The energy in MeV \( E \) is related to the range \( R \) cm in air by Geiger's formula 
\[ E = 2.1 \times 10^{-1} R \] MeV. Deduce \( E \).

**To find the range of \( \beta \)-particles in aluminium**

**Method**

The dosemeter contains a quartz fibre which is deflected when charged, the deflection being measured by means of a graduated eye-piece which reads zero when the dosemeter is fully charged. It is important that the charging voltage is gradually increased, as a rapid deflection of the quartz fibre can result in damage. You can investigate by pressing the charging knob and varying the H.T. supply that the change in deflection is proportional to the change in voltage.

Place the strontium source on the base of the ionization chamber (Fig. 2), charge the dosemeter to read zero, then time the change of deflection through one small division. Record your observations in the table on opposite page. There is no need to charge the electroscope fully every time, but it must not be allowed to be discharged below the half-way mark. The experiment should be checked, using as many sheets as necessary and available.

**Theory and Calculation**

One can consider the dosemeter as a capacitor which measures its own voltage, so that if \( dv \) is the change in voltage, \( c \) the capacitance and \( t \) time then the ionization current produced by the \( \beta \)-particles is \( c dv / t \) and hence the intensity of radiation is proportional to \( 1/t \). Plot \( \log 1/t \) against the number of sheets as in the curve shown in Fig. 5. Unlike \( \alpha \)-particles, \( \beta \)-particles have varying energies as one sees in the curving off of the graph before levelling off to the level of the background radiation.

Deduce roughly the approximate number of sheets needed to stop the \( \beta \)-particles and by weighing one sheet and measuring its area deduce the stopping power of aluminium to \( \beta \)-particles in mg cm\(^{-2} \) \((\rho)\), hence deduce the maximum energy of \( \beta \)-particles from the relation

\[ E = (0.00185\rho + 0.245) \text{ MeV} \]
To study how the intensity of radiation varies with distance from the γ-source and to find the absorption coefficient in copper

**Method**

Place the lead container of Coγ so that the opening is away from you and facing the Geiger tube (Fig. 3). Record the ratemeter reading or the number of counts per minute and the distance between the open end of the container and the top end of the tube, varying the distance from 50 cm to 5 cm, recording your observations in the usual manner.

![Fig. 3](image)

**Theory and Calculation**

As γ-rays are electromagnetic waves like light and radio waves, it is reasonable to try to find out if the inverse square law holds for them as well. Thus the intensity of radiation $I$ is proportional to $1/d^2$ where $d$ is the distance between the source and the Geiger tube. But as you cannot be certain of measuring $d$ it is best to plot $d$ against $1/\sqrt{I}$.

γ-rays are very penetrating rays and one needs metals of higher density than aluminium to absorb them. For this reason copper was chosen as the absorbing material. It can be shown that the transmitted intensity $I$ is related to the incident intensity $I_0$ from the source by the relation $I = I_0 e^{-\mu x}$ where $\mu$ is called the absorption coefficient for copper, hence

$$\log I = \log I_0 - \mu x \log e$$

$(\log e = 0.4343)$

Plot $\log I$ against $x$ and deduce the gradient $\mu \log e$ and hence $\mu$. Deduce the energies of γ-rays from the graphs given in the *American Institute of Physics Handbook*, pp. 8–93.
To find the half-life of thoron

Method

Place the dosemeter in electrical contact with the insulated central tube of the special ionization chamber (Fig. 4). Charge the dosemeter as before and force thoron into the chamber by squeezing the plastic container of thorium oxide two or three times. Time, with the help of your partner, the movement of the quartz fibre as it discharges, rapidly at first, but more slowly later, tabulating your results in the table below. Repeat the experiment as a check.

Theory and Calculation

Thorium emanation (thoron) decays by emitting α-particles, and if the dosemeter deflections (y) are plotted against time t (sec), a smooth curve results. Plot a large graph and determine the gradients dy/dt at several points on the graph which give the radioactivity at different times. Plot dy/dt against time to give a straight line sloping downwards; deduce the gradient m.

For radioactive decay \( I = I_0 e^{-\lambda t} \)

\( \log I = \log I_0 - \lambda t \log e \)

If \( T \) is the half-life of thoron then \( t = T \) (half-life) when \( I = I_0/2 \). Hence the gradient

\[ m = \lambda \times \log e \quad \text{and} \quad T = \frac{\log 2}{\lambda \log e} = \frac{0.3010}{m} \text{ sec.} \]

Comments

It should be clear from your experiment that α-particles have the greatest energy and, because of their comparatively large mass (helium nuclei, mass \( 6.6 \times 10^{-24} \text{ g} \)), they are stopped easily by matter. Hence also their short range in air. The α-particle, as it possesses energies of definite values, occupies in the nucleus definite energy levels and a rough estimate of the size of the nucleus

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can be obtained if the velocity \( v \) of the \( \alpha \)-particle and its momentum \( mv \) are calculated from its energy and an estimate is made of the de Broglie wavelength \( \lambda = h/mv \) where \( h \) is Planck's constant \( (6.6 \times 10^{-27} \text{ erg-sec}, \) or \( 6.6 \times 10^{-34} \text{ joule-sec}) \).

The continuous spectrum of \( \beta \)-energies posed a serious problem for some time, but now it is established that a neutrino, a very small neutral particle of very small variable mass, accompanies \( \beta \)-emission. Will the intensity from a \( \beta \)-source vary with distance according to the inverse square law?

Estimate the velocity of the \( \beta \)-particle from the estimate of its maximum energy \( E_{\text{max}} \) assuming the accepted value of \( e/m \) for electrons. Comment.

\( \gamma \)-energies from Co\(_{60}\) are high, in the order of one or two MeV, and have high penetrating powers which account for the health hazard from them. As the radiation falls off according to the inverse square law, \( \gamma \)-emitting sources are not only encased in lead but kept in the centre of a large box so that the energy level just outside the box is considerably lower.

Estimate from their energies the wavelength \( \lambda \) (= \( h\epsilon/E \), \( \epsilon \) being the velocity of light \( 3.0 \times 10^{10} \text{ cm sec}^{-1} \)) and consider the various ways in which they lose energy in matter.
SCHOLARSHIP EXPERIMENTS
SCHOLARSHIP EXPERIMENTS

INTRODUCTION

Experimental observations are based on the readings of the dials and scales of measuring instruments and detecting devices, none of which is perfect. Newton was fortunate in being able to base his Gravitational Law on the idealized motion of the Moon, but lesser mortals have to contend with the imperfect world of the laboratory.

The competence of the experimenter is mostly gauged by his success in dealing with these imperfections; some of the more obvious experimental errors have already been dealt with, but the one which offers the biggest challenge is the 'systematic error'. This does not reveal itself on repeating an observation, for it influences the result in the same manner every time. Only when one uses entirely different measurements or techniques can such an error be detected. Examples of this are met in calibration of instruments, faulty pointer readings, uncertainties in physical quantities used in calculations (for example, the unreliability for the coefficient of viscosity of air which Millikan used in his famous experiment to determine the electronic charge).

We have learnt that the standard error is inversely proportional to the square root of the number of observations ($\sigma_{\text{error}} = \text{standard deviation} / \sqrt{n}$) and clearly it is better to reduce $\sigma$ than to increase $n$. In an experiment involving the measurement of a short length, it is clearly better to use a pair of vernier callipers and reduce the number of observations than to use an ordinary ruler.

One therefore repeats an experiment not primarily to improve the precision of a measurement but to get an estimate of the uncertainty of the measurement (namely $\sigma$ and $\sigma_{\text{error}}$). Finally, one has to learn to squeeze the maximum possible information out of one's experimental data.

Your account of each of the experiments in the following part of the book should be modelled on that of a scientific paper. Underneath the title write a short abstract of the experiment and any important conclusions arrived at; this is to be followed by experimental details and diagrams, mentioning any special precautions taken or particular measurements used which would affect the accuracy of the final result. A tabulated summary of the observations should then follow with analysis of the results and calculations. Finally, the conclusion should include any numerical result, giving the overall limits of uncertainty, depending on the expression used for the evaluation of the result and the uncertainties in the measurements of the individual variables making up that expression.
EXPERIMENT 69

The further study of friction (a) between a cotton thread or plastic tape and a stainless steel cylinder, (b) in a liquid flowing in a pipe

Apparatus

Stainless steel cylinder (scrap gudgeon pin from any garage), pulley, clamps, light scale-pan and weights, magnetic tape, cotton thread, aspirator and accessories (Fig. 2), several capillary tubes about 25 cm long or 1–2 mm bore, travelling microscope (with six-inch objective), 100 ml measuring cylinder, mercury, spray bulb, watch glass, chromic acid, methylated spirit, cotton wool.

(a) Study of friction between a cotton thread or plastic tape and a stainless steel cylinder

Method

Clamp the steel cylinder with its axis horizontal and the pulley as shown in Fig. 1. Clean the surface of the cylinder with a wad of cotton wool dipped in methylated spirit. Find the friction of the pulley by hanging a 50 g weight from one end of a thread placed over the pulley with the other end attached to a scale-pan in which weights are added until the scale just begins to move down. Record the total weight of the pan and its contents.

Now pass a cotton thread over the cylinder and the pulley as shown in Fig. 1, and find the weight needed to cause the thread to slide over the cylinder in the direction shown. Measure the angle \( \theta \). Repeat with different angles \( \theta \).

Clean the cylinder again and hang the thread over it with 50 g attached to one end and the scale-pan to the other (the pulley is not used). Find the force necessary to cause the scale-pan to move down; repeat with different weights attached to the other end of the tape (\( \theta \) in this case is 180°).

Theory and Calculation

Using the force diagram shown in Fig. 2, it is clear that the normal reaction exerted by an element of the cotton thread on the cylinder is \( (2T + \delta T) \sin (\delta \phi /2) \) and when the limiting friction is exerted then

\[
\delta T = \mu (2T + \delta T) \sin (\delta \phi /2)
\]
as \( \delta \phi \to 0, \delta T \to 0 \) but \( \frac{\delta T}{(\sin \frac{\delta \phi}{2})} \to 2dT/d\phi \)

therefore \( \mu d\phi = dT/T, \mu = \text{coefficient of friction.} \)

![Diagram](image)

\( \text{Fig. 2} \)

Integrating and remembering when \( \phi = 0, T = W_0 \) (the weight hanging on the left) we get

\[
T = W_0e^{\mu \phi}, \quad \phi \text{ being in radians}
\]
\[
T = W_0e^{\mu \theta \pi/180}, \quad \theta \text{ being in degrees}.
\]

(1)

Verify equation (1) for cotton by plotting a suitable graph and find \( \mu \) for both cotton thread and recording tape.

\[
\log T = \log W_0 + \frac{\mu \theta \pi}{180} \log e
\]

where \( T \) is the weight of the pan and contents less the frictional force of the pulley.

(b) **Study of friction in a liquid flowing in a pipe**

**Method**

Set up the apparatus shown in Fig. 3, using one of the narrower capillary tubes after cleaning it with chromic acid, water and methylated spirit. Vary the pressure head \( h \) by raising or lowering the air inlet, and measure the mass of water flowing out of the capillary per second using the measuring cylinder and a stopwatch. Measure the mean pressure head \( h \) (the level of water in the small funnel fluctuates as the air bubbles through) each time. Record the temperature at the beginning and the end of the experiment.

Remove the capillary tube, wash it with methylated spirit and dry it out. Attach the spray bulb to one end of the capillary and, by squeezing the air out, draw an index of mercury into the tube. Find the mean length \( x \) of the index and its mass \( m \).

Repeat with a different liquid and different capillary tube.
For streamlined flow \( Q = \frac{\pi pa^4}{8\eta l} \) (Poiseuille's formula)

where \( Q \) is the volume flowing in the pipe per second
\( p \) pressure head (i.e. \( h \rho g \))
\( a \) the radius of the pipe \( (a^2 = \frac{m}{\pi x \times 13.6}) \)
\( \eta \) viscosity of the liquid in poises
\( l \) length of the pipe.

Plot \( Q \) against \( h \) and deduce \( \eta \) at the mean temperature recorded for the liquids used.

**Comments**

Inspection of equation (1) reveals that the frictional force increases very rapidly with increasing angle (i.e. when the angle is doubled, the force is squared) and this fact is used in fastening ships to bollards and supporting nylon thread as in Experiment 24.

One of the difficulties in measuring the viscosity of a liquid is the probable variation of temperature which, if not thermostatically controlled, can be more than 1° during the experiment. This alone can put a limit to the accuracy of this method. Use the widest tube to investigate the effect of increasing the speed of flow of water by increasing \( h \) and find roughly when \( Q \) ceases to be proportional to \( h \) (i.e. the onset of turbulence). Estimate the order of magnitude of Reynolds' number \( = V_c \rho a / \eta \) where \( V_c \) is the critical velocity; it is clear when \( V > V_c \), turbulence takes charge and the effect of the viscosity of the liquid becomes unimportant in comparison.

**Experiment 70**

**To study a simple rigid pendulum and the use of Kater's pendulum to determine accurately \( g \), the acceleration due to gravity**

**Apparatus**

A heavy brass bar 1 m long, a knife edge which can be clamped to the bar, a slotted steel plate on which the knife edge rests, Kater's pendulum (see *Science Masters' Book*, series 1 (Physics), page 8), a phototransistor and two 9 V dry batteries in series (any transistor will do; just scratch off the light-tight varnish and the light-sensitive pellet will be seen through the glass envelope), 6.5 V pea-bulb and holder and suitable L.T. supply, sensitive relay, P.O. magnetic counter and suitable L.T. supply, stop-watch, clamps and bosses.
Method

Clamp the steel plate to a bench, and clamp the knife edge to one end of the heavy bar, resting it on the steel plate. Allow the bar to swing through the slot through a small angle (less than $5^\circ$ on either side of the vertical). Time a suitably large number of oscillations: check the timing at least once and re-check if necessary. Measure the distance $x$ cm from the same end of the bar to a suitable fixed point on the knife edge. Vary $x$, tabulating your results in the usual manner.

Replace the heavy bar with Kater's pendulum. Since great accuracy is required in the timing, several hundreds of oscillations have to be timed at a stretch, which is not only tedious but also difficult to do without making a mistake in the counting. Place the sensitive area of the phototransistor close to the rod of the pendulum and the pea-bulb on the opposite side, so that the oscillating pendulum interrupts the light twice in a complete oscillation. Thus the relay, which should operate on 2 mA, should close the circuit of the magnetic counter twice in a single oscillation and the total number of swings (2 swings $= 1$ oscillation) will automatically be recorded. Using a good stopwatch, which should have been checked against an electric clock by running it for a full hour and then correcting for any difference, time 20 oscillations with each knife edge. If the total times differ by more than 1%, move the small mass $w$ (Fig. 1) till the times are very nearly equal. Now time the pendulum for each knife edge for 2000 swings (1000 oscillations) and calculate the period in each case, $T_1$ and $T_2$. Measure the distance between the knife edges at both the left- and right-hand sides of the knife edges twice, using a cathetometer reading to one-tenth of a millimetre. Call the mean of the distance $h$. Now balance the pendulum on a file with a triangular cross-section and determine to the nearest millimetre the distance $h_1$ of the centre of gravity from one of the knife edges. Deduce $h_2$, the distance of the centre of gravity from the other knife edge.

Fig. 1
THEORY AND CALCULATION

It must have been clear, using the heavy bar, that the period of a simple rigid pendulum is complicated and not related in a simple way to the length of the bar, as in the case of the simple pendulum. It is also obvious that the period becomes very large when the point of oscillation is close to the centre of gravity. Thus, in plotting the period against the distance \( x \), you get a curve consisting of two branches, one the mirror reflection of the other, each branch meeting asymptotically the line through the centre of gravity parallel to the time axis. There are four possible points of oscillation about which the period is the same (Fig. 2), the lengths AC and BD being equal and related to the corresponding period by a simple pendulum formula, namely
\[
T = 2\pi \sqrt{\frac{AC}{g}} \text{ or } BD/g.
\]
This should be verified. At point H the two points coincide and similarly at point K, so that HK is the equivalent length of a simple pendulum of period \( T_0 \). Points H and K are of some importance.

In the case of Kater’s pendulum the value of \( g \) can be calculated from the relation
\[
\frac{8\pi^2}{g} = \left( \frac{T_1^2 + T_2^2}{h_1 + h_2} + \frac{T_1^2 - T_2^2}{h_1 - h_2} \right)
\]
The second term in the bracket is very small so that \( h_1 - h_2 \) need not be known to a high degree of accuracy. The value of \( g \) calculated, using a checked stopwatch, should be correct to within one in a thousand.

COMMENTS

The rigid pendulum is used in mechanical clocks to regulate the time and is also used in geophysical prospecting for the accurate determination of \( g \). As the changes in \( g \) produced by the presence of certain dense mineral-bearing rocks are very small, very large numbers of oscillations have to be made to obtain adequate accuracy.

Referring to Fig. 2, AC and BD are the equivalent lengths of a simple pendulum of period \( T \). If the bar is supported at point A (or B), and it is struck a blow at C (or D), there will be no impulsive reaction at A (or B); C (or D) being known as the centre of percussion with respect to A (or B). Can you demonstrate this experiment with a cricket bat?

The distance OH or OK is called the radius of gyration, and if the mass of the bar is imagined to be concentrated at H or K, it would have the same moment of inertia about an axis through O, perpendicular to the bar, as the whole rod.
EXPERIMENT 71

The study of a bifilar suspension and the use of such a suspension in an experiment to determine the viscosity of air

Apparatus

Two metre rules, clamps and stands, cotton thread, stopwatch, mains operated turn-table, 10 in rigid playing disc suspended horizontally by means of two short cotton threads a few millimetres apart (Fig. 2), some Plasticine, thin wedge.

Method

Suspend a ruler as shown in Fig. 1, the suspension threads being equal and parallel. Find the period of oscillation for various separation distances $2a$. Next, keeping $2a$ constant, vary $l$ and find the period in each case.

To find the coefficient of viscosity of air use the apparatus shown in Fig. 2. Find first the period of oscillation of the disc, hanging freely and oscillating on its bifilar suspension (keep the amplitude of oscillation small, but make sure that the disc is horizontal, using small pieces of Plasticine as balancing weights). Next position the turn-table underneath the disc with about 2 mm air gap, start the motor and determine the angle of twist (nearest degree) of the disc due to the viscous drag on its underside. Measure the air gap at several places, using the thin wedge provided. Measure, also, the period of revolution of the turn-table. Unhook the disc and find its mass to the nearest gram and its radius.
THEORY AND CALCULATION

For small angular displacement, one can assume that the tensions in the vertical threads remain the same \((Mg/2)\) each and that the two restoring forces \((Mg/2) \times (a\theta/l)\) constitute a restoring couple \(Mga^2\theta/l\).

Therefore

\[ I\ddot{\theta} = -\frac{Ma^2g}{l} \times \theta, \]

and the period

\[ T = 2\pi \sqrt{\frac{Il}{Mga^2}}. \]

Verify this relation, deduce \(I\) and check it by weighing the suspended ruler and calculating \(I\).

In the second part of the experiment, deduce the viscous couple from Fig. 3 where the viscous force on an annular element of radius \(r\) is
2\pi dr\eta \cdot \omega / t \) (from the definition of the coefficient of viscosity \( \eta \)) and the total couple \( 2\pi \omega \int_0^\infty \eta r^2 dr \). This should be equated to the twisting couple \( \pi \theta C \over 180 \)

where \( C \) is the torque in dyne cm radian\(^{-1} \).

But \( T = 2\pi \sqrt{\frac{T}{C}} \) and \( C = \frac{4\pi^2 I}{T^2} \).

Therefore

\[
\frac{\pi \omega \eta a^4}{2t} = \frac{4\pi^2 I}{T^2} \times \frac{\pi \theta}{180}.
\]

\( I \) can be calculated, hence \( \eta \) (poises) at the recorded temperature of the room.

**Comments**

A bifilar suspension is a simple method which can be sensitive and robust yet not relying generally on the elasticity of suspending threads.

Viscosity, in common with other transport phenomena in a gas (e.g., thermal conductivity and diffusion), depends on the mean free path \( \lambda \) and simple theory would show that

\[
\eta = \frac{1}{3} \rho \bar{v} \lambda \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)
\]

where \( \rho \) is the density of the gas, \( \bar{v} \) root mean square velocity.

If the diameter of a molecule is \( \sigma \), then it would tunnel for itself in a gas a cylinder of volume \( \pi \sigma^2 \lambda \) ml and if \( n \) is the number of molecules per gram then

\[
\rho = \frac{1}{n \pi \sigma^2 \lambda} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)
\]

Thus \( \rho \lambda \) is a constant when the temperature is kept the same, so that \( \eta \) is independent of pressure (except at very low pressures). This is an unexpected result which has been verified and is a striking proof for the kinetic theory of gases.

If we assume that in liquid air each molecule effectively occupies a cube \( \sigma^3 \) ml in volume, then the density of liquid air

\[
\rho = \frac{1}{n \sigma^3} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)
\]

where \( n \) is as before the number of molecules per gram.

Use equations (1), (2) and (3) to get an estimate of \( \lambda \), \( \sigma \) and Avogadro’s number \( N \) (take \( \rho \) for liquid air to be 0.9 g/ml, \( \rho \) for air 0.0013 g/ml).

The striking behaviour of a Crooke’s radiometer is explained by the fact that the pressure of the gas inside it is so low that \( \lambda \) is comparable to the dimension of the apparatus. Use the above equation to estimate the upper limit of this pressure.
EXPERIMENT 72

To determine the modulus of rigidity of a steel wire and the moment of inertia of a body

APPARATUS

A steel wire (12 S.W.G.) bent in the manner shown in Fig. 1, two small pieces of hardboard, a solid rod 5 in long and 1 in diameter, a short piece of steel angle in which a hole is drilled as shown, small weights up to 50 g, a very light scale-pan, cotton thread, two G-clamps, set squares, stopwatch.

METHOD

Clamp the apparatus as shown in Fig. 1, suspending a plumb line from point O, and ensure that PO is as nearly horizontal as possible. Tie the light scale-pan to the point P as shown and measure as accurately as you can the vertical depression y of point P and measure also PO. Now add a small weight (say 5 g) to the scale-pan and measure the new value of the vertical depression y. Repeat the observation, increasing the weight each time, but do not exceed the elastic limit. Check your observations with decreasing loads to make sure that the arm PO returns to its original position.

To determine the moment of inertia of the solid cylinder, slide the steel wire out of the angle and clamp the short bent end of the wire between the two pieces of hardboard and the bench. Tie the cylinder or any object of unknown moment
of inertia to the long bent end of the wire so that the wire passes through the centre of gravity of the suspended body. Set the body in rotational oscillation and find the period, timing a suitable number of oscillations, and check as in all timing observations.

**Theory and Calculation**

If the mass of the scale-pan and weight is \( m \) g, PO equals \( a \) cm, then the couple twisting the wire is \( mga \cdot \cos \theta \), where \( \sin \theta = y/a \), \( \theta \) being also the angle of twist. You have learnt from theory that

\[
mga \cdot \cos \theta = \frac{1}{2} \times (\pi/l)nr^4\theta
\]

where \( n \) is the rigidity modulus, \( r \) the radius of the wire, \( l \) is the length of the wire which is twisted as shown in Fig. 1. Plot a suitable graph and deduce \( n \) (dynes cm\(^{-2}\)) making sure to measure \( r \) as accurately as you can. Give \( n \) to two significant figures.

Calculate the moment of inertia as follows:

\[
I \frac{d^2\theta}{dt^2} = \frac{n\pi r^4}{2l} \cdot \theta \quad \text{or} \quad \frac{d^2\theta}{dt^2} = -\frac{n\pi r^4}{2II} \cdot \theta.
\]

Hence the period

\[
T = 2\pi \sqrt{\frac{2II}{n\pi r^4}}.
\]

Deduce \( I \) (g cm\(^2\)).

**Comments**

The damping to the oscillation of the cylinder is only partly due to air resistance; if the steel wire is replaced by tin wire, using the same cylinder, the oscillations will damp down far more quickly, though the air resistance may not have changed. There is, in fact, damping in the wire itself similar to viscosity in liquids and the equilibrium position of the cylinder (the position of zero restoring couple) shifts during the oscillation causing the cylinder to be dragged back, thus damping the motion. The energy used in damping the oscillation of the cylinder goes to heat the wire and also produces some structural changes. This is of great importance in modern aircraft, all component parts of which are subject to continuous vibrations which can lead to metal ‘fatigue’ and subsequent accidents.

**Experiment 73**

To find Young’s modulus by bending a long rod and to check the results by finding the velocity of sound in the rod

**Apparatus**

Wooden rod of circular cross-section (\( \frac{1}{3} \) in diameter) and 5 ft long, glued at one end to a circular cork just less than 2 in diameter. Long glass
tube 2 in internal diameter, two grooved pieces of wood to fit over the rod, clamps, light scale-pan, box of weights, rulers, clamp and stand, lycopodium powder, resin, dry cloth, thermometer. 2 in cork attached to a rod to fit the glass tube tightly.

![Diagram](image)

**Fig. 1**

**Method**

Clamp the bare end of the rod, using one of the grooved pieces of wood, to the end of a rigid bench, as shown in Fig. 1. Find the depression of the other end of the rod $d$ by adding weights to the light scale-pan. The depression can best be measured by sticking a short needle or pin into the edge of the cork level with the centre of the rod. As usual with experiments on elasticity, the depressions are checked as the weights are removed. Measure $L$, and the mean diameter of the rod.

![Diagram](image)

**Fig. 2**

Clamp the exact middle of the rod to the bench using two of the grooved pieces of wood and some thin rubber packing and assemble the apparatus shown in Fig. 2. A small quantity of lycopodium powder should be introduced into the whole length of the tube, which should be perfectly dry.

Stroke the other end of the rod outwards with a dry cloth containing one or two pieces of resin and at the same time let your partner adjust the
position of the other cork, until the air in the tube resonates with the frequency of the note emitted by the stroked rod. The lycopodium dust will form striations, with heaps at the nodes, the vibrating cork at the end of the rod being also a node (see Experiments 57 and 58).

Measure the distance between the two outside antinodes, hence the mean distance between two antinodes. Repeat as a check. Note the temperature inside the tube. Weigh the long rod and measure its length.

**Theory and Calculation**

Provided that \(d\) is not large and that \(dy/dx \ll 1\) everywhere along the rod, it can be shown that \(mgL^2/3 = YAk^2d\) where \(m\) g is the mass added, \(Y\) dyne cm\(^{-2}\) is Young’s modulus, \(A\) is the cross-sectional area and \(k\) the radius of gyration of the area of cross-section about a horizontal diameter. Plot a suitable graph and deduce \(Y\).

As a check the rod is set into vibration by stroking it longitudinally, its centre being clamped, and therefore a node and the free ends antinodes. If \(l_{air}\) is the distance between two antinodes in air then

\[
\frac{l}{l_{air}} = \frac{\text{velocity of sound in wood}}{\text{velocity of sound in air}}.
\]

Deduce the velocity of sound in wood, which, as one would expect, is much greater than the velocity of sound in air (take the velocity of sound in air in the tube at \(0^\circ C = 331\) m sec\(^{-1}\)). The velocity with which a longitudinal vibration travels in a rod is \(\sqrt{Y/\rho}\) where \(\rho\) is the density of wood. From the weight of the rod and its dimensions deduce \(\rho\), hence \(Y\).

**Comments**

The expression for the depression of a loaded cantilever clamped at one end shows that the bigger the radius of gyration of the area the less is the depression, so that for the same area, an annulus has larger \(k^2\) than a circle, and weight for weight a hollow tube stands more load than a solid rod. The relation between the velocity of longitudinal waves in solids and Young’s modulus is frequently used to determine the latter, particularly if the specimen is short. A rod of small cross-section and about 60 cm long is clamped in the middle, a vibrator connected to an oscillator is pressed against one end through a small thin rubber pad glued to the rod and the rod can be set into longitudinal vibration. As the frequency is varied, resonances can be detected by using a crystal pick-up bearing at the other end of the rod connected to an oscilloscope. By determining the resonant fundamental frequency of the rod and its length, the velocity of sound in the wood can be deduced (though this is not a simple experiment as there are usually many other resonant frequencies as well).

The vibration of air in a closed pipe (Kundt’s tube: see Experiment 58) is complicated and is beautifully demonstrated by the movement of the dust particles. Air flows from antinodes to nodes along the walls of the tube and returns along the axis, in addition to some circular motion like ‘dust devils’, giving rise to the striations.
EXPERIMENT 74

To investigate the variation in the viscosity of motor oil with temperature

APPARATUS

Ostwald viscometer BS/U E277, 30 cm gas jar as a water bath, a small 2 V miniature motor to drive a stirrer, a stopwatch, 50°C thermometer, 12 V transformer, small water suction pump.

METHOD

A sketch for the suggested apparatus to be used is shown in Fig. 1. The resistance of the heating coil has to be chosen to give fairly slow heating as, for simplicity, no thermostat is provided. A rise of \( \frac{1}{3} \)°C per minute is reasonable. Allowing for heat losses, estimate the electrical power you require, decide on the gauge of wire to be used, solder the wire to the thick copper leads and assemble the apparatus.

A temperature range of 4°C to 40°C would be of interest, so chill the apparatus before use. To carry out a ‘run’, suck the oil up by means of the pump until the oil stands at a point higher than \( O_2 \) and in the other limb just below \( O_3 \). Time the interval taken by the oil to drain from level \( O_1 \) to \( O_2 \), record the mean temperature and the time, tabulating your results as usual.

Heat the water bath until the temperature reaches that of the room and carry out two ‘runs’ at room temperature to check on the reproducibility of your results. Heat the water bath further, switch off the heater, carry out another ‘run’ and record both time and mean temperature again. Continue until the temperature reaches 40°C. As the water bath cools, checks should be carried out; keep the stirrer going, even though the heating current is switched off.

THEORY AND CALCULATION

You know from theory that the viscosity at any temperature \( \eta \) equals \( k \rho T \) where \( \rho \) and \( T \) are the density and time respectively and \( k \) is a constant for any given viscometer. Though the density does change with temperature between 4°C and 40°C, decide whether the accuracy of your
method justifies determining the densities of the oil at the different temperatures (using a density bottle). If you can treat $\rho$ as constant, then $\eta$ is proportional to the time, $T$.

A study of the table of your results should convince you that the viscosity $\eta (\propto T)$ does not fall uniformly with rising temperature and you may decide that the logarithm of the time should be plotted against the corresponding temperature. Express the viscosity of the oil as a function of both the temperature and its viscosity at $0^\circ$C.

**Comments**

Though it is desirable that the viscosity of the motor oil remains reasonably constant over a wide range of temperature, in practice this is not easily achieved, as your results should show.

If the liquid is more viscous, like castor oil, an alternative method is called for. An instructive method is that using a falling sphere, which, as you know from your theory work, reaches a terminal velocity $v$ (provided turbulence does not set in) given by

$$\frac{4}{3}\pi (\rho - \sigma)ga^3 = 6\pi a\eta v$$

where $\rho =$ density of material of sphere, $\sigma =$ density of viscous liquid, $a =$ radius of sphere, $\eta =$ viscosity of liquid.

Close one end of a long glass tube, at least 3 in diameter, fill it with the viscous fluid and set it up in a vertical position. Surround the tube with a larger vessel containing water. Drop a small ball-bearing into the viscous fluid and time its passage down the middle third of the fluid column. The temperature control is most important here. The density of the liquid is best found by means of a hydrometer.

How does the variation of viscosity of castor oil with temperature compare with that of the motor oil?

**EXPERIMENT 75**

**To calibrate and use a simple refractometer**

**Apparatus**

A right-angled prism of high refractive index with a fine wire taped parallel and close to the right-angled edge (Fig. 1); a square piece of perspex (matt on one side), equal in area to either of the two faces of the prism which contain the right angle, some black matt paper, green Wratten filter, burette and stand, glycerine, 10 ml beaker, pipette, 12 W straight filament bulb and electrical supply, small chip of perspex, low power microscope. Large vertical scale forming a quadrant of a circle with a perspex movable arm pivoted at the centre of the circle, the other end of the perspex arm being curved and polished forming a cylindrical
lens with a fine line scratched on the underside. The rotating arm carries near the top a short side piece with a sighting hole.

**Method**

Cover the hypotenuse of the prism with black paper and place the prism so that the side containing the right angle which is not bearing the wire is resting on the matt surface of the perspex square as shown in the diagram. Arrange the quadrant so that its centre coincides with the level of the fine wire on the prism.

Place a drop of water between the prism face and the perspex, illuminate the gap (do not scorch the green filter) and turn the rotating arm so as to make an angle of about 40° with the horizontal. Place your eye close to the sighting hole and adjust the direction of the arm so that the view through the hole appears half dark and half bright (see diagram) with the fine wire appearing to lie exactly across the boundary. Read off the angle \( \theta \) to 1/10°.

Remove the prism, wipe off the water from the two surfaces and putting a drop of glycerine in its place, repeat the experiment. Use the pipette to put 10 ml of glycerine into the small beaker, add to it 2 ml of water from the burette and stir the mixture well, then take a drop on a clean glass rod and repeat the above observation.

Dilute the mixture by adding further quantities of water and repeat the observation several times.
THEORY AND CALCULATION

This is a simple modification of the Wollaston cube where

$$\mu_{\text{liquid}} = \sqrt{\mu_{\text{glass}}^2 - \sin^2 \theta},$$

$\mu$ being the refractive index. The apparatus is calibrated using water and $\mu_{\text{glass}}$ is calculated. The refractive indices of pure glycerine and diluted glycerine can then be calculated. Investigate the relation between the refractive index of the mixture with the concentration of glycerine (volume of glycerine per unit volume of solution).

COMMENTS

Explain with a clear diagram the view through the sighting hole when the arm is adjusted; why the sharp boundary? Predict what will happen if white light is used and check your prediction.

Look through the low power microscope at the chip of perspex immersed in glycerine with varying degrees of dilution with water until it is hardly visible. Use the calibrated refractometer to find the refractive index of the perspex.

EXPERIMENT 76

To find the refractive index of polystyrene beads and to investigate the formation of rainbows

APPARATUS

Small quantity of polystyrene beads, travelling microscope (capable of measuring to $\frac{1}{200}$ of a millimetre, objective about 8 mm), pea-bulb with holder and battery, microscope slide, circular illuminated object (2.5 cm diameter, covered by frosted glass or focusing screen), large sheet of stiff black paper, large glass sheet, ‘Flashdry’ ink marker, rulers and clamps.

METHOD

Set up the apparatus as shown in Fig. 1 and focus the microscope on the image of the circular object formed by one of the larger beads. As the diameter $d$ of the image is small (0.1 mm), great care should be taken in measuring the diameter, and it should be done more than once. Rack the microscope down and focus on the bead itself. The circular outline should appear sharp and dark against a bright background (why?). Measure the diameter $2R$ (0.3 to 0.4 mm). Select at least two other beads and repeat the observations. Measure the distance $U$ (Fig. 1) and the diameter $D$ of the circular object.

To investigate the formation of rainbows, spread polystyrene beads thinly but uniformly on the black stiff paper provided. Draw several concentric circles of various radii on the glass sheet, using the ink marker. Set
up the apparatus as shown in Fig. 2. The pea-bulb should be clamped, but the glass sheet can be made to rest on the base of the pea-bulb holder.

Adjust the distance between the bulb and the black paper till on looking down you see a circular rainbow just coinciding with the first marked circle, diameter δ. Measure the distances h and H and repeat for the other marked circles.

**Theory and Calculation**

Let O be the object (Fig. 3), I₁ be the first image produced by the refraction of the light at the first surface of the spherical bead (V₁ from the first surface). Let I₂ be the final image after refraction at the second surface (V₂ is the image distance from the second surface).

For the first refraction \[ \frac{1}{U} + \frac{\mu}{V_1} = \frac{\mu - 1}{R} \]
but \( U \gg V_1 \), therefore

\[
V_1 = \frac{\mu R}{\mu - 1} \tag{1}
\]

For the second refraction

\[
-\frac{\mu}{V_1 - 2R} + \frac{1}{V_2} = \frac{\mu - 1}{R}.
\]

Substituting for \( V_1 \) from equation (1) and solving for \( V_2 \) you get

\[
V_2 = \frac{(2 - \mu)R}{2(\mu - 1)} \tag{2}
\]

also the object distance \( V_1 - 2R = \frac{(2 - \mu)R}{\mu - 1} \).

Now the overall magnification \( \frac{d}{D} = m_1 \times m_2 \),

and \( m_1 = \frac{V_1}{\mu U} \), \( m_2 = \frac{\mu V_2}{V_1 - 2R} \),

therefore

\[
\frac{m_1}{d} = \frac{R}{U(\mu - 1)} \quad \text{and} \quad m_2 = \frac{\mu}{2}.
\]

Therefore \( \frac{d}{D} = \frac{\mu R}{2U(\mu - 1)} \). Hence \( \mu \).

For the rainbow part of the experiment, it can be shown (see standard textbook of light) that a ray of light, suffering minimum deviation on being refracted into a sphere, internally reflected and refracted out again, makes an angle of incidence \( i \), given by

\[
\cos i = \sqrt{\frac{\mu^2 - 1}{3}} \quad \text{and} \quad \sin i = \mu \sin r
\]

and the angle of deviation

\[
2\pi - 2\theta = 2\pi - 2(2\pi - i) = 2\pi - 4\pi + 2i.
\]

From Fig. 2 and Fig. 4,

\[
2\theta = 2 \tan^{-1} \frac{\delta}{2(h + H)}
\]

Using the value of \( \mu \) you have found from the first part of the experiment, verify the above relation.

**Comments**

The accuracy of the above method for determining \( \mu \) is limited by the accuracy of the measurement of \( d \) and \( 2R \); what accuracy do you hope to obtain by this method? Use the vertical movement of the travelling microscope to verify equation (2) above.

If each bead can be used as a miniature lens as shown above, why is it that a comparatively thin layer of polystyrene beads is almost opaque to light?
EXPERIMENT 77
The study of the phenomena of polarization and some of its applications

Apparatus
Two polaroids, one 2 in square, the other circular and fitted into the centre of a circular protractor, a pile of 2 in square glass slides, ray box, semi-circular protractor, green Wratten filter, photoelectric cell, Scalamp, some cellophane and Sellotape, sucrose, apparatus shown in Fig. 2, Plasticine, small white screen.

![Diagram](image)

Fig. 1

Method
Use the arrangement shown in Fig. 1 to find the polarization angle for one of the square glass slides (preferably in a darkened room). The glass slide can be held vertical by Plasticine and the reflected ray viewed on a small white screen. You will note that the reflected light is cut off for a definite angle when the polaroid is placed in front of the ray box with its plane of polarization vertical (what about the transmitted light? Determine whether it is polarized). You should be able to obtain the Brewster angle $\theta$ to one degree; use the pile of glass slides to obtain the refractive index $\mu$ of the glass.

Connect the photoelectric cell, which should have a sensitive circular area of diameter 4 cm, to a Scalamp, and use the apparatus shown in Fig. 2 to investigate the intensity of light transmitted through two polaroids (for this part of the experiment you do not need the green filter). Start with the polaroids parallel and record the maximum intensity of the light,
turn the analyser through increasing angles recording the transmitted intensities until the polaroids are crossed (90°), record the 'dark' current and subtract its value from each of your other readings. Record the intensity without the analyser.

The apparatus shown in Fig. 2 can be used as a saccharimeter (without the photoelectric cell), but it must be clear to you by now that the arrangement as it stands would hardly be sensitive enough to detect any small optical rotation. If we start with two polaroids crossed and then introduce the sucrose solution (containing a known mass of sucrose dissolved in 100 ml of water) the angle which is turned through by the analyser to extinguish the transmitted light would be difficult to determine, as the intensity of the transmitted light varies little with θ at this setting. Cut narrow
(\frac{1}{4} \text{ in}) \text{ strips of cellophane parallel to one side of a cellophane sheet and also cut strips at 45° to that side; lay them out alternately side by side on a 2 in square glass slide, cover them with another slide and bind them together with Sellotape (Fig. 3).}

Use a green filter underneath the polarizer and turn the analyser until the polaroids are crossed; put the slide with the strips of cellophane below the analyser and turn the slide round until the strips appear equally bright; fix it in position with Plasticine.

Now introduce some of the sucrose solution through the side tube, and you will note that the strips appear alternately dark and bright; turn the analyser round to regain their uniform brightness, recording the angle turned and the depth of the liquid. Repeat by pouring more liquid into the tube and investigate the relation between the total angle of rotation and the depth.

Repeat with different sugars.

**Theory and Calculation**

For the first part verify Brewster's Law $\mu = \tan \theta$ where $\theta$ is the polarization angle.

For the second part the transmitted intensity $I = I_0 \cos^2 \theta$ where $\theta$ is the angle between the planes of polarization of the polaroids. Verify this graphically.

It is clear that \[ \frac{dI}{d\theta} = -I_0 \sin 2\theta \]

or \[ dI = -I_0 \sin 2\theta \cdot d\theta \]

which means to get a maximum change in the intensity with change in $\theta$, $2\theta = \pi/2$ or $\theta = \pi/4$ (if $\theta$ is nearly $\pi/2$, $dI$ is nearly zero for any finite change in $\theta$).

The optical rotation $\phi = s \cdot l \cdot q$ where $s$ is the specific rotary power, $l$ the depth of the liquid in decimetres and $q$ the mass of active substance in 100 ml of water. Deduce $s$ in each case.

**Comments**

There are many double-refracting substances for you to investigate, particularly cellophane (as well as Sellotape), mica and quartz. Investigate the transmitted light (white as well as monochromatic, using the red and green Wratten filters) using Sellotape or cellophane of different thicknesses between the two polaroids. Explain the interference thus produced. Use the pile of glass slides provided as a polarizer and compare its efficiency with that of the polaroid using the photoelectric cell. What is the fraction of the transmitted light when one polaroid is used? Can you find any use for that in photography?
EXPERIMENT 78

To use a diffraction grating for the measurement of the Balmer series for hydrogen

APPARATUS

Spectrometer, sodium lamp, transmission diffraction grating (about 14,000 lines/inch), Geissler hydrogen tube, induction coil and battery.

METHOD

Adjust the spectrometer so that the telescope is focused for infinity (after focusing the eye-piece on the cross-wire) and the collimator is transmitting parallel rays (see Experiment 31). Place the grating with its carrier on the turn-table so that its plane is perpendicular to the two levelling screws AB (Fig. 1). With the telescope and collimator at right angles, rotate the turn-table so that an image of the slit (suitably illuminated with sodium light) is reflected into the telescope. Adjust either screw A or B so that the image is central. Now set the grating at right angles to the collimator and turn the telescope to receive the first-order spectrum. Adjust the third screw C until the image of the slit is central. The plane of the grating should now be parallel to the axis of the spectrometer.

To set the plane of the grating so that it is perpendicular to the axis of the collimator, you must first align the telescope and collimator (without the grating), noting down the position of the telescope. Turn the telescope through 90°, replace the grating and rotate the turn-table until the image of the slit reflected into the telescope coincides with the vertical cross-wire. Note the position of the turn-table, which should now be rotated through 45° towards the collimator. Thus the collimated light should now be falling perpendicular to the grating.

Narrow the slit and note the positions of the first- and second-order spectra on either side of the central maximum. Replace the sodium lamp by the Geissler tube, with the middle, slender, part of the tube (where the light is more bright) close to the slit (you may find it necessary to widen the slit and work in subdued light). Turn the telescope to receive the first-order spectrum and you will note immediately the three brightest lines, in this order: $H_a$ (red), $H_\beta$ (greenish blue), $H_\gamma$ (violet). $H_a$ is in the extreme violet and you may find difficulty in locating it. Determine the mean positions of each of these lines on both sides of the central image.

THEORY AND CALCULATION

For light falling normally on a grating, the diffracted ray makes an angle $\theta$ with the normal, given by the relation $e \sin \theta = n\lambda$ (see Experiment 60)
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where $e$ is the element of the grating, $n$ is the order of the spectrum, and $\lambda$ is the wavelength.

Use your values for $\theta$ for the sodium light to calculate as accurately as you can $e$, taking $\lambda = 5.893 \times 10^{-5}$ cm, hence evaluate $\lambda$ for H$_{a}$, H$_{b}$, H$_{\gamma}$ and H$_{\delta}$ (if possible). Plot $1/\lambda$ against $1/n^2$ where $n$ is 3, 4, 5 and 6 respectively. Deduce from your graph the gradient and the intercept on the $1/n^2$ axis (where $1/\lambda = 0$).

Comments

Balmer had successfully produced an empirical formula to account for some of the H$_{\alpha}$ lines you have just investigated (called the Balmer series), but the theoretical explanation came later, with the advent of the quantum theory and the Bohr-Rutherford atom.

Bohr originally assumed that the single electron of the hydrogen atom revolves uniformly around the nucleus (with angular velocity $\omega$), obeying the ordinary laws of motion. It is accelerated towards the centre by $\omega^2 r$ where $r$ is the radius of the orbit; therefore, for equilibrium

$$\omega^2 r = \frac{e^2}{mr^2} \quad \ldots \ldots \ldots \ldots \ldots \ldots (1)$$

(where $e^2/r^2$ is the electrostatic attraction between the electron and the nucleus and $m$ is the mass of the electron). Bohr assumed further that following Planck’s theory, the angular momentum of the electron is a multiple of $h/2\pi$ where $h$ is Planck’s constant. Hence

$$moar^2 = \frac{nh}{2\pi} \quad \ldots \ldots \ldots \ldots \ldots \ldots (2)$$

Eliminating $\omega$ from equations (1) and (2) we get

$$r = \frac{n^2 h^2}{4\pi^2 me^2} \quad \ldots \ldots \ldots \ldots \ldots \ldots (3)$$

It can also be shown that the total energy of the electron in an orbit is $-\frac{2\pi^2 me^4}{h^2 n^2}$ where $n = 1, 2, 3$, etc., are called the quantum numbers and designated by the letters K, L, M, N, O, P. The frequency $\nu$ of the radiation emitted by the hydrogen atom is given by

$$h\nu = \frac{2\pi^2 me^4}{h^2} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right).$$

But $\nu = c/\lambda$ where $c$ is the velocity of light. Therefore

$$\frac{1}{\lambda} = \frac{2\pi^2 me^4}{h^2 c} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right), \quad n_2 > n_1.$$

From the graph of $1/\lambda$ against $1/n^2$ which you have plotted and which should be a straight line, you will probably find that the intercept $1/n_1^2$ (when $1/\lambda = 0$) is about 0.25, indicating that $n_1 = 2$ and that the Balmer series is given by

$$\frac{1}{\lambda} = \frac{2\pi^2 me^4}{h^2 c} \left(\frac{1}{4} - \frac{1}{n^2}\right). \quad \ldots \ldots \ldots \ldots \ldots \ldots (4)$$
From the gradient deduce a value for \( h \), using the values you have already obtained for \( e \) (coulombs e.s.u.) and \( m \) (g), taking \( c = 3 \times 10^{10} \) cm sec\(^{-1}\).

For M.K.S., express \( \lambda \) in metres, \( m \) in kg, \( e \) in coulombs and the right-hand side of equation (4) is divided by \( 16\pi^2\varepsilon_0^2 \) where \( \varepsilon_0 \) is space permittivity.

Use equation (3) to find an estimate for the atomic diameter of hydrogen.

**EXPERIMENT 79**

**The study of absorption spectra**

**Apparatus**

Direct vision spectroscope (or ordinary spectrometer) with small 45° prism covering one half of the slit to enable one to compare two spectra simultaneously. Helium and hydrogen discharge tubes, sodium and potassium salts, platinum wire and holder, Bunsen burner. 12 V 60 W tungsten filament lamp and holder, 12 V auto-transformer, A.C. voltmeter (0–12 V) and ammeter (0–5 A), long Bunsen burner, several sodium chloride pencils or long block of common salt. P.O. box and accessories.

![Spectroscope diagram](image)

**Method**

Look at the sky or at a cloud with the spectroscope and let your partner test some salts in the Bunsen flame, set at right angles to the axis of the spectroscope so that the light is reflected at the small 45° prism into the instrument. You will be able to identify some of the dark lines which cross the solar spectrum with emission lines of light elements like hydrogen, helium, potassium, sodium and others. You should be able to recognize the Balmer series of hydrogen (see Experiment 78).

Set up the apparatus shown in Figs. 2(a) and 2(b). To get good
absorption, the light should be filtered through an appreciable depth of the sodium flame. Place the auto-transformer close to hand and let your partner look after the instruments and the flame. With the lamp fully lit, a dark line appears at the position corresponding to the emission D line of sodium. As you dim the light a point is reached when the light merges into the background spectrum. Ask your partner to record the voltmeter and ammeter readings. This should be repeated starting with the light dimmed and the yellow line bright against a dark background spectrum. Unfortunately the sodium flame is never steady, so that several checks are necessary. Find the resistance of the bulb at room temperature using the P.O. box.

Theory and Calculation

The explanation of the dark lines in the solar spectrum (first studied in detail by Fraunhofer and called after him) is absorption, and in fact the Fraunhofer lines constitute absorption spectra of the cooler vapours or gases found in the solar atmosphere.

The purpose of the second experiment is to explain the phenomenon of absorption, the cooler sodium vapour absorbing from the white light the
quantum which will lift electrons from the normal state to higher energy levels so that absorption lines corresponding to the principal series of sodium are observed. On the average, an atom remains in the excited state only for a very short period \((10^{-3} \text{ sec})\) so that there is not enough of them at any one time to absorb more energy for higher levels and the absorptions of secondary series of sodium are not observed.

The energy first absorbed is radiated back in all directions so that in the direction of the white light the line appears dark by contrast; when the temperature of the flame approaches that of the filament, the re-emitted energy equals that emitted from the flame, provided both the filament and the flame have equal emissivities. From the ratio of the resistance of the hot and the cold filament and using the graph (Fig. 2 of Experiment 51), estimate the temperature of the flame.

The Balmer series (see Experiment 78) corresponds to transitions between first excited states \((n = 2)\) and higher energy levels. How does one explain its occurrence in view of the statement above? The reason must be that the very high temperature of the solar system allows a few excited hydrogen atoms to exist.

**Comments**

The above experiment is intended more as a demonstration of absorption spectroscopy than anything else. This phenomenon when applied to the infra-red region yields a wealth of information about molecular structure, because transitions between energy levels associated with vibrations and rotations of molecules correspond to emissions in the infra-red region \((\lambda > 1 \mu \text{ (10}^{-3} \text{ mm)}, \text{ the limit of visibility for the human eye). The general principle of infra-red spectroscopic analysis is to allow a continuous spectrum to pass through a sample under test (which can be a gas, liquid or solid) and to detect both the absorption lines or bands and their degree of absorption using infra-red detectors, the results being recorded automatically. This technique, helped by advances in quantum mechanics, has become indispensable to the experimental chemist.

**EXPERIMENT 80**

To use the Fresnel biprism to determine accurately the wavelength of sodium light

**Apparatus**

Fresnel biprism; optical bench consisting of an adjustable slit, biprism holder with both lateral and vertical adjustments, micrometer eye-piece, short-focus lens \((f = 7.5 \text{ cm})\). Sodium lamp and test-tube, spectrometer, glass block and Sellotape, glass slide, travelling microscope.
Method

Place the prism in its holder less than 15 cm in front of the illuminated but fairly wide slit and look through the prism without using the eye-piece. Adjust the tilt and the lateral position of the prism until you see two parallel images of the slit when you look along the axis. Put in the eye-piece, narrow the slit and make final fine adjustments until you see sharp bright fringes (Fig. 1).

Place the short-focus lens as near to the prism as possible and move it slowly towards the eye-piece until a sharp and magnified image is formed of two slits which are the two virtual images of the single one seen through the prism previously. If no such image is formed, the slit must be moved nearer to the prism and the whole adjustment repeated. Measure $s_1$ the separation between the two slits and the position of the lens on the optic bench. Now move the lens nearer to the eye-piece and find another position giving sharp but diminished images of the virtual slits. Determine the new position of the lens and hence the distance between the two positions $d$, also the separation $s_2$ between the slits in the new image.

Without disturbing the prism, eye-piece and slit remove the lens and measure the average separation $x$ between consecutive bright fringes. Measure the distance $D$ between the slit and the plane of the reticule of the eye-piece, and the distance $l$ between slit and the centre of the prism; also determine as accurately as possible the focal length of the lens $f$. Repeat the experiment at least once with different distances.

Adjust the telescope and collimator of the spectrometer as described in Experiment 31 and measure the small acute angle $\alpha$ between the two inclined faces of the prism; also find the angle of deviation $\delta$ produced by each prism.

Now place the prism with its flat side up and parallel to the glass block by raising one edge of the prism with a strip or two of Sellotape (Fig. 2). Use the arrangement in Fig. 3 for viewing the two sets of parallel fringes produced between each prism and the surface of the glass block. Measure the average separations $x_1$ and $x_2$ between consecutive bright fringes for
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each set of fringes, using the travelling microscope. Repeat as a check, using different settings. Use the travelling microscope to find the apparent

![Diagram of a mounting semi-silvered mirror and a lens](image)

and real thicknesses of the two prisms for measured distances from the central ridge on each side of the ridge.

**Theory and Calculation**

Using the notation discussed in Experiment 59

\[ x = \frac{D\lambda}{s} \]  

(1)

where \( s \) is the separation between the virtual images of the slit, \( D \) the total distance between the slit and the eye-piece, \( \lambda \) the wavelength of the sodium light. One can deduce \( s \) from the measured distances \( s_1 \) and \( s_2 \) by the relation \( s = \sqrt{s_1 s_2} \), also \( D \) is known from direct measurement and from the two positions of the lens. If \( d \) is the distance between the two positions of the lens, then \( D = f + \sqrt{4f^2 + d^2} \) from which \( \lambda \) may be found.

Also \( s \) can be calculated from the knowledge of the angular separation of the two virtual slits at the centre of the prism and the distance of the slit from the centre of the prism. The spectrometer data give \( 2\delta \) for the angular separation, so that \( s = 2\delta \). \( \delta \) can also be checked from the spectrometer data using the relation \( \delta = (\mu - 1)a \), treating the Fresnel biprism as two thin prisms. \( \mu \) can be determined from the real and apparent depth measurements and the profile measurements. It can be shown that the separation between consecutive parallel fringes of an air wedge is \( \lambda/2\theta \) where \( \theta \) is the angle of the wedge, or

\[ a = \frac{2}{4} \left( \frac{1}{x_1} + \frac{1}{x_2} \right) \]  

(2)

Use the value of \( a \) calculated above and deduce \( \lambda \) from equation (2).

**Comments**

Two examples of interference are demonstrated in the above experiment: in the Fresnel biprism the wave trains from the slit are divided up into two wave fronts by the two prisms and then made to overlap at the eye-piece; on the other hand, in the air wedge the amplitude of the incident wave is resolved into two components travelling in opposite directions.
which are subsequently recombined. Apart from this the different methods suggested for determining some of the quantities illustrate the way to avoid and detect systematic errors.

EXPERIMENT 81
To find the resolving power of a telescope

Apparatus
Discarded light-tight envelope (supplied with photographic paper), clean pin, Amateur Photographer 'lens testing chart', sodium lamp, adjustable iris, two telescopes of different diameter objectives, long measuring tape, sheet of tracing paper (or diffusing screen), travelling microscope, light boxes and clamps.

Introduction
You have observed in Experiment 60 that the diffraction pattern changes when a distant narrow slit is viewed through a slit of variable width. If the near slit is sufficiently narrow, you would observe a central bright fringe flanked by a series of parallel fringes which get further spaced out as the slit is narrowed down.

When looking at a distant point object through a telescope, the edge of the objective acts like a circular diaphragm producing a diffraction pattern.

Method
Cut a convenient strip of the light-tight envelope (usually red on the inside and black on the outside); on the red side draw two fine inclined lines 4 cm long, intersecting at one end and only 2 mm apart at the other end. Now place the strip on some paper and make a series of pairs of fine circular pin-holes along the lines as shown in Fig. 1 (clean holes of equal diameter 0.3 mm can be made with a red hot pin or needle), and at the intersection of the two lines make a single hole. Clamp the perforated strip of paper in front of the sodium lamp with the diffusing screen in between and view it with one of the telescopes at a distance of 9–10 m. Place the iris in front of the objective of the telescope (see Fig. 2). Note the improved definition as the iris is stopped down (why?). Look at the single

Fig. 1

Fig. 2
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hole as the iris is stopped down further and notice how the small circular patch appears to spread and to be surrounded by circular fringes.

You should have noticed how each pair of holes seems to merge together as the iris is progressively stopped down; find the mean diameter of the iris at which the telescope just fails to resolve each pair of holes in turn. Measure the distance between the perforated paper and the objective and the distances between the holes. Repeat with the other telescope.

Replace the lamp and the paper by the testing chart, uniformly well illuminated (the chart consists of a series of black and white lines vertical, horizontal and diagonal becoming progressively finer, 48 lines per inch being the finest). Find the nearest number of lines per inch which each telescope resolves. Find the nearest distance at which you can resolve 48 lines per inch with your unaided eye—do you find this affected when the illumination is reduced? How do you account for this?

Theory and Calculation Comments

Rayleigh's criterion for resolution is that two point sources are just resolved if the central maximum of one coincides with the first minimum of the other. For circular apertures the smallest angle resolved is $1.22\lambda/D$ radians (where $\lambda$ is the wavelength of the light and $D$ cm is the diameter of the diaphragm). Verify this.

How does the angular resolution thus found compare with the angular separation of the finest lines resolved; can you account for any difference? How do the resolutions of the two telescopes compare?

What is your visual acuity? (The angular separation of two points which the eye can just resolve.)

Comments

When the iris is fairly wide, the fringes surrounding the image point are too close to be resolved by the eye. Estimate the angular separation between the centre of the image and the first bright fringe when one of the telescopes is used at full aperture.

If the angular resolution of a diffraction grating is $1/Nn$ radians, where $N$ is the total number of lines effectively illuminated and $n$ the order of the spectra, how would you propose to use a variable slit and grating spectrometer to verify this relationship, using a sodium lamp as a source of light?

Experiment 82

To find the specific heat of air at constant pressure

Apparatus

This consists of three concentric copper tubes assembled as shown in Fig. 1, the inner two having fins to ensure thorough mixing of the flowing
stream of air. The heating element is from a mains soldering iron, of about 25 W. A.C. voltmeter and ammeter, two mains lamps in parallel, with a switch. Simple motor driven suction pump.

*Warning.* As mains voltage is used, care should be taken not to have any bare terminals. The ammeter, voltmeter and lamps should all be plugged into well marked positions.

![Diagram](image)

*Fig. 1*

**Method**

Assemble the apparatus as shown, plug in and start the pump going straight away, as it takes the apparatus nearly an hour to reach steady conditions. Switch on both lamps in parallel so that the heating coil will be receiving maximum power. The position of the top of the float in the rotameter should be read as well as measured and the inlet and outlet temperatures; record also the pressure on the gauge, which should be small or otherwise there is obstruction inside the apparatus. Record the voltmeter and ammeter readings.

The rate of flow is now reduced by screwing in the clip, switching off one of the lamps at the same time so that the power supplied to the heating element is almost halved. By careful adjustments of the screw the temperature rise can be kept very nearly the same. Record all the observations as before.
THEORY AND CALCULATION

The manufacturers provide a graph giving the rate of flow in litres/min, $V_a$ corresponding to the float reading of the rotameter at $15^\circ$C and 760 mm of mercury. The true flow is then

$$V = V_a \sqrt{\frac{\text{density at 15}^\circ\text{C}}{\text{density at the outlet temperature}}}.$$  

(Why?) Taking the density of air at S.T.P. = 1.29 g/litre, deduce the mass of air $m$ g flowing per second in each case.

If $V_1$, $I_1$, $\theta_2^\circ$C and $\theta_1^\circ$C are the voltage, current, outlet and inlet temperatures respectively then

$$V_1I_1 = m_1C_p(\theta_2 - \theta_1) + k \quad . \quad . \quad . \quad (1)$$

where $C_p$ is the specific heat at constant pressure joule $g^{-1}$ deg. $\text{C}^{-1}$ and $k$ is the heat loss per second to the outside.

As the experiment is repeated with much the same temperatures, $k$ will be the same; we have therefore

$$V_2I_2 = m_2C_p(\theta_2' - \theta_1') + k \quad . \quad . \quad . \quad (2)$$

and subtracting equation (2) from equation (1) deduce $C_p$.

COMMENTS

This apparatus is capable of refinements so that the electric power and temperatures can be measured more accurately. A correction is necessary for the fact that the inlet pressure is slightly higher than the outlet pressure, which means that the air must have expanded slightly more than if the pressure is kept the same. If $v_1$ ml is the volume of 1 g of air at atmospheric pressure $\pi$ dynecm$^{-2}$ and $\theta_2^\circ$C and $v_2$ at the pressure $\pi - \delta \rho$, then we can assume that 1 g of gas has expanded by a further $v_2 - v_1$ ml at an average pressure $\pi - \frac{\delta \rho}{2}$, so that the extra work done by the gas is

$$\left(\pi - \frac{\delta \rho}{2}\right)(v_2 - v_1) \text{ ergs, which means extra heat had to be supplied,}$$

$$\frac{1}{10^3}\left(\pi - \frac{\delta \rho}{2}\right)(v_2 - v_1) \text{ joules. When gases are used in cooling as in air-}$$

cooled car engines or in heat transfer by carbon dioxide in nuclear power stations, the specific heat required is $C_p$, but as gases are not as dense as liquids, the cooling gases have to be circulated at a high speed or high pressure or both (as in nuclear power stations), hence the massive pressure vessels required.

The rotameter consists of a tube tapering slightly downwards, so that a constriction is formed between the float and the tube and the difference in pressure thus established provides the required force to balance the weight of the float.
EXPERIMENT 83

To determine accurately the specific heat of a liquid (Ferguson's method)

Apparatus

A calorimeter placed inside a water-jacketed outer calorimeter and insulated from it. The inner calorimeter has a close fitting lid with an efficient stirrer, a sensitive 50°C thermometer and a heating coil of eureka (approximately 3 ohm); water from a tap circulates in the outer jacket and maintains it at constant temperature. 8 V D.C. supply, standard resistor (1 ohm) and Weston cadmium cell. Potentiometer, accumulator, tapper,
sensitive centre zero galvanometer and high resistor of 1000 ohm. Stopwatch, various liquids: distilled water, glycerine (but not electrolytes). Throw-over switch, rheostat, P.O. box and switches.

Method

Find, using the P.O. box, the resistance $R$ of the coil to three significant figures. Weigh the calorimeter empty and when half filled with distilled water and deduce the mass of water used. It will save time if you heat the water in the calorimeter to about 35°C before assembling the apparatus as shown in the figure. Switch on the current and adjust it so that the temperature of this water is maintained at a steady value ($\pm 0.1$ deg C), about 15 deg C higher than the temperature of the water in the jacket (it helps matters if an ammeter is included in the circuit as an indicator).

Calibrate the potentiometer using the standard cell and then balance the p.d. across the 1 ohm standard resistor against the potentiometer wire. This should be done frequently while the temperature is steadying, as a check that the current is constant. The stirring should be done regularly and effectively. Measure the temperature of the circulating water. Now increase the current for a short time, thus raising the temperature a few more degrees, switch off the current, stir the water until the temperature begins to fall and start taking a cooling curve at one minute intervals for about 25 minutes. Check again on the temperature of the circulating water.

The experiment is now repeated using other liquids in the calorimeter.

Theory and Calculation

An empirical law of cooling can be found in the form

$$\frac{d\theta}{dt} = -K\theta^n$$

where $\theta$ is the excess temperature. Ferguson and others found that $n = 5/4$. Integrating equation (1) $\theta^{-1/4} - \theta_0^{-1/4} = \frac{1}{4}Kt$, $\theta_0$ being the initial temperature. Plotting $\theta^{-1/4}$ against $t$ you should get a straight line, which is a proof of the correctness of the cooling law. Determine $K$ accurately from the gradient of the slope (if $t$ is plotted in minutes, the slope is $K/240$).

If $\theta_1$ is the excess steady temperature when a current $I$ A is passed through the circuit, then $-(d\theta/dt)(Ms + w) = I^2R$ where $M$ is the mass of the water, $s$ the specific heat in joule g$^{-1}$ deg C$^{-1}$, $w$ the thermal capacity of calorimeter, coil and thermometer in joule deg C$^{-1}$.

But from (1)

$$\frac{d\theta}{dt} = -K\theta_1^{4/5},$$

hence

$$K\theta_1^{4/5}(Ms + w) = I^2R.$$ (The current $I$ is found from the balance-length of the potentiometer wire against the p.d. of $R$ ($l_1$), and when a Weston cadmium cell is used, $l_2$. Thus $R/I = 0.0183 = l_1/l_2$. Hence $I$.)

Devise a method for estimating $w$, which is usually small, say 2% of $Ms$, so it need not be known to more than one or two significant figures. Hence calculate $s$.  

COMMENTS

The above method is accurate enough for all practical purposes (about 2%), but unfortunately not suitable for electrolytes unless the heating coil is electrically insulated from the liquid. An interesting investigation is the variation of the specific heat of brine with concentration.

EXPERIMENT 84

To determine the thermal conductivity of air at different pressures

APPARATUS

An air-tight copper tube about 50 cm long and 1 cm internal diameter along the axis of which is stretched a tungsten wire (about 26 S.W.G.), tensioned at each end (see Fig. 1). The tube is connected, by a side-arm, to a rotary high vacuum pump, preferably in series with a vacuum gauge. (Provisions are made for electrical contacts at both ends of the tungsten
wire.) Potentiometer, 0.1 ohm standard resistor (4 terminals), two rheostats, ammeter (0-5 A), throw-over switch, sensitive centre zero galvanometer, 6 V. D.C. supply, one 2·0 V accumulator, switches. 0–100°C thermometer, steam jacket made in two halves to fit around the copper tube.

**Method**

Connect up the circuit as shown in Fig. 1 and compare the resistance of the tungsten wire at room temperature with that of the 0·1 ohm standard resistor, using the potentiometer. (It is important that the current passed through the wire is small—less than 0·1 A.) Record the temperature by means of the thermometer inserted into the side arm (which is not yet connected to pump). Enclose the copper tube in the steam jacket, so that

![Fig. 2](image)

the side arm protrudes (Fig. 2). Small losses of steam here are of no consequence. Pass steam from a generator for a while until the temperature is steady, as recorded by the thermometer inserted into the side arm. Again compare resistances using the same current through the wire as before and record the corresponding temperature.

Now connect the side arm to the vacuum pump. Pass a suitably large current, say 1 A; compare resistances as before and record the value of the current. This is repeated with increasing values of current up to 3 A. Reduce the pressure of the air inside the tube and repeat the whole experiment. When the pressure of the air is less than 0·1 mm of mercury, it will
be found that the resistance of the wire rises rapidly, even with small currents. Adjust the current so that the resistance corresponds roughly to the values obtained previously with higher air pressures. Measure the pressure of the air or else estimate it by replacing the copper tube by a discharge tube and noting the state of the discharge.

Theory and Calculation

The heat conducted per second through a gas from hot wire $l$ cm long and radius $a$ cm, placed coaxially in a cylinder of radius $r$ cm,

$$Q = \frac{2\pi k\theta}{\ln r/a} = \frac{2\pi k\theta}{2.303 \log r/a} \text{ joule sec}^{-1},$$

where $k$ is the thermal conductivity in joule sec$^{-1}$ cm$^{-1}$ deg C$^{-1}$ and $\theta$ is the temperature difference in deg C.

The electrical power supplied to the wire is $I^2R$ where $I$ A is the current through the wire and $R$ ohm is resistance, but not all this power is lost by conduction through the gas. Some is lost by convection, some by radiation and some by heat conduction through the contacts. As the temperature of the wire is never very high (<150°C) the effect of radiation is small, the effect of convection is also small, particularly when the air pressure is about 10 cm of mercury. The temperature difference can be calculated from the increase in resistance $R$ over that at room temperature, using the steam point calibration and assuming that the resistance increases uniformly with temperature. A useful check on the steam point calibration is possible if a 50 cm specimen of the tungsten wire is available. Measure its resistance by the potentiometer method and calculate the increase in resistance corresponding to the observed temperature rise, using the accepted value for the temperature coefficient of resistance of tungsten. You will note that the resistance of the wire in the tube at room temperature is not the same as that of the specimen wire, though both are of equal length. Comment on this.

If the current needed to maintain the temperature and hence the resistance of the wire at $R$ when the tube is almost completely evacuated is $I_0$, then $I_0^2R$ is the heat lost by conduction at the contacts, radiation and the feeble heat conduction at the very reduced pressure. Then

$$Q = I^2R - I_0^2R = \frac{2\pi k\theta}{2.303 \log r/a}$$

Plot $Q$ against $\theta$ and deduce $k$ correct to one significant figure.

The above experiment should have convinced you that the heat conducted through the gas is almost independent of pressure until the latter is reduced to a fraction of a millimetre of mercury, when a pronounced and significant drop in heat conduction occurs—why?

Heat conduction like other transport phenomena in a gas (i.e. diffusion, viscosity, etc.) depends on the transport of heat energy by the molecules themselves. The number of molecules/ml is directly proportional to the pressure, on the other hand the heat power transported also depends on the ‘clear run’ of the molecules (i.e. the mean free path $\lambda$) before colliding with other molecules and ‘spilling’ the transported energy. As $\lambda$ is inversely
proportional to the pressure, \( k \) remains constant. When the pressure is low enough so that \( \lambda \approx r \), \( k \) will depend on the number of molecules/ml, which means that \( k \) is proportional to the pressure. This fact enables one to measure very low pressures with a Pirani gauge.

Show that \( k = \frac{1}{3} C_v \bar{C} \rho \lambda \) where \( C_v \) is the specific heat of air in joule g\(^{-1}\) deg C\(^{-1}\) at constant volume, \( \rho \) the density of air and \( \bar{C} \) the root mean square velocity of the air molecules. Deduce and estimate the pressure of the air when \( \lambda = r \).

The above experiment, next to the Brownian movement, provides the most convincing evidence for the kinetic theory of gases.

**EXPERIMENT 85**

**To determine the coefficient of thermal conductivity of a long copper rod (Forbes bar)**

**Apparatus**

A copper rod, one metre long and about 12 mm in diameter, about 7 cm of which at one end is bent at right angles to the rest of the rod and about 5 cm of which at the other end is heavily shellacked and wound with a suitable heating coil of about 4\(\frac{1}{2}\) ohm. The surface of the coil is shellacked and wrapped round with a suitable insulating material of adequate thickness to ensure negligible heat loss through it. A close-fitting loop of eureka of, say, 30 S.W.G. is wrapped round the rod for measuring the temperature, and a similar wire is soldered to the tip of the bent and exposed rod. Also required: 50 ohm resistor, Pye Scalamp, galvanometer, rheostat, 12 V D.C. supply electric water heater, large copper calorimeter and ring clamp to support it, (0–12 V) voltmeter and (0–2 A) ammeter, switch, 50° and 100°C thermometers, two tripods and two small asbestos sheets.

**Method**

The eureka loop, the copper rod and the second eureka wire soldered to the tip of the rod form a thermocouple, and it is necessary first to calibrate it. This is done by using circuit 1, shown in Fig. 1. Heat the water in
the calorimeter and make sure that the eureka loop is slid as near as possible to the heating coil end of the rod. Choose a suitable sensitivity so that the galvanometer deflection is just over half the full scale. When conditions are steady take the thermometer readings and galvanometer readings. For the main experiment circuit 2 is used as shown in Fig. 2, remembering to interchange the connections to the galvanometer. Make 15 faint scratches on the rod to mark the points at which the temperatures are read.

![Fig. 2](image)

The points should be closer together near to the heating coil and farther spaced out towards the other end of the rod, the distances (x) from all these points to the end of the heating coil A being measured. Support the rod horizontally by resting the heater end on a small asbestos sheet placed on top of a tripod, and the other end (removed from the water) on a similar sheet resting on the other tripod.

Now connect the low tension supply, adjust the rheostat, giving about 6 V across the heating coil, and allow conditions to become steady (which may take nearly an hour). Then record the galvanometer reading, with the loop still as near to the heating coil as possible, giving a maximum reading on the galvanometer. Take similar readings as the loop is moved along to each mark on the rod until the cold end of the rod is reached. Re-check the observations (in reverse order) as the loop is moved along from the cold end back to A. The two sets of observations should agree very closely; otherwise steady conditions have not been reached. Repeat the experiment using a higher voltage for the heating coil, noting the power W supplied in each case.

**Theory and Calculation**

Consider an element of the rod $dx$, cross-sectional area $A$, distance $x$.

![Fig. 3](image)
from the hot end, and let \( \theta \) be the temperature in excess of the room temperature (see Fig. 3).

Heat current passing through \( P = -KA \frac{d\theta}{dx} \) where \( K \) is the thermal conductivity of copper.

Heat current through \( Q = -KA \frac{d}{dx}(\theta + d\theta) \) joule sec\(^{-1}\)

Difference in heat currents \( = -KA \frac{d\theta}{dx} - KA \frac{d^2\theta}{dx^2} \cdot dx + KA \frac{d\theta}{dx} \)

\( = -KA \frac{d^2\theta}{dx^2} \cdot dx \)

\( = \) heat lost to the outside by the element

\( dx = -2\pi r \theta \beta dx \)

(assuming Newton's law of cooling) where \( \beta \) is the emissivity of the copper surface in joule sec\(^{-1}\) cm\(^{-2}\) deg C\(^{-1}\) excess and \( r \) the radius of the rod.

Integrating after multiplying both sides by \( d\theta/dx \),

\[ \frac{d^2\theta}{dx^2} \cdot \frac{d\theta}{dx} = \frac{2\pi r \beta \theta}{KA} \cdot \frac{d\theta}{dx} \]

\[ \left( \frac{d\theta}{dx} \right)^2 = a \theta^2 \text{ where } a = \frac{2\pi r \beta}{KA} \]

therefore

\[ \frac{d\theta}{dx} = -\sqrt{a} \theta \] \hspace{1cm} (1)

The negative sign is chosen, as the heat flows along the downward gradient of the potential of the temperature.

Integrating \( \ln \theta = -\sqrt{a}x + \text{constant} \). If \( x = 0 \), \( \theta \) equals \( \theta_{\text{max}} \)

\[ \ln \theta = -\sqrt{a}x + \ln \theta_{\text{max}} \text{ or } \theta = \theta_{\text{max}} e^{-\sqrt{a}x} \] \hspace{1cm} (2)

Now the power supplied in joule sec\(^{-1}\) = \( W \) = heat current flowing at point A (see Fig. 3) = \( -KA(d\theta/dx) \), but \( d\theta/dx = -\sqrt{a} \theta_{\text{max}} \), therefore

\[ W = KA \sqrt{a} \theta_{\text{max}} \] \hspace{1cm} (3)

Plot \( \ln \theta \) against \( x \) and you should get a straight line, the gradient of which is \( -\sqrt{a} \). \( \theta_{\text{max}} \) is deduced from the calibration part of the experiment and from the value of the maximum galvanometer reading. You may assume that the thermoelectric e.m.f. is proportional to the temperature difference between the two junctions. Hence calculate \( K \) from equation (3) (in practice you can plot log \( \theta \), or better still the logarithm, to the base ten, of the galvanometer readings, but the gradient is now \( \log e \sqrt{a} \) or 0.4343 \( \sqrt{a} \)).

Comments

The importance of this experiment is that it gives an exercise in heat transfer virtually in one dimension and shows the kind of mathematics required to meet such a problem. Discuss critically the sources of error.
For example, how accurately can one determine the temperature of the hot end of the rod? How does this method compare with that given in Experiment 37?

**EXPERIMENT 86**

To compare accurately various resistors of nominal value 1 ohm each, using the Carey-Foster bridge, to find the resistance per centimetre of a metre bridge and to determine the internal resistance of an accumulator

**Apparatus**

Two accumulators, a third accumulator of unknown internal resistance, 60 ohm rheostat, 50 ohm fixed resistor, four 1 ohm resistors (one of which is a standard), 5 ohm resistor, 0.5 ohm standard resistor, manganin metre bridge, tapper, centre zero galvanometer with shunt (to render it less sensitive).

**Method**

Connect up the circuit shown in Fig. 1, let $R$ be the standard resistor of 1.00 ohm, and $P$, $Q$ and $S$ the other nominal 1 ohm resistors; find the balance length $l_1$. Interchange $S$ and $R$ and repeat; let the new balance length be $l_2$, as measured from the same end of the wire. Put $R + 0.5$ ohm in series and repeat the whole experiment. Interchange $S$ with each of the two nominal resistors in turn and repeat the observations.

To find the internal resistance of the accumulator, use the circuit shown in Fig. 2 with $X$ being 1 ohm. Adjust the rheostat so that the balance point (with the galvanometer shunted) is close to A when the key is closed. Now find the balance lengths $m$ and $n$ accurately when the key is open and closed respectively. Increase $X$ and repeat up to $X = 9$ ohm.
**SCHOLARSHIP EXPERIMENTS**

Fig. 2

**Theory and Calculation**

If \( \rho \) is the resistance per cm of the manganin wire, then for the first part of the experiment

\[
\frac{P}{Q} = \frac{1.00 + l_1\rho}{S + (L - l_1)\rho}
\]

Interchanging \( S \) and \( R \) we get

\[
\frac{P}{Q} = \frac{S + l_2\rho}{1.00 + (L - l_2)\rho}
\]

\[
\frac{S + (L - l_1)\rho}{1.00 + (L - l_2)\rho} = \frac{1.00 + l_1\rho}{S + l_2\rho} = \frac{S + 1.00 + L\rho}{S + 1.00 + L\rho} = 1
\]

\[
\Rightarrow S = 1.00 + (l_1 - l_2)\rho
\]

When 0.5 ohm is added to \( R \) (1.00 ohm) in series, we get

\[
S = 1.50 + (l_1' - l_2')\rho
\]

Solve equations (1) and (2) and find \( \rho \) and \( S \). Do the same for the other two resistors and obtain further checks for \( \rho \).

For the last part of the experiment, if the internal resistance is \( r \), then

\[
\frac{50 + \rho m}{50 + \rho n} = \frac{r + X}{X} = 1 + \frac{r}{X}
\]

hence

\[
\frac{\rho (m - n)}{50 + \rho n} = \frac{r}{X}
\]

Plot a suitable graph from which \( r \) can be deduced as a gradient.

**Comments**

The Carey-Foster bridge measures the difference between two comparable resistors accurately, so that even if \( \rho \) is found to 1% only, the accuracy...
of $S$ becomes very high, provided $R$ is known accurately. The ratio of $S/R$ would be known very accurately even if $R$ is not known to more than 1%. Use the above bridge to investigate the end correction of the bridge and the uniformity of the resistance wire.

Why is manganin wire used for the metre bridge?

**EXPERIMENT 87**

To measure resistance and potential without reference to any electrical standards (absolute measurements)

**Resistance**

**Apparatus**

Unknown resistor $X$ (value about 0.25 ohm); rectangular coil consisting of 1000 turns, measuring $4\frac{1}{2}$ in $\times$ 3 in, mounted horizontally with a large pulley and commutator on one common shaft. Large Helmholtz coils (see Experiment 44), variable speed electric motor with a driving pulley, 12 V D.C. supply, sensitive centre zero galvanometer, a loop of string, stopwatch, reversing switch, ammeter (not essential but useful as an indicator), rheostat.

**Method**

The apparatus is connected as shown in Fig. 1 (a current of about 1 A is found suitable). First ascertain the number of revolutions of the rotating coil corresponding to one complete revolution of the driving loop of string (there should be a small knot on the loop). Now switch on the motor and the Helmholtz coils circuit, find the polarity of the induced e.m.f. and connect the leads from the brushes on the commutator correctly across the unknown resistor (common positive or negative as in a potentiometer). Adjust the speed until the centre zero galvanometer shows no deflection and simultaneously find the time it takes the loop to make 20 revolutions (two people are required for this experiment). Reverse the current and repeat the experiment.

Repeat the experiment with a higher value of current.

**Theory and Calculation**

This is a modification of the Lorenz method and the principle is to balance the e.m.f. induced in the rotating coil against the p.d. across an unknown resistor.

If $n_1$ is the mean number of revolutions of the loop per second (for both directions of current) and if there are $n_2$ revolutions of the coil to one revolution of the loop, then the number of revolutions per second of the coil is $n_1n_2$. 
C.G.S.

If the magnetic field at the mid-point of the Helmholtz coils is \( H \) oersted, \( H = kI \) where \( k \) is a constant dependent on the dimensions of the coil and \( I \) is the current in amps. \( k \) can be shown to be equal to \( \frac{16\pi N}{25\sqrt{5}r} \) where \( N \) is the number of turns in each coil and \( r \) is the radius of the coil in cm. If the number of turns in the rotating coil is \( v \) and the mean area of each coil is \( A \) cm\(^2\), then the induced e.m.f. is

\[
\frac{4n_1n_2AvkI}{10^8} \text{ V}^* 
\]

and when balanced against the p.d. across the unknown resistor \( X \), we get

\[
XI = \frac{4n_1n_2AvkI}{10^8} \text{ ohms} \quad \text{or} \quad X = \frac{4n_1n_2Avk}{10^8} \text{ ohms} 
\]

M.K.S.

The magnetizing force at the mid-point of the Helmholtz coils is \( KI \text{ A m}^{-1} \) where \( K = \frac{8N}{5\sqrt{5}r} \), \( N \) being the number of turns in each coil, \( r \) the radius in metres. The flux density \( B = \mu_0H \), thus the induced e.m.f. in the rotating coil = \( 4n_1n_2Ap\mu_0H^* \) where \( A \text{ m}^2 \) is the mean area of the coil, and \( v \) the number of turns.

When this is balanced against the p.d. across the unknown resistor \( X \), we get

\[
XI = 4n_1n_2Ap\mu_0KI \quad \text{or} \quad X = 4n_1n_2Ap\mu_0K \text{ ohms} 
\]

\* Mean e.m.f. = \( \frac{2E_{\text{max}}}{\pi} \)

angular velocity of the coil = \( 2\pi n_1n_2 \text{ rad sec}^{-1} \)

**Potential**

**A p p a r a t u s**

Simple attracted disc electrometer (details shown in Fig. 2), lamp and scale. E.H.T. supply (3000 V or higher).

Before assembling the apparatus, find the centre of gravity of the attracted disc and its extension and measure its distance \( h \) from the points of pivot; also find the mass \( M \) of the suspended system. Assemble the apparatus as shown in Fig. 2. Record the zero position of the spot on the scale.

Connect the E.H.T. leads to both the suspended system and the guard ring; place the earthed plate about 6·0 cm from the attracted disc and switch on the E.H.T. supply. Record the deflected position of the spot. Now switch off the E.H.T. supply, measure the distance \( t \) between the earthed plate and the disc in its deflected position, using the screw provided through the centre of the earthed plate. Find also the distance \( D \) between the galvanometer mirror and the scale, and the distance \( l \) between the centre of the disc and the points of suspension. Repeat this experiment with different values of \( t \) and potential \( V \).
THEORY AND CALCULATION

If the deflection of the spot on the scale is \( d \), then the angle turned by the disc is \( d/2D \) radians and the restoring couple is \( Mgh \cdot (d/2D) \) approximately. If \( V \) is the potential of the disc in volts:

\[
\begin{align*}
\text{C.G.S.} & \\
\text{the potential in e.s.u. is } V/300 & \text{ and the force on the disc is} \\
& \frac{V^2A}{8(300)^2\pi t^2} \\
\text{where } A \text{ cm}^2 & = \text{area of disc}, \ t \text{ cm} \\
& = \text{separation between the plates.} \\
\text{This force acts at the centre of the disc which is } l \text{ cm below the points of} \\
& \text{suspension. Then for equilibrium} \\
Mgh \cdot \frac{d}{2D} & = \frac{V^2Al}{8 \times (300)^2\pi t^2} \\
\text{Hence } V.
\end{align*}
\]

\[
\begin{align*}
\text{M.K.S.} & \\
\text{the force of attraction on the disc} & \frac{1}{2} \cdot \frac{\varepsilon_0 V^2A}{t^2} \\
\text{where } \varepsilon_0 & = \text{the space permittivity (F m}^{-1}), \ A \text{ m}^2 = \text{area of disc}, \ t \text{ m} = \\
& \text{separation between the plates. This force acts at the centre of the disc which is } l \text{ m below the points of} \\
& \text{suspension. Then for equilibrium} \\
Mgh \cdot \frac{d}{2D} & = \frac{\varepsilon_0 V^2Al}{2t^2} \\
\text{M in kg, } h \text{ in m and } g & = 9.81 \text{ m sec}^{-2}. \text{ Hence } V.
\end{align*}
\]

COMMENTS

1. What is the purpose of reversing the current and averaging \( n \)?
2. What is the purpose of the guard ring? Repeat the experiment without it and comment on your results.

EXPERIMENT 88

To use A.C. bridges to compare (a) capacitances (DeSauty's), (b) self-inductance with capacitance (Owen's), and (c) mutual inductance with capacitance and resistance (Heydweiller's)

APPARATUS

Two 100 ohm decade dial resistance boxes, 2000 ohm variable (plug type) resistance box, 6 \( \mu \)F and 1 \( \mu \)F condensers, \( \cdot 05 \) \( \mu \)F condenser (1500 V working with high leakage resistance), crystal ear-piece of high impedance. Two identical circular coils, 4 in diameter, each 500 turns, of low resistance, which can be mounted coaxially as a mutual inductance. 1000 c/s A.C. supply.

INTRODUCTION

The Wheatstone bridge can be used with A.C. voltage and each arm can then be replaced by an impedance \( Z \). As \( Z \) consists of a real and an imaginary component, a balance point will satisfy two independent equations.
That is why two variable components (in this experiment variable resistors are used) are needed to balance the bridge. Each practical air-cored inductance will be treated as an ideal inductance in series with a pure resistor and each condenser (except in Owen's bridge) as an ideal capacitor in series with a pure resistor \((r)\) (because of leakage).

**Method**

Connect up the circuit shown in Fig. 1, making one of the resistors (say \(R_3\)) the variable one. As you will find later from theory, for balance \(C_1 R_3 = C_3 R_4\), if \(C_1 > C_3\) then \(R_4 > R_2\), so that if you use the plug box for \(R_4\), you should be able to use the 100 ohm decade dial resistor for \(R_2\), \(R\) being the other decade resistor. Adjust \(R_3\) for minimum sound and then \(R\) to get the best balance. Record all observations and repeat with different values of \(R_4\).

![Fig. 1](image)

Now connect up the circuit shown in Fig. 2, using one of the coils as the unknown inductance \(L\), \(C_1\) being 1 \(\mu F\), \(C_3\) 6 \(\mu F\), and \(R_3\) and \(R\) the decade resistors and \(R_2\) the plug box. Two independent equations result from a complete balance so that two variable resistors are needed, \(R_3\) and \(R\). Adjust \(R_3\) for minimum sound and \(R\) for a better balance. Record your observations and repeat with different values of \(R_2\), \(R_3\), and \(R\).

Repeat the experiment with the other coil, also with the two coils placed coaxially and touching with their magnetic fluxes (\(a\) reinforcing, and \(b\) opposing each other.

In the circuit shown in Fig. 3, \(C_1 = 0.05\) \(\mu F\), \(R_3\) is the plug resistance box, \(R_1\) and \(R_2\) the dial resistance boxes and the two coils are connected in opposition (which you can find by turning one of them round to hear if the sound in the ear-piece is increased). Adjust \(R_2\) first and then \(R_1\) for balance, recording all observations. Repeat with different values of \(R_3\).
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THEORY AND CALCULATION

For general balance $\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4}$. Consider DeSautey's bridge where $\omega = 2\pi f$, $f$ being the frequency.

$$\frac{r_1 + \frac{1}{j\omega C_1}}{R_2} = \frac{R + r_3 + \frac{1}{j\omega C_3}}{R_4}$$

and equating real and imaginary terms respectively, we get $\frac{r_1}{R_2} = \frac{R + r_3}{R_4}$, $C_1R_2 = C_3R_4$, the balance being independent of $\omega$ or the frequency.

Owen's bridge: $\frac{1}{jC_1\omega} = \frac{R_2}{R + R_1 + j\omega L}$, $R_1$ being the resistance of the coil, from which one can deduce $L = C_1R_2R_3$ and $R + R_1 = C_1R_2/C_3$ (the leakage resistors have been neglected as they will complicate the results, which the limited accuracy hoped for does not justify). If $L_1$ is the inductance of the first coil, $L_2$ the inductance of the second coil, $M$ the mutual inductance, then when the two coils are connected in conjunction, the inductance is $L_0 = L_1 + L_2 + 2M$, while when in opposition

$$L_0 = L_1 + L_2 - 2M \quad \text{so that} \quad M = \frac{L_0 - L}{4}.$$

Heydweiller: The p.d. between A and B must be zero for both branches when balance is attained. Hence

$$R_2I_1 + jL_2I_1 = jM\omega(I_1 + I_2)$$

and

$$(R_1 + r_1)I_1 + \frac{I_1}{j\omega C_1} = R_3I_2$$

therefore

$$\frac{R_2 + j\omega(L - M)}{1} = \frac{jM\omega}{R_3}$$

$$R_1 + r_1 + j\omega C_1$$

$$R_2R_3 + j\omega R_3(L - M) = j(R_1 + r_1)M\omega + \frac{M}{C_1}$$

and

$$M = C_1R_2R_3, \quad \frac{L}{M} = 1 + \frac{R_1 + r_1}{R_3}$$

thus $L$ must be $> M$ for balance.

COMMENTS

The value of $C$, $L$ and $M$ can be checked with 50 cycle A.C. supply using ammeter and voltmeter. In the case of the mutual inductance, which is small (of the order of a few millihenries), the ammeter is placed in the primary to measure $I$ and a high resistance sensitive voltmeter connected to the secondary to measure $V$ where $V = M\omega I$. 
This experiment reveals some of the imperfections of both practical capacitors and inductors. In the case of the inductor another defect is self-capacitance, a defect which becomes serious when higher frequencies are used, as will be discussed in Experiment 89.

EXPERIMENT 89

The study of electrical resonance and forced oscillations

APPARATUS

Two similar circular coils, each consisting of 500 turns of copper enamelled wire, and of low total resistance (see Experiment 88), two 1 μF condensers (with very high leakage resistance), variable audio-frequency oscillator up to 20 kc/s, small oscilloscope, sensitive A.C. ammeter (0–100 mA), two 0-0005 μF tuning condensers, a mutual inductance consisting of long solenoid (as the primary, and 1000 turn coil wound over its centre (see Experiment 53 as a secondary). Thin lubricating oil. 100 ohm non-inductively wound resistor. 50 μμF condenser.

METHOD

Connect up the circuit shown in Fig. 1 using one of the coils, find the resonance frequency $f_0$ roughly when the current is at its lowest, then tabulate your observations of current $I$ and the corresponding frequency between the range $f_0/2$ to $2f_0$, taking several observations in the region of resonance.

![Fig. 1](image1)

![Fig. 2](image2)

Repeat with the circuit shown in Fig. 2 using the second coil, except that this time the resonant frequency $f_0$ corresponds to maximum current, with and without the 100 ohm series resistor.

The primary in the circuit of Fig. 3 is exactly the same as in Fig. 2 but without the resistor, the secondary uses the first coil with the 1 μF condenser and the oscilloscope in parallel, the two coils being held coaxially with their planes parallel. Obtain a series of observations of current in the
primary $I$; the voltage in the secondary is given by the length of trace $y$ on the oscilloscope and the frequency $f$.

Fig. 3

Theory and Calculation

The first and second methods are examples of resonant circuits, being parallel and series resonance or 'rejeter' and 'acceptor' circuits respectively. Plot $I$ against $f$ for all the observations obtained and comment on them. Note the difference between the curves, with and without the series resistor in the 'acceptor' circuit of the second method. The effect of the resistance is to reduce the sharpness of resonance. If a condenser is charged and then discharged into a circuit containing an inductance, the resulting current does not die away rapidly (as when we measure capacitance with a ballistic galvanometer, see Experiment 53) but oscillates, the amplitude decreasing at a rate depending on the resistance in the circuit, on $L$ and on $C$ and is expressed by the letter, $Q$, which is called the 'quality factor' or the 'circuit magnification factor'. $Q$ can be shown to be $\frac{1}{R\sqrt{\frac{L}{C}}}$, the symbols having their usual meaning, and there are many ways of calculating it for a circuit. For example, find from your graph the values $f_1$ and $f_2$ corresponding to $I = \frac{I_{\text{max}}}{\sqrt{2}}$ then $f_1 - f_0 = \frac{f_0^2 - f_2}{f_0}$ (this is not difficult to prove theoretically); check by another method.

In the third method, the effect of coupling two tuned circuits is shown. Note the effect of varying the distance (i.e. the coupling) between the coils. This is important in the tuning of radio receivers—why?

Comments

One application of parallel resonance is to find an estimate for the dielectric constant (relative permittivity) of oil. First calibrate one of the tuning condensers, with a circular protractor attached to the shaft, against the 50 $\mu$F condenser, then connect it, when set at maximum, in parallel with the secondary of the mutual inductance as shown in Fig. 4. The oscillator is set for resonance with the coil in the secondary and the capacitance and self-capacitance in parallel, as shown by the large trace on the oscilloscope. Without altering the oscillator clip on the leads of the second tuning
condenser, in parallel with the first but set to have a small capacitance. Note how the oscilloscope trace shrinks markedly. Decrease the capacitance of the first condenser until the circuit is in resonance again and from this deduce the capacitance $C_0$ of the second tuning condenser, which is suspended inside a beaker. Now turn the first tuning condenser back to maximum, re-adjust the oscillator for resonance and pour oil into the beaker; again without touching the oscillator, decrease the capacitance of the first tuning condenser to restore resonance and deduce $C_1$, the capacitance of the second tuning condenser in oil. Deduce the dielectric constant (relative permittivity) from $C_1/C_0$. By the same method you could have calibrated the graduations of the tuning condenser against the capacitance of the 50 $\mu\mu F$ condenser.

**EXPERIMENT 90**

**To investigate the relationship between thermionic emission and temperature and to estimate the work function of tungsten**

**Introduction**

It was thought that electrons in a metal behave like those in a gas with an average energy equal to that of the host atoms of the metal. This classical concept of equipartition of energy was abandoned in favour of the quantum theory as a result of quantitative experiments on photoelectric and thermionic emissions.

**Apparatus**

Ferranti GRD7 diode (see Experiment 63), L.T. supply (not exceeding 8 V), four accumulators in series, 0–30 mA milliammeter, 200 V H.T. supply, two 8 ohm rheostats, two keys, one two-way switch, travelling microscope, potentiometer wire (about 2 ohm), centre zero galvanometer, 10 ohm resistor (5 W), 0·5 ohm resistor, P.O. box, slider.

**Method**

Remove the valve from its holder and use the travelling microscope to measure the length of the tungsten filament proper. Replace the valve and measure the filament resistance at room temperature using the P.O. box; this will include the contact and lead resistance as well as the resistance of the filament proper. Measure also the resistance per cm ($\rho$) of the potentiometer wire.
Connect up the circuit shown in Fig. 1 with the filament current adjusted to give a current just measurable on the milliammeter. Record the value of this current.

First balance the p.d. across the filament against that of the 10 ohm resistor and the potentiometer wire, adjusting the rheostat so that the balancing length \( l_2 \) is small. Next, balance the p.d. across both the filament

and the 0.5 ohm resistor against that of the 10 ohm resistor and the potentiometer wire, the balancing length this time being \( l_2 \).

Increase the filament current and measure the anode current in each case. Repeat as many times as you can (the p.d. across the filament should never exceed 6.3 V, so that the rheostat in series with the filament should never be cut out altogether).

**Theory and Calculation**

From the knowledge of the length of the filament wire and its diameter (given by the manufacturers, but this can be checked) and assuming the resistivity of tungsten at room temperature to be 5.0 \( \mu \) ohm cm, calculate the resistance of the filament \( R_F \). This you will find to be less than the resistance found from the P.O. box. The difference \( R_X \) is due to contact and lead resistances (which we shall assume to remain constant throughout the experiment).

Simple calculation will show you that the resistance of the filament \( R_T \) at any temperature \( T^\circ K \) is \( \frac{0.5(10 + l_1\rho)}{(l_2 - l_1)\rho} \). The ratio of the resistances of the filament proper at \( T^\circ \) and at room temperature is

\[
\frac{0.5(10 + l_1\rho)}{(l_2 - l_1)\rho} - R_X
\]

\[
R_F
\]
PRACTICAL PHYSICS

Use the graph given in Fig. 2 of Experiment 51 to deduce \( T^\circ K \) in each case.

With 200 V applied to the anode, the anode current \( I_a \) represents the saturation current in each case.

It can be shown, using the quantum theory (as embodied in the Fermi-Dirac statistics), that \( I_a = C T^2 e^{-\frac{\phi}{kT}} \) where \( \phi \) is the work function (eV) (eV = the work done when one electron falls through one volt), \( k \) is Boltzmann's constant = 1.38 \( \times 10^{-23} \) joule deg K\(^{-1} \) and \( C \) is a constant. Therefore \( \log \frac{I_a}{T^2} = \log C - \frac{\phi}{kT} \log e \), where \( \log e = 0.4343 \).

Plot \( \log \frac{I_a}{T^2} \) against \( 1/T \) and show that it is a straight line and from the gradient deduce \( \phi \) in electron volts. Taking \( e = 1.6 \times 10^{-19} \) coulomb deduce the voltage barrier which electrons have to overcome to escape from the metal.

COMMENTS

The remarkable thing about thermionic emission is that pure metals emit hardly any electrons when their temperature is much lower than 2000\(^\circ \)K, suggesting that their work function is comparatively high, a fact confirmed from photoelectric experiments (with thoriaed tungsten—a so-called dull emitter—emission is much more copious at a considerably lower temperature).

The agreement between theory and experiment above confirms the hypothesis that electrons in a metal occupy certain quantum states and have energies much higher than was once thought. These energies are hardly dependent on temperature so that the contribution of electrons to the specific heat of metals at low temperatures is negligible—a fact confirmed by the low specific heat of metals at low temperatures.

EXPERIMENT 91

A further study of the triode as (a) an amplifier, (b) an oscillator

APPARATUS

Miniature valve ECC82 on a suitable base. Peg-board, 6-3 V and H.T. supplies (the latter about 70 V; a dry battery would do), variable 50,000 ohm and 3000 ohm resistors, small cathode ray oscilloscope (or alternatively a valve voltmeter), small 1.5 V bias battery, one 50 µF electrolytic condenser, audio-frequency oscillator. Pair of dividers, switch.

Milliammeter (0–10 mA), microammeter (0–100 µA), two circular coils mounted coaxially at a variable distance apart (see Experiments 88 and 89). The following condensers: two 1 µF (paper, 200 V), two 0.01 µF,
one 0.05 μF, one 0.001 μF (all mica), grid leak resistor (value to be determined by the student), crystal ear-piece, 50 cm ruler.

(a) The triode as an amplifier

Method

(It is assumed that the student has already done Experiment 65.) Connect up the circuit shown in Fig. 1(a), set the output of the oscillator for 0.5 V and the frequency for 1000 c/s. If the cathode ray oscilloscope has a built-in isolating condenser, there will be no need for the 1 μF condenser. Set R for a low value, and if the length of the vertical trace on the cathode

ray is too small to measure, increase the output of the oscillator to 1 V (there is no need to use the time base or the amplifier, the connection should be made direct to the tube itself).

Vary and record R and the corresponding length of the y-trace on the cathode ray tube. If the deflection sensitivity of the tube is not known, it should be found by connecting the variable output of the oscillator to the tube and calibrating the latter accordingly.

The circuit in Fig. 1(b) differs from that in Fig. 1(a) by the absence of the bias battery. Instead, a resistor is inserted between the cathode and earth, shunted by a large electrolytic condenser of 50 μF (the condenser must be connected with the correct polarity). A milliammeter is also inserted in the anode circuit. Note the effect on the anode current and the length of the trace as the resistor in the cathode circuit is varied. Note also the effect of disconnecting the shunting condenser on the length of the trace.

Theory and Calculation

From the static characteristics of the triode shown in Experiment 65, Fig. 4, you have already deduced the voltage amplification factor $\mu = (dv_a/dv_g)$ anode current being constant, or $\left(\frac{dv_a}{dv_g}\right)_{i_a}$, but more information can be obtained from the equal slopes of the straight portions of the graphs. Each is equal to $\left(\frac{di_a}{dv_a}\right)_{v_a}$, which has the dimension of
conductance (the reciprocal of resistance) and is denoted by $g_m$, usually expressed in mA/V.

We also learn from these graphs that provided a small alternating voltage is applied to the grid, the resulting alternating current $i_a$ in the anode depends on $v_g$ and $v_a$, where $v_a$ is the alternating voltage superimposed on the H.T. due to the p.d. across $R$ in the anode circuit, and the relation can be expressed as $i_a = \alpha v_g + \beta v_a$ (this is justified because of the straight portions of the graphs). By differentiating, $(di_a/dv_g) = \alpha = g$ and $(dv_a/dv_g) = -\mu = -(a/\beta)$ (where $dv_a$ is positive, $v_g$ has to be reduced to make $i_a$ the same, so $dv_g$ is negative), hence

$$i_a = g_m v_g + \frac{g_m}{\mu} v_a \tag{1}$$

If $v_g$ is kept constant and the anode current is varied by varying the anode voltage, one gets graphs with some straight portions whose slopes are equal to $(dv_a/di_a)_v_g$ having the dimensions of resistance. This is the A.C. resistance of the valve and is denoted by the $\rho$. By differentiating (1) we get

$$(dv_a/di_a)_v_g = \mu g_m = \rho. \text{ Hence}$$

$$i_a = \frac{1}{\rho} (\mu v_g + v_a) \tag{2}$$

In the above experiment $v_a = -Ri_a$ (for if $i_a$ is positive then the potential of the anode is decreased by $Ri_a$). Hence $i_a \left(\frac{\rho + R}{\rho}\right) = \frac{\mu v_g}{\rho}$ and the output voltage $i_a R$ measured by the cathode ray tube is $\frac{\mu v_a R}{R + \rho}$ and the stage voltage amplification is $\frac{R}{R + \rho} \cdot \mu$. Verify. What is the function of the condenser across the H.T. supply?

From your measurement of the output voltage $i_a R$ or $v_a$ deduce the output power $v_a^2/R$, plot the output power against $R$ and comment on and prove your result (see Experiment 47).

Comments

The anode current provides a negative bias for the grid through the resistor shown in Fig. 1(b), making the cathode slightly positive with respect to the grid, the condenser offering a low impedance shunt for the alternating signal so that no A.C. voltage is impressed on the cathode with respect to the grid. Calculate the value for the cathode resistor giving the same bias as that of the battery previously used (is there any other advantage in the use of automatic bias?). Disconnecting the shunting condenser decreases the amplified signal—why? This is an example of voltage feedback which, in this case, opposes the input signal and effectively reduces it. It is called negative feedback (is negative feedback always undesirable?).
(b) The triode as an oscillator

**Method**

Connect the circuit diagram shown in Fig. 2 with $C = 0.05 \mu F$, using the grid bias battery at first. Bring the two coils together and note the readings on the instruments (if nothing happens, turn over one of the coils). Both the grid and anode currents will suddenly increase and at the same time you will hear the oscillations through the ear-piece. Measure the distance between the coils and record the value of $C$, replace the ear-piece with the oscilloscope and, using the time base (previously calibrated using the oscillator), deduce the frequency of oscillation.

![Circuit Diagram](image)

Repeat with $C = 0.1 \mu F$ and $C = 0.001 \mu F$ (you may find the frequency too high to hear with the latter condenser). Disconnect the grid bias battery and replace it by a variable high resistor (about 2 Meg ohm) shunted by a 0.1 $\mu F$ condenser. With the two coils well apart note the effect on the anode and grid currents as the resistance is gradually increased. Set the resistor for zero grid current and bring the two coils together, noting the onset of oscillation on the two currents; why does the anode current decrease this time?

Get the two coils to touch and increase the resistance in the grid to maximum. Do you hear the oscillation coming through in pulses as if it is stopping and starting at regular intervals? (If this does not happen, increase the capacitance of the shunting condenser.)

**Theory and Calculation**

The anode in the above experiment is connected to a rejector circuit (see Experiment 89), so that the mere switching on of the circuit sets up electrical oscillation which left to itself would be damped down quickly.* When however another coil connected to the grid is placed near it, an induced oscillating e.m.f. is set up across it and this is amplified. If the coil is suitably connected the amplified signal will provide the energy to maintain

* Can you demonstrate this experimentally?
the oscillations in the anode. This is one of the simplest ways of using positive feedback to maintain steady oscillations.

Verify that the frequency of oscillation \( f = \frac{1}{2\pi \sqrt{LC}} \), having found \( L \) from either Experiment 88 or 89, also verify that the condition for oscillation is \( M \geq \frac{L + \frac{\rho R}{\mu}}{\mu} \) where \( M \) is the mutual inductance between the coils and \( R \) is the resistance of the coil in the anode. \( M \) can be deduced (see Experiment 56) from the variation of the mutual inductance with different separations between the coils, \( R \) can be measured using a P.O. box.

Replacing the grid bias battery by a grid leak resistor offers a built-in limit to the amplitude of the oscillation and gives more stable oscillations. Knowing the mean grid current and the desired bias for the grid, deduce the value for the leak resistor.

Without the shunting condenser there will be no steady negative potential on the grid; on the other hand with the condenser alone (without the resistor) the condenser will be charged to such an extent (i.e. the grid potential becomes very negative) that the oscillations stop altogether. However, intermittent oscillations take place if \( RC \) is too big (why?) (see Experiment 51, \( RC \) has the dimension of time). Now suggest a value for \( C \).

**COMMENTS**

The triode when used as an amplifier with the grid biased sufficiently negative with respect to the cathode has infinite input impedance (for low frequencies anyway). This simplifies the equations relating to the behaviour of the valve down to one single equation (equation (2)), containing only two independent parameters characteristic of the valve.

Construct a two stage amplifier, using an \( R-C \) coupling (i.e. a resistor replacing the C.R.O. or the crystal ear-piece in Figs. 1(a) and 1(b)), remembering that the second valve will have to handle larger swings of voltage; you may have to use another miniature valve with a smaller value for \( \rho \). Where would you place a volume control and how would it be connected?

**EXPERIMENT 92**

The further study of the transistor as an amplifier (grounded emitter)

**INTRODUCTION**

(It is assumed that the student has already attempted Experiment 65.) Unlike the thermionic triode the transistor has an input which is neither infinite nor constant nor independent of the output impedance. Therefore we need two equations to relate alternating input and output of current \( i \) and voltage \( v \) due to a small signal, the coefficients again being slopes of the static characteristics which have already been plotted in Experiment 65.
(Subscript 1 refers to the input and 2 to the output, it is also usual to add a dash ′ for a grounded emitter but this will be omitted as we are using only one configuration.)

\[
\begin{align*}
v_1 &= h_{11}i_1 + h_{12}v_2 \\
i_2 &= h_{21}i_1 + h_{22}v_2
\end{align*}
\]

(1)

\(h_{11}\) is the input impedance when \(v_2 = 0\) (short-circuited to A.C., \(i_2 = 0\) by shunting the output with a large capacitance (see Fig. 2)). One can obtain \(h_{11}\) or \(Z_{11}\) as the slope of a tangent of the curve \(v_b = i_b\) (\(v_b\) constant—see Experiment 65). \(Z_{11}\) is clearly not a constant but is of the order of 1000 ohm. \(h_{21}\) is the current amplification factor \(a\), \(h_{22}\) is output conductance when \(i_1 = 0\) (equals \(\dfrac{1}{Z_{22}}\)).

If \(R\) is the load resistance in the collector circuit when the transistor is used as an amplifier, then clearly \(v_2 = -i_2R\), and substituting in equation 1, we get the current amplification factor \(\dfrac{i_2}{i_1} = \dfrac{aZ_{22}}{Z_{22} + R}\) which is less than \(a\), but as \(Z_{22}\) is of the order of 30,000 ohm \(\gg R\) the reduction is not very great.

Similarly the voltage amplification is \(\dfrac{v_2}{v_1} = \dfrac{a}{\Delta R + (h_{11}/\Delta)}\)

where

\[
\Delta = h_{11}h_{22} - h_{21}h_{12}
\]

as \((h_{21}, h_{12})\) is small,

\[
\Delta \approx \dfrac{Z_{11}}{Z_{22}}
\]

and the voltage amplification \(A_v\) becomes

\[
A_v = \dfrac{aZ_{22}}{Z_{11}} \cdot \dfrac{R}{R + Z_{22}}
\]

(2)

and the stage power gain \(\rho\) is

\[
\rho = \dfrac{a^2Z_{22}^2}{Z_{11}} \cdot \dfrac{R}{(R + Z_{22})^2}
\]

(3)

**APPARATUS**

Transistor OC71, four accumulators, milliammeter (0–10 mA), peg-board, 10 ohm and 5000 ohm variable resistors, the following fixed resistors, 10 ohm, 30 ohm, 100 ohm, 1000 ohm, and 400,000 ohm, decade variable 100 ohm resistance box, 1000 c/s oscillator of variable output, small oscilloscope, crystal ear-piece, 50 μF electrolytic condenser, two 1μF isolating condensers, pair of dividers.

**METHOD**

The circuit shown in Fig. 1 is for measuring the current amplification factor. Adjust \(X\) for a suitable value of \(I_c\) (say 3 mA), set the oscillator output for a few millivolts and find the balance (for minimum sound in the ear-piece) by adjusting \(R_2\). Record \(I_c\) and \(R_2\). Decrease \(I_c\) by 0.5 mA each time and repeat.

The circuit shown in Fig. 2 is for measuring the input impedance \(Z_{11}\).
Again adjust $X$ to give a suitable value for $I_e$ and the oscillator to give a few millivolts and find the balance point by adjusting $R_4$. Record $I_e$ and $R_4$ and repeat for different values of $I_e$.

The circuit shown in Fig. 3 is for measuring the voltage amplification factor. The A.C. signal injected into the base is about 10 mV. The amplified voltage, which is not large enough to be measured directly by the cathode ray tube, has to be connected to the amplifier input of the oscillo-
scope and the amplifier gain adjusted so that the amplifier is not overloaded when $R$ is maximum (i.e. 5000 ohm). Without disturbing the amplifier, measure by means of the pair of dividers the vertical height $y$ of the trace (no need to use the time base, just as in Experiment 91). Record $y$ and $R$ and repeat for a different set of values. Calibrate the oscilloscope using the variable output from the oscillator.

**Theory and Calculation**

From Fig. 1, the p.d. across $R_1$ is $i_1 R_1$ and across $R_2$ is $-i_2 R_2$, therefore for balance $i_1 R_1 = i_2 R_2$ and $a = \frac{i_2}{i_1} = \frac{R_1}{R_2}$. In Fig. 2, the input impedance of the transistor $Z_{11}$ ($i_s = 0$) forms the second arm of a Wheatstone bridge so that when the bridge is balanced $\frac{R_1}{R_2} = \frac{R_s}{R_4}$, hence $R_s \approx Z_{11}$. As $Z_{11}$ depends on the input base steady current $I_b$, it can be deduced from the static characteristics (Experiment 65), as $V_c = -2V$, $I_c$ is known, hence $I_b$. Plot $Z_{11}$ against $I_b$.

The output impedance $Z_{22}$ can best be deduced from the slopes of the $I_c - V_c$ curves plotted in Experiment 65 for different values of $I_b$ and another graph connecting $Z_{22}$ and $I_6$ drawn, preferably on the same graph paper as the $Z_{11} - I_6$ curves, with a different scale for $Z_{22}$.

Deduce the voltage amplification $A_v$ from the ratio of the output to the input voltage and verify relation (2) (plot $1/A_v$ against $1/R$). Deduce $i_s$ and hence investigate the variation of power gain with load resistance $R$.

**Comments**

The maximum power gain from (3) can be found to be when $R \approx Z_{22}$ but if another amplifier stage is to follow, using $C-R$ coupling (see Experiment 91), then the input impedance of the following transistor $Z_{11}$ is in parallel with $R$, and since $Z_{11}$ is of the order of 1000 ohm, then the load resistance becomes effectively equal to $Z_{11}$ and therefore the power gain is $\approx a^2$ ($Z_{22}$ can be written for $(R) + Z_{22}$) as $R$ would be small compared with $Z_{22}$). Express the power gain in decibels (a ratio when very large is best expressed on a logarithmic scale, so that $10 \times \log_{10}$ of the ratio is the gain in decibels).

What is the purpose of the 400,000 ohm resistance in Fig. 3? Measure or estimate the current flowing in it. Construct as in Experiment 91 a two stage $R-C$ coupled amplifier and compare the practical performance (power and voltage gain) with those expected theoretically. Can you account for the differences, if any?
EXPERIMENT 93

A quantitative study of some properties of 3 cm microwaves and a determination of the dielectric constant (relative permittivity) for glass and paraffin wax at 10,000 Mc/s.

APPARATUS

Horn-transmitter and receiver of 3 cm microwaves. Associated power supplies for Klystron and modulator. 10,000 Mc/s probe diode, crystal ear-piece, Scalamp, 200 ohm plug resistance box, small turn-table. Mounted paraffin wax plano-convex lens, 12 in diameter, radius of curvature of curved side about 10 in (see Fig. 2). Thin 4 in sheets of glass and paraffin wax, two 10 in square sheets and a 10 in × 1 in strip of metal, all supported to stand vertically.

To investigate the variation of the strength of the field of the horn-transmitter with distance and angle

METHOD

Use the arrangement shown in Fig. 1. Set the Klystron modulation on D.C. so that the rectified output could be fed to a D.C. meter such as the Scalamp, set at a suitable range, the strength of the field being proportional to the deflection $I$. Keep the height of the diode probe level with the centre of the horn and vary the distance $D$ between the probe and an arbitrary reference point on the transmitter (say the position of the $\frac{1}{4}$ wavelength radiating dipole inside the waveguide), recording the distance and the microammeter reading. Next keep the distance of the probe from the transmitter fixed but raise or lower the probe and move it sideways and investigate the divergence of the beam radiating from the transmitter.

THEORY AND CALCULATION

Plot $1/\sqrt{I}$ against the distance $D$ and also plot the intensity $I$ in horizontal and vertical planes and estimate the angle of divergence of the transmitted beam (take the limit of the beam when $I < I_{\text{max}}/10$).

To determine the dielectric constant of paraffin

METHOD

Use the paraffin wax lens shown in Fig. 2 and determine the focal length of the lens using the radiating dipole in the transmitter as your object, locating the image using the diode probe with a small light half-cylinder copper reflector (which can be clipped on to the probe); the position of the image can be located by listening into a crystal ear-piece connected to the
probe. As the diameter of the lens is only 10 wavelengths across, some diffraction effects and spherical aberration tend to degrade the image (the aperture is \( f/2 \)) so that the locating of the image has to be done while moving the probe inwards and again on moving it outwards and averaging the two points thus located. Find the radius of curvature as with optical lenses, by measuring the thickness of the lens \( t \) and the diameter \( d \).

**Theory and Calculation**

Find \( f \) by plotting \( 1/U \) against \( 1/V \) and determine the refractive index \( \mu \) from \( 1/f = (\mu - 1)(1/R) \) where \( R \) is the radius of curvature of the curved surface \( [(2R - t)t = d^3/4] \). Now \( \mu^2 = k \), the dielectric constant of paraffin wax (M.K.S. \( \mu^2 = \varepsilon_r \mu_r \) where \( \varepsilon_r \) is the relative permittivity and \( \mu_r \) the relative permeability of wax). Deduce \( k \) (or \( \varepsilon_r \)) for paraffin wax at 10,000 Mc/s.

**To locate the direction of the electric vector**

**Method**

You may already have seen demonstrated that the transmitted microwave is plane-polarized. The apparatus for this experiment is shown in Fig. 3. Fix the transmitter so that the short side of the wave-guide is vertical and place the probe vertical and about 50 cm from it, put the turntable, with the sheet of glass held vertical on it by means of Plasticine, close to the probe. On rotating the turntable you will find two positions of the sheet at which the wave transmitted by the sheet is minimum. Measure 2\( \theta \), the angle between the two positions, as shown. Repeat using a thin sheet of paraffin wax. What happens if both the transmitter and receiver are turned through a right angle?

The arrangement shown in Fig. 4 is to measure the intensity \( I \) of the wave detected by a horn receiver as it is rotated through an angle \( \theta \). With the two horns facing each other, with their guides level and parallel, record the position of the pointer in relation to a protractor fixed to the back of the receiver. Unscrew the holding screw and rotate the receiver
through an angle $\theta$, recording the new level of the intensity of the receiver, and repeat this until $\theta = 90^\circ$. Check by turning the receiver in the opposite direction.

**Theory and Calculation**

We have learnt from Experiment 77 that when light is reflected off a sheet of glass at an angle $\phi$ where $\tan \phi = \mu$ (refractive index), the reflected wave is plane-polarized and the transmitted light is plane-polarized at right angles to it. This experiment shows that when the electric vector is normal to the plane of incidence (i.e. vertical in our experiment, for the electric vector is parallel to the short side of the guide or parallel to the
dipole), very little energy is transmitted at Brewster’s angle \( \phi \) and most of it is reflected, proving that in the reflected wave, at Brewster’s angle, the waves are plane-polarized with the electric vector at right angles to the plane of incidence.

As \( \theta = \frac{\pi}{2} - \phi \), therefore \( \cot \theta = \mu \)—a surprising result, as it shows that \( \mu \) for glass is about 2.5. This is explained by the fact that \( k \) (or \( \varepsilon_r \)) is dependent on the frequency; when visible light is used the frequency is about \( 5 \times 10^{14} \) c/s while for this experiment the frequency is \( 10^{10} \) c/s. It is clear that generally \( k \) (or \( \varepsilon_r \)) gets less as the frequency increases.

When the two horns in Fig. 4 are parallel (i.e. \( \theta = 0 \)) the intensity \( I \) recorded by the galvanometer is maximum. Plot \( I \) against \( \cos^2 \theta \) and comment on the result.

**The positions of maxima due to stationary waves**

**Method**

Place one of the large metal sheets as a reflector in front of the transmitter, and locate with the probe the positions of maxima due to stationary waves. Determine the mean distance between two consecutive maxima \( (\lambda_o/2) \).

![Fig. 5](image)

Repeat the experiment using the arrangement shown in Fig. 5, keeping the two sheets about 2 cm apart and using the long strip as a reflector at the far side. Again determine the mean distance between two consecutive maxima \( (\lambda_a/2) \). Increase the distance between the plates and repeat.

**Theory and Calculation**

Here is another surprising result: the wavelength \( \lambda_a \) when the two plates are 2 cm apart is not the same as the \( \lambda_o \) when a single plate is used. Does this mean that the electromagnetic waves travel faster between the plates than in free space? Does this contradict the theory of relativity?

The answer to the first question depends on what is meant by the velocity of a wave. It may mean the speed with which a front of the *same* phase travels and this is called the ‘phase’ velocity. If the distance between the plates is comparable to the wavelength, waves shuttle back and forth
between the plates, establishing interference patterns and the speed of the
crests, say, is greater than the speed of the individual waves. In fact
\[
\frac{1}{\lambda^2} = \frac{1}{\lambda_0^2} - \frac{1}{4a^2}
\]
(try to prove this relation theoretically if you can). Verify.
The velocity of pulses of waves sent between the plates is actually less than
in free space; it is only by this velocity, called ‘group’ velocity, that actual
information can be transmitted and this is the one velocity the theory of
relativity is concerned with. Thus the answer to the second question is that
the theory of relativity is not contradicted.

Comments

The arrangement of the two plates about 2 cm apart in the fourth
method constitutes the principle of a wave-guide, which is similar to the
light tubes made of very fine transparent fibres which can guide visible
light. Compare the intensity detected by a probe at a distance of a yard or
two with and without a long row of parallel plates, and note the dimensions
of the actual guide used in the transmitter.

The mode of propagation of electromagnetic waves in a guide depends
on how the waves are introduced into the guide and the most simple
method is probably the one you have just used, transmitting what are
called \( H_{10} \) waves. Progressive electric waves are transmitted at right angles
to the direction of propagation along the guide but parallel to the short
side of its rectangular section. In any section there is also a stationary elec-
tric wave forming half a sine wave, being zero at each of the conducting
sides. The magnetic vector can be imagined to be at right angles to the
electric vector. The flaring, by using a horn at the end of the guide, per-
mits the waves to be radiated as a divergent beam without any lobes or
variations which would have been objectionable in this experiment.

EXPERIMENT 94

To study some aspects of ferromagnetism

Apparatus

Two slender specimen anchor rings of iron and hard steel respectively,
about 5 in mean diameter, each with 200 turns of cotton-covered stout
copper wire wound uniformly over it (see Fig. 1) and 25 turns of secondary
coil. A board with 6 resistors (indicated in Fig. 1) wound on empty cotton
reels with 5 switches as shown. Ammeter (0–5 A), 6 V accumulator,
ballistic galvanometer (Scalamp), 0–1000 ohm resistance box, shorting
switch, reversing switch.

Various ferroxcubes of different Curie points, ticonal magnet, special
brass tube (supplied by Mullard), oil bath, 0–300°C thermometer, sand
tray, Bunsen burner and stirrer.
METHOD

The specimen ring must first be demagnetized by connecting the coil to an A.C. supply, starting with a large current then decreasing it to zero.

Use the iron specimen first and connect the circuit as shown in Fig. 1 with all the switches out but with the magnetizing coil wound over the specimen disconnected. Let \( R \) be, say, 100 ohm. Quickly switch the coil into the circuit and adjust \( R \) to give a throw corresponding to half the total scale and do not alter it during the rest of the experiment. Again demagnetize the specimen and quickly switch the coil into the circuit, noting this time the throw of the ballistic galvanometer \( \theta_1 \) as well as the current \( i_1 \). Now close switch 5 and note the throw \( \theta_2 \) and repeat until all the switches are closed. Reverse the connections to the ballistic galvanometer (why?) and find the throw as switch 1 is opened and repeat with switches 2, 3, 4 and 5 finally when the coil is disconnected altogether.

Repeat the experiment with the magnetizing current reversed in the coil (this completes the cycle). Repeat the whole experiment using the steel specimen, without altering \( R \).

The Curie point of various ferroxcubes can be found using the apparatus in Fig. 2. The specimen and the ticonal magnet should be immersed in oil, the specimen being attracted to the magnet. Heat the oil and note the temperatures when the specimen breaks away and when it is attracted again as the oil is cooled. Repeat with different specimens.
THEORY AND CALCULATION

The ballistic throw $\theta$ is proportional to the change in flux density $dB$ and the change in the current is proportional to the change in the magnetizing field $dH$. At any moment, $\Sigma \theta \propto B$ and $i \propto H$. Plot the algebraic sum of $\theta$ against the current $i$ A over the whole cycle for both specimens.

An estimate can be made of the remanance $B_{rem}$ and the coercive force $H_c$ if the ballistic galvanometer is calibrated and the area of cross-section of the anchor ring, the mean diameter of the ring $D$, the primary $N$ and secondary $n$ number of turns are known. The magnetizing field is $2Ni/5D$ oersted (M.K.S.: $Ni/\pi D$ A m$^{-1}$) and the change in the flux density $dB$ can be found from the throw as was done with the search coil in Experiment 53.

Take the Curie points as the mean of the two temperatures for each specimen.

COMMENTS

Iron, steel, and ferrites possess, at room temperature, spontaneous magnetization, though, because it is fragmented into domains, it is not easily detectable without aligning the domains by applying a small magnetizing field. Thus a fraction of an oersted (1 oersted = 40 A m$^{-1}$) is sometimes sufficient to produce an enormous change in the magnetic flux, though the equivalent fields which would align the electron spin in the domains are of the order of $10^7$ oersted.

Ferrites, which are usually complex chemically, possess more than one magnetic moment and they are aligned antiparallel $\downarrow\uparrow$ to each other in each domain, so that the observed magnetization is the difference between them. All ferrite-like substances are said to display ferrimagnetism (as distinct from ferromagnetism).

EXPERIMENT 95

To study the characteristics of a Geiger-Müller tube, to estimate the maximum energy of $\beta$-particles by using magnetic deflection and to investigate the phenomena of $\beta$-back-scattering

APPARATUS

Halogen quenched Geiger-Müller tube (Mullard MX168), scaler with variable E.H.T. (300–500 V), $Sr_{90}$ (1 $\mu$C) $\beta$-source, stopwatch. Electromagnet with about 25 mm pole pieces, 0–10 A D.C. ammeter, 12 V D.C. supply, rheostat, circular search coil 1 in diameter containing a few turns $N$, ballistic galvanometer (calibrated in $\mu$ coulombs), resistance box (0–1000 ohm), large steel plate with $\frac{1}{2}$ in hole drilled in its centre, switch, clamps and stand. 1 in thick foam polystyrene sheet, $\frac{1}{8}$ in thick metal
plates (4 in side), made of aluminium, iron, copper, lead and zinc. Thin aluminium foil (used in cooking) cut in squares 4 in side, 2 in brass rod, 1 in diameter.

**Method**

Clamp the source at a suitable distance from the Geiger tube with the E.H.T. set at minimum. Increase the E.H.T. slowly until the tube begins to count (threshold potential), tabulate the recorded counts per minute against the potential of the tube, increasing the E.H.T. by 10 V steps until the counting rate begins to increase appreciably (sparking potential) when it should be stopped.

Set the E.H.T. at a potential half-way between the threshold and sparking potentials and set up the arrangement shown in Fig. 1. Find the background count without the β-source in position. Connect the electromagnet to the 12 V D.C. supply in series with a switch, rheostat and ammeter. Record the count per minute against the magnetizing current as the latter is increased to maximum. Measure the length of the magnetic path \( l \), distance \( a \) and the diameter \( d \) of the hole in the steel plate. Without disturbing the pole pieces use the ballistic galvanometer, series resistor and the search coil to calculate the magnetic field corresponding to different magnetizing currents (for details see Experiment 53).

Now set up the arrangement shown in Fig. 3. It is important that the pieces of apparatus be clamped well above the bench and that the large polystyrene sheet should be supported in such a way that the clamp stands themselves do not scatter any radiation. Find the background count per minute without the source, then with the source. The increase in count is due to the β-particles being scattered from the polystyrene sheet, but this
increase should not be high (not more than two or three times the background count) or else the tube is receiving β-particles direct from the source or the source is too near the bench.

![Diagram](image)

Fig. 3

Investigate the *increase* in count when a thin sheet of aluminium foil is placed directly below the tube and the source. (Pencil the corners of the sheet on the polystyrene foam so that it can always be replaced correctly.) Increase the number of sheets until you get steady counts, recording all observations (it is important to remember that as the counts are comparatively small, at least two minutes or more should be taken over each count).

Repeat these observations using the sheet of metal provided, checking each observation at least once.

**Theory, Calculation and Comments**

The Geiger-Müller tube contains a central fine insulated wire inside a metal cylinder in an atmosphere of gas at reduced pressure (a few mm of mercury), the wire being kept at a high positive potential with respect to the cylinder. When ionizing particles or photons enter the tube, the ion pairs produced are accelerated by the electric field, which will cause further ionization (the field is particularly strong near the wire—see Experiment 38). The ionic mobilities however are not the same for positive ions and electrons (at high gas pressures the electrons tend to ride 'pick-a-back' on neutral atoms, hence the mobilities are more nearly equal), and because of this disparity in mobilities, a positive space charge envelops the wire, reducing temporarily the potential difference between the wire and the cylinder. This cloud of positive ions takes nearly 100 µsec to reach the cylinder, hurling themselves at it, and unless slowed down sufficiently beforehand, will raise a cloud of electrons on impact. This slowing down or quenching is produced by some added halogen molecules which will be
'excited' but not ionized by the onrush of positive ions (for the distinction between excitation and ionization, see Experiment 96). Thus the tube does not count for a short time after each event (called 'dead' time) which should be allowed for in fast counting but which we shall ignore in our work. Plot the counts per minute against the potential. Note how the count increases very little over a range of 100 V, usually expressed as a percentage change in the count per volt. As the voltage increases further, spontaneous sparking takes place which would harm the tube and should be avoided at all costs.

You have already estimated the maximum energy $E_{\text{max}}$ of $\beta$-particles emitted by Sr$_{90}$ in Experiment 68 and the second part of this experiment not only provides a rough check of $E_{\text{max}}$ but is most revealing in another way. Consider the path of the electron in a magnetic field directed at right angles to the plane of the paper (see Fig. 2). If $R$ is the radius of curvature and if the final direction of the path when produced backwards meets the initial direction at O, then $PO = \left( \frac{a + \frac{l}{2}}{l} \right)$. If $d$ is the lateral displacement of the electron then $\frac{d}{a + \frac{l}{2}} \approx \theta = \frac{l}{R}$. Hence $R$.

C.G.S.  \hspace{1cm} M.K.S.

If $H$ is the field in oersted, then in air

$$H_{\text{ev}} = \frac{mv^2}{R}$$

$$H.e/m = \frac{v}{R} \quad \cdots \quad (1)$$

If $B$ (weber m$^{-2}$) is the flux density then

$$B_{\text{ev}} = \frac{mv^2}{R}$$

$$B.e/m = \frac{v}{R} \quad \cdots \quad (1)$$

Plot the magnetizing current against the count per minute (see Fig. 2) and estimate the magnetizing current which just about stops most of the particles from reaching the tube; as a rough estimate take $d$ to be equal to the diameter of the hole in the steel shield. From the calibration curve connecting the magnetic field and the current deduce the corresponding magnetic field and calculate $v$ from equation (1). You should find that $v$ is greater than the velocity of light, $c = 3 \times 10^{10}$ cm sec$^{-1}$ ($= 3 \times 10^8$ m sec$^{-1}$) which is clearly impossible. If we take $v$ to be slightly less than the velocity (say 0.95$c$) then $R$ is much smaller than we observe! Has the mass increased? Einstein's special theory of relativity states that the mass of a moving particle is $= \frac{m_0}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}}$ where $m_0$ is the rest mass of the particle, and equation (1) is now modified

$$H.e/m = \frac{v}{R\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\left[ \begin{array}{c} B.e/ m_0 = \frac{v}{R\sqrt{1 - \frac{v^2}{c^2}}} \\ \end{array} \right. \quad \text{M.K.S.}$$

Deduce $v$ and $E_{\text{max}}$ ($= \frac{1}{2}m_0v^2/\sqrt{1 - \frac{v^2}{c^2}}$). Express $E_{\text{max}}$ in Mev.

This increase in mass as $v \rightarrow c$ considerably modifies particle accelerators, as it becomes more and more difficult to deflect these particles or
maintain them in a curved path when their energies approach thousands of Mev (Gev).

The last part of the experiment deals with the scattering of \( \beta \)-particles from a metal sheet. First you were asked to investigate how this scattering depends on the thickness of the metal sheet and it clearly increases with thickness to a maximum: has this any relation to the range of \( \beta \)-particles in that metal? This has great application as a thickness gauge and can be used in the automatic control of thickness in a continuous operation.

Lastly, the scattering of particles from a thick metal sheet must depend on the coulomb field produced by their nuclei (viz. their atomic number \( Z \)). Investigate the relation between \( Z \) and scatter. This is quite different from the absorption of \( \beta \)-particles by a metal sheet, which, at least for light elements, is dependent only on the mass per \( \text{cm}^2 \) of the absorbing medium and not on the atomic number or atomic weight.

**EXPERIMENT 96**

To estimate the excitation and ionization potentials of mercury vapour

**Apparatus**

Mercury vapour tetrode G.E.C. valve E2780 with octal base (B8-0), 6.3 V heater filament supply, 14 V D.C. supply connected to a potentiometer divider providing a variable potential, voltmeter (0–15 V), sensitive microammeter (Scalamp; 1 \( \mu \text{A} \) mm\(^{-1}\)), telemicroscope (magnification about \( \times 30 \)) with micrometer eye-piece focused on the fine line of the light spot of the Scalamp. Two-way switch, low resistance rheostat, four accumulators.

**Method**

Connect up the circuit shown in the Fig. 1. The purpose of the rheostat in the filament circuit is to prevent a large initial current, and it is cut out as the filament gets hot.

![Circuit Diagram](image)

**Fig. 1**
The anode is earthed using the two-way switch, a trial 'run' is made by increasing the grid potential $V_{g1}$ from 0 to 12 V, and observing the fine line of the light spot through the telemicroscope. At first there is no deflection until $V_{g1}$ is about 5 V. Then a small deflection is observed which increases slowly until it reaches a maximum at about 11 V; further increase of $V_{g1}$ causes a rapid reduction in the deflection and its ultimate reversal. On investigation you should find that the initial anode current is negative (i.e. due to the flow of electrons towards the anode through the valve) and changing to positive as $V_{g1}$ is increased.

Now connect the anode to the 8 V battery ($V_a = -8$ V) and repeat the whole experiment. You will again find that no deflection is detectable until $V_{g1}$ is about 5 V, but the current this time is positive and it shoots up rapidly as $V_{g1}$ reaches 11 V.

These trial 'runs' will help you to decide the best way to arrange the 'zero' on the micrometer scale (i.e. the position of the fine line when no current flows), so that you can carry out the experiment recording the deflection and the corresponding first grid voltage $V_{g1}$. Plot the deflection ($I_a$) against $V_{g1}$ and thus check any doubtful observations (Fig. 2). In this experiment we are not concerned with the actual values of the current, hence the deflections in micrometer divisions are sufficient, provided that neither the telemicroscope nor the Scalamp is moved during either of the two sets of observations.

**Theory and Calculation**

Let us consider what happens in the valve when $V_a = 0$, $V_{g1} = -4$ V and $V_{g1}$ is increased from zero. Electrons are accelerated by the positive
potential of the first grid \( (V_{g1}) \), but these electrons will soon lose their energy by collision and are further prevented from reaching the anode by the negative potential of \( g_2 \). Therefore, initially \( I_a \) is zero, but when \( V_{g1} \) reaches an excitation potential, the accelerated electrons possess sufficient energy to raise electrons from the ground state to an excited state, on collision with atoms of mercury. As a result these excited atoms emit characteristic radiation when the excited electrons fall back to the ground state, causing the emission of photoelectrons from both \( g_2 \) and the anode. Those from the anode are repelled back by the negative potential of \( g_2 \), but those from \( g_2 \) are attracted to the anode and a negative current \( I_a \) ensues. As \( V_{g1} \) is further increased the accelerated electrons venture farther between \( g_2 \) and anode, the number of excited mercury atoms increases and so does \( I_a \). Between 6 V and 7 V there is a change in the slope of the graph as another excitation potential is reached; \( I_a \) will increase further until the mercury atoms become ionized when positive ions will reach the anode reversing the direction of the anode current.

Now consider repeating the experiment with \( V_a = -8 \text{ V}, \ V_{g1} = -4 \text{ V} \). When \( V_{g1} = 5 \text{ V} \), the photoelectrons from the anode are accelerated to \( g_2 \), but those from \( g_2 \) are prevented from reaching the anode, so that a positive anode current results. When ionization occurs we have in addition positive ions reaching the anode, hence the rapid increase in \( I_a \).

Either by inspection of the two graphs or by plotting \( \log I_a \) against \( V_{g1} \), deduce from the changes of the slopes the two excitation potentials and the ionization potential of mercury vapour.

Estimate the wavelength \( \lambda \) corresponding to each excitation potential (from the relation \( hc/\lambda = Ve \)) where \( V \) is the excitation potential, \( e \) the electronic charge, \( h \) Planck's constant and \( c \) the velocity of light.

**Comments**

This experiment, first carried out by Davis and Goucher in 1917, gives a revealing picture of the critical potentials of mercury vapour. The fact that reversing the voltage between \( g_2 \) and the anode (i.e. from \(-4 \text{ V} \) to \(+4 \text{ V} \)) reverses the anode current, and that each curve in the graph above is a mirror image of the other, supports the view that the anode current is photoelectric up to ionization. Further, the two wavelengths corresponding to the excitation potentials when looked up in the spectrum of mercury vapour will be found to belong to the 'principal series' (i.e. corresponding to a transition from a ground state to an excited state).

How can you check on the value of the ionization potential from the data of the arc spectrum of mercury vapour?

A simpler method for finding the critical potentials of xenon gas has been published by Messrs Mullard using their thyratron EN91. This valve like the one used above is a tetrode, but xenon filled, and the circuit diagram used is shown in Fig. 3 for estimating the excitation potential. \( R \) in the grid \( g_1 \) is to limit the current in case of accidental ionization and is in the order of a few thousand ohms.

Unlike the experiment above the anode is made only slightly negative with respect to the accelerating grid \( g_1 \) by virtue of the small current flowing in the valve, but \( g_2 \) is kept at the potential of the cathode, so that
electrons reaching the anode would have to contend with a retarding electric field. While in the above experiment with mercury vapour no thermionic electrons actually reach the anode, with the xenon tetrode they are the only current carriers inside the tube to the point of ionization.

Initially when $V_{g1}$ is small thermionic electrons reach the anode without difficulty and a current $I_a$ of the order of a few microamps is registered; a larger current $I_{g1}$ flows to the grid $g_1$. As $V_{g1}$ is increased both currents increase at first, but soon $I_a$ starts to drop as a few energetic electrons with adequate energy to excite the atoms lose all of it by collision and cannot make the 'potential hill' to reach the anode. When $V_{g1}$ is between 8 V and 9 V, the excitation potential of xenon, $I_a$, reaches a minimum. As $V_{g1}$ is further increased both $I_a$ and $I_{g1}$ increase rapidly and an estimate can be made of the excitation potential from the change in slope of each graph.

The circuit for measuring the ionization potential is shown in Fig. 4; the

anode is made negative with respect to the cathode so that only positive ions could reach the anode. A variable resistor is included in the filament circuit to reduce the filament current and thus limit the ionization current. As before $V_{g1}$ is increased, but no anode current would flow until ionization is started so that the ionization potential can be estimated, subject to the usual 'contact potential' error.
EXPERIMENT 97

To compare conductivities of electrolytes using a simple conductivity cell and to use a simple H.F. technique to monitor a titration

APPARATUS

(a) Simple conductivity cell (see Fig. 1(b)) with platinum electrodes, 500 ohm and 100 ohm non-inductive resistance boxes, 100 ohm decade dial resistance box. Normal solution of potassium chloride, saturated solution of sodium chloride, beaker placed in a constant temperature water bath. Crystal ear-piece, thermometer, burette, distilled water. 1000 c/s oscillator of variable voltage output, matching transformer (if necessary).

(b) A single-valve H.F. crystal-controlled oscillator [7 megacycles] (see Fig. 2(b) for detail of circuit), germanium diode. Scalamp. 1 ml pipette, 10 in glass tube closed at one end and about 1 cm internal diameter, two similar glass tubes external diameters just under 1 cm to fit inside the first glass tube, 0.4 N hydrochloric acid and 0.02 N sodium hydroxide solution. Switch, aluminium foil, litmus solution.

METHOD

(a) The circuit shown in Fig. 1(b) is used and the N potassium chloride solution first used as electrolyte, great care being taken to keep the temperature of the electrolyte constant at 18°C. First find an approximate

Fig. 1(a)
value of the resistance of the electrolyte and adjust the ratio arm $R_1/R_2$ so that $R_3$ is less than the maximum resistance of the box. There should be no difficulty in determining $R_3$ to three significant figures. Failure to do so suggests a mis-matching between the oscillator output and the bridge circuit or that the output of the oscillator is too low.

Rinse out the conducting cell with distilled water and then rinse and fill with the saturated solution of sodium chloride and determine the resistance in the same way. Now dilute the salt solution using the burette filled with distilled water and repeat the resistance measurements with different concentrations of sodium chloride solution.

(b) Wrap two strips of aluminium foil round the wide glass tube as shown in Fig. 2(a) and connect the circuit shown. Switch on the H.F. oscillator and adjust to maximum output. The lamp connected across the output of the oscillator should glow. It is most important to keep the Scalamp and accessories as far away as possible from the conductimetric cell, as the cell is not screened and any earthed object brought near to it can affect the output voltage picked up by the lower aluminium strip.

The two narrower glass tubes are now partially filled with a known volume of 0.02 N sodium hydroxide (about 10 ml, in fact) so that the level of the liquid is just visible above the top aluminium foil when each is inserted into the wider tube. A suitable sensitivity is chosen for the Scalamp giving a large deflection with either of the tubes, one of which is now kept as a reference tube and the other used for titration purposes.

Using the micro-pipette add about 0.1 ml of 0.4 N hydrochloric acid to the sodium hydroxide solution in the titration tube, which when inserted into the wider tube should cause a decrease in deflection. Note the new reading. This is repeated adding about 0.1 ml of acid each time, checking with the reference tube the output of the oscillator, which should be adjusted if necessary before another reading is taken. This is continued past the lowest reading (end-point) until the reading is about the same as before, with the solution acidic.

It is useful to add a drop of a suitable acid-base indicator to the sodium
hydroxide at the beginning of this experiment. The colour change should coincide with the minimum deflection in the Scalamp.

**Theory and Calculation**

(a) Calculate the resistance of the electrolyte using the usual relation for a balanced Wheatstone bridge \( R_1/R_2 = R_3/R_4 \), hence deduce the conductance as the reciprocal of \( R_4 \). Using the N potassium chloride as a
standard, deduce the conductivity of saturated sodium chloride solution and compare it with the accepted value given in standard reference books. Deduce the equivalent conductance \( \Lambda \) by multiplying the conductivity (or specific conductance) by the number of millilitres containing the gram-equivalent \( v \) of sodium chloride for each concentration used, and plot \( \Lambda \) against concentration (g NaCl per litre). Comment on the graph (Fig. 3).

(b) Plot the galvanometer reading against the number of ml of 0.4 N hydrochloric acid added to 0.02 N sodium hydroxide and you will note that this gives two straight lines intersecting at the end-point (Fig. 3). As the initial volume of 0.02 N sodium hydroxide is known, compare this titration result with that obtained using an acid-base indicator.

**Comments**

One would expect the equivalent conductance to remain constant for different concentrations, as specific conductance \( \sigma \) depends on the product
of number of ions per millilitre \( n \) and the ionic mobility \( u \). With decreasing concentration \( n \) decreases by the same factor that \( v \) increases, so that \( \sigma v \) or \( A \) should remain constant, unless \( u \) changes with concentration. At high concentrations we have the equivalent of a space charge surrounding each ion, and when the ion moves by the action of the applied electric field, it is affected by the presence of the other ions resulting in an additional but retarding electric field, as if each ion is dragging some 'camp followers' of opposite sign with it. This reduces \( u \) and hence the equivalent conductance. The remarkable increase in conductivity with temperature is dependent partly on the decreased viscosity with increasing temperature (see Experiment 74).

With very high frequency fields, the effect of high concentration on mobility is negligible, as the alteration is too rapid to allow the ions to move any distance before the field changes direction, so that the resulting polarization is negligible and the conductivity is higher than when a lower frequency is used.

There are numerous applications of H.F. technique in conductivity measurements but the one chosen is most suitable when a very simple oscillator is available. It uses the conductance of the solution as an indicator in titrations and gives a high degree of accuracy even with the crude arrangement described. By using the concentrations recommended, the volume of hydrochloric acid added is too small in relation to the volume of sodium hydroxide to alter sodium ion concentration \( n \) appreciably during the titration. Therefore the differences in conductivity from that at the end-point observed are proportional to hydrochloric acid added, hence the two straight lines observed. If the sodium ion concentration changes as well, one should get two curves with a discontinuity at the end-point.

The difference in the slopes of the two lines confirms the known fact that the mobility of the hydrogen ion is greater than the hydroxyl ion (one can assume that the mobilities of the sodium and chlorine ions are comparable), a fact which is explained by imagining the hydrogen ion moving in the solution by bouncing from one water molecule to another.*

---

**EXPERIMENT 98**

**To determine the ionic mobilities in (a) an electrolyte and (b) a metallic conductor**

(a) Ionic mobility in an electrolyte

**Apparatus**

A U-tube about 10 in high, internal diameter (4 mm) as shown in Fig. 1, two platinum electrodes to fit into the U-tube, 200 V D.C. supply with a safety resistor in series, milliammeter (0–15 mA), 0.1 N KCl solution to

* Use the above apparatus to distinguish between tap water and distilled water.
which mercuric chloride is added, 0·1 N KI solution, pipette, high resistance voltmeter (0–200 V), stopwatch, ruler.

Method

Pour the KI solution into the U-tube until it is three-quarters full, then gently pipette the KCl solution into one of the limbs of the U-tube until it is almost full. There will be a diffused disc of mercuric iodide where the two liquids meet (see Fig. 1) which becomes thinner as some of it dissolves in excess KI solution. Connect up the electrical circuit as shown in Fig. 1 with the polarities as indicated. You will note that the disc appears to move slowly towards the anode.

Note the position of the disc every minute, the p.d. across the U-tube, and the temperature of the electrolyte and the total length of the U-tube from electrode to electrode.

Theory and Calculation

The yellow disc marks the position of the advancing front of iodine ions I\(^-\) as they move in the electric field. Deduce the ionic velocity \((c \text{ cm sec}^{-1})\) of the iodine ion I\(^-\) and hence the ionic mobility, \(u = \frac{c}{E}\) where \(E\) is the electric field in volt cm\(^{-1}\) (you may assume that the field is uniform along the whole of the U-tube, hence \(E = \frac{\text{p.d. (volt)}}{\text{length of tube (cm)}}\)).
(b) Ionic mobility in a metallic conductor

Apparatus

A copper strip 4 in long, 1 in wide and $\frac{1}{8}$ in thick, sandwiched between two strips of wood; one long copper wire is soldered to the mid-point of the bottom edge of the copper strip and a similar copper wire is soldered to a small movable screw contact on the upper edge of the copper strip opposite the first wire (see Fig. 2); a stout copper wire is also soldered to each end of the copper strip. A wooden clamp holds the copper strip vertically with its long side horizontal in the gap of a strong permanent magnet, which can be removed and replaced without disturbing the wires. A Scalamp microvoltmeter of high sensitivity ($1 \mu V \text{ mm}^{-1}$), a small telemicroscope with a micrometer eye-piece (giving a linear magnification of about $\times 35$), ammeter (0–10 A), high duty rheostat, plug switch, tapper switch, 6–7 V D.C. supply (Nife batteries if possible).

![Diagram](image)

Fig. 2

Method

Connect up the circuit as shown in Fig. 2. Focus the telemicroscope on the fine dark line of the galvanometer’s light spot, and note the number of
millimetre divisions in the eye-piece which corresponds to one millimetre on the scale of the microvoltmeter.

Pass the largest current possible through the copper strip and close the microvoltmeter circuit by means of the tapper, noting the deflection of the dark line. Adjust the position of the variable contact screwed to the upper edge of the copper strip so that on pressing the tapper there is a negligible deflection of the line which should appear in the middle of the micrometer eye-piece. Slide in the magnet and then press the tapper again; you will note that the line has moved. Record its new position on the micrometer scale. Then reverse the magnet and the line will move this time in the opposite direction. Again record its position, and note the current in the copper strip.

Repeat the above experiment with different currents and measure the thickness of the copper strip.

Measure the magnetic field in the gap of the magnet (see Experiment 53), also the conductivity of the copper strip (see Experiment 48).

**Theory and Calculation**

The action of the magnetic field on the current carriers in a conductor is to deflect them in a direction mutually at right angles to both the magnetic field and current (the motor effect). This deflection sets up a potential difference in a tranverse direction to the flow of current so that electric force on the carriers balances the force produced by the magnetic field.

C.G.S. Let the width of the conductor be \( b \) cm, thickness \( d \) cm, current \( I \), transverse potential difference \( v \) V, magnetic flux density \( B \) gauss, the charge on the current carrier \( q \) coulombs e.m.u., the ionic velocity \( c \) cm sec\(^{-1} \), the number of ions ml\(^{-1} \) \( n \), then

\[
\frac{I}{10} = nqbd \quad \ldots \ldots \quad (1)
\]

and

\[
\frac{10^7 \times v}{b} q = Bqc \quad \ldots \ldots \quad (2)
\]

Eliminating \( c \) from equations (1) and (2) we get

\[
n = \frac{IB}{10^8 qvd} \quad \ldots \ldots \quad (3)
\]

Show that the electrical conductivity \( \sigma \) (ohm\(^{-1} \) cm\(^{-1} \)) is \( nqu \) where \( u \) is the ionic mobility. Hence

\[
u = \frac{\sigma}{nq} = \frac{10^8 qvd}{IB} \text{ cm}^2 \text{ sec}^{-1} \text{ V}^{-1}
\]

M.K.S. Let the width of the conductor be \( b \) metres, thickness \( d \) metres, current \( I \) A, transverse potential difference \( v \) V, magnetic flux density \( B \) weber m\(^{-2} \), charge on the carrier \( q \) coulombs, ionic velocity \( c^* \) m sec\(^{-1} \), number of ions m\(^{-3} \) \( n \), then

\[
I = nqbd \quad \ldots \ldots \quad (1)
\]

and

\[
\frac{vq}{b} = Bqc \quad \ldots \ldots \quad (2)
\]

\( c^* \)
Eliminating $c$ from equations (1) and (2) we get

$$n = \frac{IB}{qd}$$

(3)

Show that the electrical conductivity $\sigma$ (ohm$^{-1}$ m$^{-1}$) is $nqu$ where $u$ is the ionic mobility.

Hence

$$u = \frac{\sigma}{nqu} = \frac{\sigma nd}{IB} \text{ m}^2 \text{ sec}^{-1} \text{ V}^{-1}.$$

Estimate $n$ from equation (3) by assuming $q$ to be equal to an electronic charge. Hence, by assuming that there is one conducting electron for every atom of copper, deduce a value for Avogadro's number $N$; taking the atomic weight of copper to be 63.5 g and its density 8.9 g cm$^{-3}$.

Comments

The first method gives some idea of the order of magnitude of ionic mobility, though these vary for different ions and different temperatures. Colloidal particles are frequently charged, and so behave like ions. This fact is used in the analysis of complex colloidal constituents by a technique called 'electrophoresis'.

Fill the above apparatus with a dilute solution of Aquadag and investigate the effect of applying a D.C. potential of about 20–50 V to the two electrodes.

The second method is in effect a study of the Hall effect and when combined with conductivity measurements enables one to determine ionic mobilities and concentrations; furthermore by noting the direction of the potential difference $v$ one can determine whether the carriers are negatively or positively charged. (Establish the direction of $v$ in Experiment (b) from the polarity of the terminals of the microvoltmeter and show that the carriers in copper are in fact negatively charged.) It was found, however, that some metals seem to have positive carriers. This remained a puzzle which was cleared up when it was realized that a 'hole' vacated by an electron will appear to move backwards as each neighbouring electron advances forward to fill it, leaving another 'hole' to be filled in turn by another electron and so forth all along the conductor.

The Hall effect is more pronounced in semi-conductors such as a germanium to which a controlled but small quantity of impurity is added (see Experiment 65). This becomes clear from equation (3) above which shows that the transverse potential $v$ is inversely proportional to $n$ (number of ions ml$^{-1}$). As $n$ is equal to the number of impurity atoms ml$^{-1}$ which is very small, $v$ is therefore large and can be of the order of millivolts if the same magnet is used as in Experiment (b) above.

Messrs Mullard have supplied pairs of $n$-type and $p$-type germanium crystals which can be studied using the second method above. Before assembly, the conductivity of each marked crystal and its dimensions, measured and recorded, should be determined in the usual way, then the two crystals are assembled as shown in Fig. 3. First, the two crystals are cemented to a perspex base, making sure of good electrical contact be-
tween the adjoining faces by inserting a layer or two of gold foil. Eight electrical contacts are provided by copper wires which have previously been anchored rigidly to the perspex base, each wire being arched beforehand with tweezers to make springy firm contact with the crystal. The wires are numbered and soldered to suitable leads and the crystals protected by a transparent cover.

![Fig. 3]

The Hall effect is studied for each crystal, using the circuit diagram shown in Fig. 3. $R$ is rheostat (about 200 ohm) used as a potentiometer divider, the sliding contact being adjusted until the voltmeter reads zero when $I$ is switched on. The magnet is next brought in, so that the magnetic field is at right angles to the plane of the paper. Both $v$ and $I$ are recorded. Next, $I$ is varied by altering the number of accumulators in the circuit. One should plot $v$ against $I$ and deduce both $n$ and $c$. The experiment is repeated with the second crystal. Let the direction of $I$ be reversed through the crystals. Does one get the same current with the same number of accumulators? This should be investigated in the light of the comments on Experiment 65.

**EXPERIMENT 99**

**To determine $g$ by free fall, using a dekatron counter**

**Apparatus**

Panax transistorized scaler type 102ST, Mullard photo-diode OAP12, simple apparatus shown in the figure consisting of an electromagnet the height of which can be adjusted, 6 in hacksaw blade, 2-2 V pea-bulb (with built-in converging lens), the whole apparatus mounted in wood and fixed
to a base with levelling screws, with sand in a tray to break the fall of the blade. L.T. supply and switch.

**Method**

The scaler has a 1000 c/s supply which can be switched on, the 2.2 V pea-bulb is connected to two terminals on the back of the scaler, the photo-diode is connected to two sockets marked ‘make to stop’ and the two sockets marked ‘make to count’ should be connected together. Press the counting switch and on interrupting the light the counter should begin counting and stop when the diode is illuminated again (if the counting is fitful, interchange the photo-diode leads in the two sockets).

Start the experiment with the electromagnet energized. The blade attached to the electromagnet and the scaler should be read. Release the blade and record the new scaler reading after the blade has obstructed the light in its downward flight. Repeat twice and you should get the same difference in scaler readings \( n \) each time.

The distance \( s_0 \) should be estimated to a fraction of a millimetre by holding a steel ruler rigidly vertical close to the bottom of the suspended blade, reading off its position, then lowering the blade (keeping it vertical all the time) until the light is just interrupted and the scaler begins to count. The new position of the blade is recorded. Lower the blade farther until the scaler just fails to count, reading its position again. From these three readings deduce \( s_0 \) and \( s_1 \); repeat as a check.

Raise or lower the electromagnet a few millimetres and repeat the experiment. This is repeated at least five times.

**Theory and Calculation**

If \( v \) is the velocity of the blade as it reaches the centre of the photo-diode then

\[
v = \sqrt{2gs_0} \quad (1)
\]

and

\[
s_1 = vt + \frac{1}{2}gt^2 \quad (2)
\]

where \( t = \frac{n}{1000} \).
Combining equations (1) and (2)

\[ s_1 = \sqrt{2gs_0} \cdot t + \frac{1}{2}gt^2 \]

which is a quadratic in \( \sqrt{g} \).

Therefore

\[ g + \frac{2\sqrt{2s_0}}{t} \cdot \sqrt{g} - \frac{2s_1}{t^2} = 0 \]

or

\[ \sqrt{g} = \frac{2\sqrt{2s_0} \pm 2\sqrt{2(s_0 + s_1)} - \sqrt{2s_0}}{2t} = \frac{1}{t}( - \sqrt{2s_0} \pm \sqrt{2(s_0 + s_1)} ) \]

as \( \sqrt{g} \) must be positive

\[ \sqrt{g} = 1000 \times \frac{\sqrt{2(s_0 + s_1)} - \sqrt{2s_0}}{n} \]

As \( n \) can only be counted to a whole integer, a mean of several readings is taken with different values of \( s_0 \), thus

\[ \sqrt{g} = 1000 \times \frac{\Sigma(\sqrt{2(s_0 + s_1)} - \sqrt{2s_0})}{\Sigma n} \]

Hence \( g \).

**Comments**

The accuracy of this experiment is limited by the accuracy of the 1 kc oscillator (stated by the manufacturers to be correct within \( \pm 2\% \)). As \( t \) is squared an accuracy better than 5\% cannot be hoped for. The response time of the photo-diode is short enough for 1 kc signals to be counted. A useful application of the above arrangement is to test speeds of between-lens camera shutters. For this the photo-diode has to be connected to the ‘make to count’ sockets, the other two sockets being left disconnected.

**EXPERIMENT 100**

**An elementary study of the statistics of random errors**

**Apparatus**

A simple modification of Galton’s quincunx, consisting of an adjustable inclined plane (12 in \( \times \) 12 in) bearing 16 rows of gramophone needles, each row containing 25 equally spaced (\( \frac{3}{8} \) in apart) steel gramophone needles as shown in Fig. 1. Each pin is held vertically by a pair of pliers and then hammered into position on the stout and flat square strip of wood. 50 steel ball-bearings (\( \frac{3}{8} \) in diameter). The balls are rolled down the inclined plane from an adjustable curved funnel and are collected into 25 channels at the bottom, the balls being removed by lifting a long narrow trap door at the bottom end of the channels. Levelling screws are
fitted so that no bias is given to the rolling balls and the whole board is covered by a transparent sheet of cellulose acetate to protect the needles from being disturbed. Small alnico magnet.
Method

The inclined plane is set at a few degrees to the horizontal and adjusted so that its lower end is perfectly level, and the curved funnel is set in the position marked 0. 60 runs of the 50 balls (making a total of 3000 ball runs) are made and the number of balls rolling into each of the channels recorded on a long sheet of graph paper. This is headed by a horizontal list of the numbered channels $-12, -11, \ldots, 0, +1, \ldots, +12$, and each entry consists of one vertical stroke in the appropriate vertical column, each fifth stroke being drawn diagonally (i.e. $\parallel\parallel$), this making a bunch of five strokes. The next bunch in the same column is entered below the last so that a quick count of the frequency in each column can be made. The experiment is repeated by moving the chute to another position, say $+2$, and similar observations are made.

Now the inclined plane is set at about $60^\circ$, the chute set back at 0 and the whole experiment repeated.

Theory and Calculation

Every time the ball rolling down the inclined plane encounters a needle, it has an equal chance of being deflected to the left or the right (i.e. probability $\frac{1}{2}$), so that the definite channel it rolls into after rolling down past 16 rows of needles becomes uncertain, though the starting point is exact enough.

You should note that as the number of observations increases, the frequency distribution becomes symmetrical (if the board is correctly levelled and the surface perfectly flat), ranging from $-8$ to $+8$ channels. Assuming ideal conditions one can deduce the probable distribution, called the binomial distribution, by expanding

$$\left(\frac{1}{2} + \frac{1}{2}\right)^{16} = \frac{1}{2^{16}} (1 + 16C_1 + 16C_2 + \ldots 16C_r \ldots + 1) = 1.$$  

Each term represents the probability of the ball rolling into a definite channel, for example, the first term of $\frac{1}{2^{16}}$ represents the probability of the ball being deflected consistently to the left or the right thus rolling into the $\pm 8$ channel, the second term of $\frac{1}{2^{16}} \times 16C_1$ or $\frac{16}{2^{16}}$ gives the probability of the ball rolling into the $\pm 7$ channel and so on.

If $n$ is the total number of ball runs then the theoretically expected distribution is given by $\frac{n}{2^{16}} (1 + 16C_1 + 16C_2 + \ldots 16C_r + \ldots 1)$. Compare graphically the two distributions and comment on them. It is possible to test whether there is significant agreement between the two distributions by calculating

$$\chi^2 = \sum \frac{(f_0 - f_e)^2}{f_e},$$

where $f_0$ is the observed frequency and $f_e$ is the calculated frequency for each channel, treating for the purpose of this calculation the three extreme
channels at each end \((\pm 8, \pm 7, \pm 6)\) as one channel, so that the total number of frequencies is reduced to 13; as the two distributions are made to have the same total \(n\), the degrees of freedom \(N\) are reduced to 12. Look up the \(\chi^2\) table (Statistical Methods for Research Workers by R. A. Fisher, page 112) and read off the probability \(P\) for your calculated \(\chi^2\) against \(N = 12\). If \(P > 0.05\) then the result is significant.

Plot the distribution when the balls are rolled from a different starting position and compare it with the one obtained when the chute was set at 0. Can you infer confidently from these distributions that the starting point has been moved?

When the angle of tilt of the board is increased, the speed of the rolling ball increases, and skidding takes place or further multiple deflections occur so that the distribution is no longer a binomial one but approximates to the normal distribution (see Experiment 16). This will clearly be seen as channels up to \(\pm 12\) will become occupied, as if the number of rows is being effectively increased and the distribution becomes close to that of the normal distribution.

This hypothesis can be tested using the \(\chi^2\) tables, but a rough test can be made if a frequency distribution curve is plotted, with the numbered channels as the horizontal axis and the frequency \(f\) as the vertical axis. If the distribution is symmetrical about channel 0, then the mean (where \(f\) is also maximum) coincides with \(x = 0\) and it is possible, if the standard deviation \(\sigma\) is calculated, to compare the ratio \(f/f_{\text{max}}\) when \(x = \sigma, 1.5\sigma, 2\sigma\), etc., with that given in standard books on statistics.

**COMMENTS**

As stated earlier in Experiment 16 physical measurements are subject to random error, so that repeated measurements do not necessarily give the same value, in the same way that consecutive balls do not necessarily roll into the same channel. The purpose of the experiment is to show how the results from repeated observations can be grouped into a distribution, first deduced theoretically by De Moivre and later shown by Gauss to fit the accidental errors involved in physical measurements.

This experiment also illustrates very simply the idea of scattering, as, for example, when particles pass through gold foil; one can easily visualize certain modifications to the apparatus making for closer analogy, explaining, for example, large-angle scattering.

Finally, this experiment also illustrates very simply the idea commonly used in wave mechanics where a sub-atomic particle, say an electron, is imagined to occupy not a definite point but a region defined by a probability function. The ball-bearing before rolling occupied a definite point and this is more or less the way things are defined in the macroscopic world, but on being rolled down its subsequent position becomes uncertain and its final position can only be predicted by a probability curve which in essence is equivalent to the distribution curve you have already plotted.

Not all experimental results are exact quantities. Some observations are subject to random fluctuations on which the experimenter has no control. We have seen an example of that in radioactivity, where the measure of
activity is registered by a scaler connected to a Geiger tube. Here the results—say, the number of counts per minute—are in fact derived from a sample depending on the duration of the count and therefore subject to sampling errors. The smaller the sampling error the greater is our confidence in our result.

We have already learnt from Experiment 16 that the sampling error depends on the size and standard deviation of the sample. We shall now try to investigate the effect of the size of the sample on the sampling error.

Connect a Geiger tube to a scaler and with the help of a stopwatch record the background count every 20 seconds for a full hour. From the knowledge of the total count $X$ and the total time calculate the number of counts/20 sec, $\bar{X}$ (equals $X/180$). Your total observation thus represents 180 samples each of 20 second duration and it is possible to calculate the standard error (or deviation) $\sigma_1$ between samples or better still the variance $\sigma_1^2$ from the relation

$$\sigma_1^2 = \frac{\sum(X_1 - \bar{X})^2}{n}$$

where $X_1$ is the number of counts for any 20 sec period deduced from the running total of counts recorded and $n$ the number of samples, namely 180.

Now add two counts in consecutive intervals and divide by two to give the average count per 20 sec worked out over the double period (namely 40 sec). You will have only 90 samples but you can calculate the new variance $\sigma_2^2 = \frac{\sum(X_2 - \bar{X})^2}{n}$ where $X_2$ is the average of each sample and $n$ is 90. This is repeated grouping 3, 4, 5 and 6 counts together, and calculating the corresponding variances $\sigma_3^2$, $\sigma_4^2$, $\sigma_5^2$ and $\sigma_6^2$ respectively. You will note that the variance decreases with increasing size of sample; plot $\sigma^2$ against $1/\nu$ where $\nu$ is the size of the sample in arbitrary units, being 1 for 20 sec sample, 2 for 40 sec sample, 3 for 60 sec sample, etc. You should get a straight line passing through the origin proving that $\sigma^2$ is proportional to $1/\nu$ or $\sigma_{\text{error}} \propto (1/\sqrt{\nu})$, namely that the standard error is inversely proportional to the square root of the sample size.

Show theoretically that $\sigma_{\text{error}} = \sigma/\sqrt{\nu}$ where $\sigma$ is the standard deviation of the population, a result you have already used in Experiment 16.
APPENDIX 1

Simple statistical treatment of random errors

If \( x_1, x_2, x_3, \ldots \), are successive observations subject to random error, then the arithmetic mean \( \bar{x} \) is defined as

\[
\bar{x} = \frac{\sum x}{N}
\]

(1)

where \( N \) is the total number of observations.

Equation (1) can be written as

\[
\Sigma x - N\bar{x} = 0
\]

or

\[
\Sigma (x - \bar{x}) = 0
\]

(2)

Equation (2) signifies that the algebraic sum of the deviations from the arithmetic mean is zero.

Now if \( x_0 \) is chosen so that the sum of the squares of the deviations of each observation from \( x_0 \) is minimum (i.e. least squares) then \( \Sigma (x - x_0)^2 \) is minimum. This is the case when \( \frac{\partial}{\partial x_0} \Sigma (x - x_0)^2 \) is zero, or \(-2\Sigma (x - x_0) = 0\), thus

\( x_0 = \bar{x} \).

It follows therefore that \( \bar{x} \) has the additional property that it makes the sum of the squares of the deviations least.

Let \((x_1 : y_1), (x_2 : y_2), (x_3 : y_3), \ldots\), be pairs of \( N \) observations to be fitted to a straight line \( y = mx + c \) where \( m \) and \( c \) are the gradient of the line and the intercept of the \( y \)-axis respectively.

From the graph (Fig. 1) the deviation of point 1 from the fitted line is \( y - mx_1 - c \), and point 2, \( y_2 - mx_2 - c \), etc.

If the line is to be chosen so that the algebraic sum of the deviations is zero then

\[
\Sigma (y - mx - c) = 0
\]

or

\[
\frac{\Sigma y}{N} = m\frac{\Sigma x}{N} + c
\]

Therefore

\[
\frac{\Sigma y}{N} = m\frac{\Sigma x}{N} + c
\]

or

\[
\bar{y} = m\bar{x} + c
\]

(3)

Thus the line of best fit must pass through a point the co-ordinates of which \( \bar{y} \) and \( \bar{x} \) are the mean ordinates and abscissa respectively (see Experiment 2).

Equation (3) is by itself not sufficient to determine \( m \) and \( c \) and we need another equation. This is provided by the additional condition that the sum of the squares of the deviation be minimum (least squares), namely \( \Sigma (y - mx - c)^2 \) is minimum when

\[
\frac{\partial}{\partial c} \Sigma (y - mx - c)^2 = 0 \quad \text{and} \quad \frac{\partial}{\partial m} \Sigma (y - mx - c)^2 = 0
\]

Thus

\[
\Sigma (y - mx - c) = 0
\]

(4)

\[
\Sigma x(y - mx - c) = 0
\]

(5)

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APPENDIX 1

Equation (4), as we have seen, leads to equation (3) which we have already discussed.

Equation (5) gives

\[ \Sigma xy = m \Sigma x^2 + \Sigma xc \]

or

\[ \frac{\Sigma xy}{N} = \frac{m}{N} \Sigma x^2 + cx \]  \hspace{1cm} \ldots \ldots \ldots (6)

\[ \text{DEVIATION} = y - mx - c \]

\[ y = a + bx + cx^2 + \ldots \]

by solving the number of simultaneous equations

\[ \Sigma y = Na + b \Sigma x + c \Sigma x^2 + \ldots \]

\[ \Sigma xy = a \Sigma x + b \Sigma x^2 + c \Sigma x^3 + \ldots \]

\[ \Sigma x^2y = a \Sigma x^2 + b \Sigma x^3 + c \Sigma x^4 + \ldots \]

\[ \ldots = \ldots + \ldots + \ldots + \ldots \]

\[ \ldots = \ldots + \ldots + \ldots + \ldots \]

\[ \ldots = \ldots + \ldots + \ldots + \ldots \]

Hence \( m \) and \( c \) are deduced by solving (3) and (6) as simultaneous equations. This method can be extended to fit any curve which can be expressed as a polynomial in \( x \):
APPENDIX 2

List of suppliers of specific apparatus. Further construction hints.

Experiment 2
The long glass tube which is to contain the ether vapour is curved at the bottom and sealed at the top. It is then filled completely with mercury and inverted over a shallow tray and set up like a barometer. A small quantity of ether is injected by means of a curved pipette, then the curved tube is joined by a rubber tubing to the other open vertical glass tube which is now slowly filled with mercury, carefully releasing any trapped air.

Experiment 5
The timer supplied by Philip Harris Ltd (Ludgate Hill, Birmingham 3) (P 10040/6) was modified to have further guides and a roller made of rubber, which made it possible to use it horizontally. The weight used was 500 g.

Experiment 11
The motor used was war-surplus but Philip Harris Ltd provide an excellent general motor (P 7138).

Experiment 12
It was found helpful to cover all wires with polythene sleeving to reduce damage to the wires as the junctions are transferred. To prevent entangling the wires, all thermocouples were bound together close to the cold junctions.

Experiment 17
It was found that a pair of P.S.S.C. trolleys with strong spring plunger demonstrated the principle well. The subsequent velocity was proportional to the square root of the distance covered by each loaded trolley.

Experiment 23
The rotameter (Metric 14A) was supplied by the Rotameter Manufacturing Co. Ltd (Purley Way, Croydon, Surrey). Only the tube and the float were bought.

Experiment 29
The semi-silvered mirror was supplied by New Clear Ltd (392 Commercial Road, Portsmouth).

Experiment 32
The brass calorimeter is cylindrical, 1 in diameter and 1½ in high, a hole ¾ in diameter is drilled into it to a depth of 1 in. The brass plug is turned on a lathe to fit the hole closely allowing for the thickness of the heating coil. A hole is drilled for the hot junction of the thermocouple.
Experiment 36
The precision tube was supplied by Chance Brothers Ltd (Glass Works, Smethwick 40, Birmingham). The ball-bearing was supplied by Hoffmann Manufacturing Co. Ltd (Chelmsford, Essex).

Experiment 45
The MetroSil disc was supplied by A.E.I. (Rugby).

Experiment 49
The thermocouple used was supplied by A.E.I. (Rugby), but Griffin and George Ltd (Alperton, Middlesex) (L 88–150) can also be adapted for the purpose.

Experiment 52
The apparatus supplied by Philip Harris Ltd (P 7708) is similar to the one used.

Experiment 54
The motor-generator was supplied by Newton Bros Ltd (Alfreton Road, Derby).

Experiment 55
The coil used and the two cores were supplied by Philip Harris Ltd. The coil is (P 7804), the laminated iron core (P 7806), the cast iron core (P 7808).

Experiment 58
The oscillator used was type H1 and the vibrator V1, both supplied by Advance Components (Roebuck Road, Hainault, Ilford, Essex).

Experiment 62
The telemicroscope used was supplied by Griffin and George Ltd.

Experiment 66
On going to press it was learnt that Mullard DG 4–2 was discontinued and is replaced by DG 7–6. Unfortunately in the new tube the deflecting plates are not visible from the outside, but the Educational Services of Mullard supply on request the internal structure of the tube from which all measurements can be made.

Experiment 68
The spark detector was supplied by Griffin and George Ltd (L 91–025).

Experiment 82
The same rotameter is used as in Experiment 23.

Experiment 84
Speedivac 2 S 20 was used, supplied by Edwards High Vacuum Ltd (Manor Royal, Crawley, Sussex).


Experiment 89
The mutual inductance was supplied by Philip Harris Ltd (P 7700).

Experiment 93
The microwave transmitter and receiver were supplied by Unilab Division of Rainbow Radio (Blackburn), Blackburn.

Experiment 99
The dekatron was SA/102/S supplied by Panax Equipment (Holmethorpe Industrial Estate, Redhill, Surrey).